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Petr FERFECKI[*]**, Zdeněk PORUBA**[**]

ALGORITHM FOR INVESTIGATION OF IMPACTS OF ROTORS SYSTEMS SUPPORTED BY RADIAL ACTIVE MAGNETIC BEARINGS

ALGORITMUS PRO ZKOUMÁNÍ RÁZŮ V ROTOROVÝCH SOUSTAVÁCH ULOŽENÝCH V RADIÁLNÍCH AKTIVNÍCH MAGNETICKÝCH LOŽISKÁCH

Abstract

The active magnetic bearings (AMBs) are consist of quite complicated and expensive electromechanical parts and due to this fact they are protected by the auxiliary bearings. If the rotor system design is made so that the clearance between the disc and the stationary part is smaller than between the auxiliary bearing and the shaft journal the contact between those two parts can occur due to the overloading of the rotor system. The influence of radial AMBs and of the impacts in rotor system is usually investigated separately. Eventually, the shaft is modelled as rigid or elastic but massless body. Due to this it is necessary to develop a procedure which makes possible to analyze their mutual behaviour on the rotor system discretized by the finite element method. The proposed procedure is verified on the test rotor system and the influence of Rayleigh coefficient of external damping and of vibration character during the impacts occurring between the disc and the stationary part is investigated by means of numerical simulations.

Abstrakt

Radiální aktivní magnetická ložiska se skládají ze složitých a drahých elektromechanických částí a proto jsou chráněna pomocnými ložisky před mechanickým poškozením. Jestliže je konstrukční návrh rotorové soustavy proveden tak, že mezera mezi diskem a stacionární části je menší než mezi pomocným ložiskem a stacionární části, může během přetížení rotorové soustavy dojít ke kontaktu mezi těmito částmi. Obvykle je vliv radiálních aktivních magnetických ložisek a rázů v rotorových soustavách zkoumán odděleně. Případně je hřídel modelován jako tuhé nebo pružné, ale nehmotné těleso. Proto je důležité vyvinout proceduru, která umožňuje analyzovat jejich vzájemné působení na rotor diskretizovaný metodou konečných prvků. Navrhnutá procedura je ověřena na testovací rotorové soustavě a numerickými simulacemi je zkoumán vliv Rayleigho koeficientu vnějšího tlumení na charakter kmitání během rázů nastávající mezi diskem a stacionární části.

1 INTRODUCTION

The radial AMBs present a new technology which has many advantages compared to conventional bearing designs (rolling element bearings and fluid film bearings). The most important advantages are: non-contact operation, very low power consumption and very long life because there is no mechanical friction or wear, possibility of high speed revolutions and quiet operation, etc. During the excessive lateral rotor vibration (during start up and shut down of rotor, undesirable operating conditions, etc.) the contact between the rotor and the inner ring of an auxiliary bearing or between the disc and the stationary part can occur in the rotor system supported by AMBs. The contact of the rotating part with the stationary part causes several physical phenomena, such as friction in the contact area, deformation of both colliding bodies, modification of the rotor system stiffness, impacts and converts the mechanical energy into heat. Friction at the contacting areas produces tangential force in the di-

[*] Ing., Ph.D., Structural Integrity & Materials Design, Center of Advanced Innovation Technologies, VSB-TUO, 17. listopadu 15, Ostrava-Poruba, tel. (+420) 59 732 5752, e-mail petr.ferfecki@vsb.cz

[**] Ing., Ph.D., Department of Mechanics, Faculty of Mechanical Engineering, VSB-TUO, 17. listopadu 15, Ostrava-Poruba, tel. (+420) 59 732 3228, e-mail zdenek.poruba@vsb.cz

rection opposite to the angular speed of a shaft and its magnitude depends on normal force and surface properties. Therefore for investigation impact problems in the area of a rotor system supported in radial AMBs effective tool such as computer modelling method is applied. In this study, a computationally efficient impact model between the disc and the stationary part is proposed.

The development of a two efficient approaches for the dynamics of a rotor system with impacts occurs between discs and the stationary part describes paper [5]. Both approaches are based on the assumption that the discs and the stationary part are considered as absolutely rigid bodies. The first approach is based on the application of Newton's laws. In the second approach, direct computation of the local flexibility and damping of the colliding bodies are considered. Applicability of the designed procedures has been verified by means of computer simulations. The aim was to analyze the steady-state response on the unbalance force excitation and kinematic excitation with impacts between the disc and the stationary part of the rotor system supported in two hydrodynamic bearings. Compared to the first developed approach the second one provides information about magnitude and time history of the impact force.

In the article [4] is investigated the dynamic response of a rotor landed on auxiliary bearings in the radial AMBs supported rotor. The simple motion equation of a disc is built up in stationary coordinate system and has two degrees of freedom. The equation motion of rotor system includes a switch function to indicate contact/non-contact events. The friction forces between the auxiliary bearing and the shaft journal is of a Coulomb type. The shaft and auxiliary bearing is assumed as massless. The shooting method is used to determine the effects of the various parameters of the rotor system. On the base of performed numerical simulations the preliminary recommendations for auxiliary bearings design are stated.

Impacts occurring between the shifted auxiliary bearing centre and the rotor are described in the contribution [1]. The motion equation assumes the rigid rotor supported by the AMB. The auxiliary bearing is rigid and with circular cross section. The contact force is determined from Coulomb law and is implemented into the motion equation by nonlinear force coupling. Only sliding contact type is considered in this study even different kinds of impact are possible. The parameters studies are carried out for the values of the friction and the impact restitution coefficient. Numerical simulations show the different types (periodic, quasi-periodic and chaotic) of rotor vibration possible under impacts.

In order to get a better understanding of the rotor system supported by means of radial AMBs under undesirable operating conditions an impacts between the stationary part and the disc are investigated. Compared to the common approach when the shaft is assumed as massless, in some cases also rigid, the shaft is discretized into the finite elements in this work. The discs and the stationary part are considered to be absolutely rigid. To determine the impact forces Hertz theory is applied. The radial AMBs and the impacts between the disc and the stationary part are incorporated into the motion equation by means of nonlinear force coupling vectors. The aim contribution is to present a computational procedure for analysis of impacts in a rotors supported by radial AMBs. By the mentioned procedures it is possible to calculate the response on centrifugal forces excitation caused by unbalances of the rotor system discs if the impacts between the shaft journal and the auxiliary bearing or between the disc and the stationary part occur during transient or steady-state response. The appropriate direct integration method has been employed to obtain steady-state response of the rotor system on the excitation by unbalance force. Finally, simulation results for proposed computational procedure are presented. The influence of Rayleigh coefficient of external damping on characteristics the different types of vibration is discussed.

2 MOTION EQUATION OF ROTOR SYSTEM WITH IMPACTS

The following properties are assigned to investigated rotor system: (i) the shaft is flexible, linear elastic and in the computation model the shaft is represented by a flexible beam-like body that is discretized into finite elements, (ii) the shaft element is modelled on the basis of Bernoulli-Euler beam theory, (iii) the stationary (i.e. non-rotating) part is considered to be absolutely rigid and fixed,

(iv) the rotor is coupled with the stationary part through the radial AMBs, (v) the discs are considered to be absolutely rigid axisymmetric bodies, (vi) inertia and gyroscopic effects of the rotating parts are taken into account, (vii) material damping of the shaft and other kinds of damping are regarded as linear, (viii) the rotor is loaded by its self-weight and forces with periodic time histories and (ix) the rotor rotates at constant angular speed.

The motion equation of lateral vibration of the rotor system supported by the radial AMBs [2] have in stationary co-ordinate system the following form

$$\mathbf{M}\,\ddot{\mathbf{q}}(t)+\left(\mathbf{B}+\eta_{\mathrm{V}}\mathbf{K}_{\mathrm{SH}}+\omega\mathbf{G}\right)\dot{\mathbf{q}}(t)+\left(\mathbf{K}+\omega\mathbf{K}_{\mathrm{C}}\right)\mathbf{q}(t)=\mathbf{f}_{\mathrm{M}}(\mathbf{q},\mathbf{i})+\mathbf{f}_{\mathrm{I}}(\mathbf{q},\dot{\mathbf{q}})+\mathbf{f}_{\mathrm{A}}(t)+\mathbf{f}_{\mathrm{V}}\,, \tag{1}$$

where $\mathbf{M}$, $\mathbf{B}$, $\mathbf{G}$, $\mathbf{K}_{\mathrm{C}}$, $\mathbf{K}_{\mathrm{SH}}$, $\mathbf{K}$ are the mass matrix, the damping matrix (external damping and damping of material), the gyroscopic effects matrix, the circulation matrix, the stiffness matrix of the shaft and the stiffness matrix of the rotor system respectively, $\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}$ are vectors of generalized nodal displacements, velocities and accelerations respectively, $\mathbf{f}_{\mathrm{M}}$, $\mathbf{f}_{\mathrm{I}}$, $\mathbf{f}_{\mathrm{A}}$, $\mathbf{f}_{\mathrm{V}}$, $\mathbf{i}$ are vectors of electromagnetic forces, impact forces, generalized forces exerting on the rotor system (external and constraint forces) and control currents passing by core of electromagnets respectively, while ω is the angular speed of a shaft, η_{V} is the coefficient of viscous damping of a material and t is the time.

Two kinds of the damping are incorporated into the motion equation (1), the external damping (corresponding to the multiple of the Rayleigh coefficient of the external damping and to the mass matrix) and the damping of material (corresponding to the multiple of the coefficient of viscous damping of a material and the stiffness matrix of the shaft) respectively.

The components of the electromagnetic force acting on the shaft journal in horizontal $f_{\mathrm{M,y}}$ and vertical $f_{\mathrm{M,z}}$ direction of vibration are according to [2] given by following relations

$$\begin{aligned} f_{\mathrm{M,y}}(i_{\mathrm{y}}, y) &= \frac{1}{4}\cos(\alpha_0)\mu_0 N^2 S_{\mathrm{M}}\left[\left(\frac{I_0-i_{\mathrm{y}}}{c_0-y}\right)^2-\left(\frac{I_0+i_{\mathrm{y}}}{c_0+y}\right)^2\right], \\ f_{\mathrm{M,z}}(i_{\mathrm{z}}, z) &= \frac{1}{4}\cos(\alpha_0)\mu_0 N^2 S_{\mathrm{M}}\left[\left(\frac{I_0-i_{\mathrm{z}}}{c_0-z}\right)^2-\left(\frac{I_0+i_{\mathrm{z}}}{c_0+z}\right)^2\right], \end{aligned} \tag{2}$$

where y, z are the generalized components of displacements in horizontal and vertical direction of vibration, c_0 is the size of air clearance between the rotor and stator, i_{y}, i_{z} are the controlling currents (in horizontal and vertical directions), I_0 is the bias current, N is the number of winding of the electromagnet coil, S_{M} is the area of the pole of electromagnet in the air part of magnetic circuit, μ_0 is the permeability of vacuum, α_0 is angle between the electromagnetic force and the vertical direction.

The control equation of current PD controller for rotor system placed in two identical AMBs is

$$\mathbf{i} = \mathbf{D}[k_{\mathrm{p}}]\,\mathbf{q} + \mathbf{D}[k_{\mathrm{d}}]\,\dot{\mathbf{q}}\,, \tag{3}$$

where $\mathbf{D}[k_{\mathrm{p}}]$ and $\mathbf{D}[k_{\mathrm{d}}]$ are the diagonal matrixes with elements of proportional and derivation constants of the controller.

3 DETERMINATION OF COMPONENTS OF IMPACT FORCES

The impacts are implemented into the mathematical model by means of nonlinear force coupling, which is composed of its normal and tangential component (Fig. 1). The contact between the disc and the stationary part is modelled as a direct computation of the local flexibility and damping of the colliding bodies, occurring between two rigid bodies except a small neighbourhood of the contact point. Consequently it is assumed that friction force acting on the disc in the circumferential direction, friction is Coulomb type and the impacts do not influence the angular speed of the shaft rotation.

The radial component impact force is a function of the impact penetration and the penetration velocity. The radial component of the impact force must be compressive and radial displacement of the shaft centre in the place of disc is greater than the radial clearance between the disc and the stationary part during the contact. The magnitude of the radial impact force f_{Ir} can be written as follows

$$
\begin{aligned}
& f_{\mathrm{Ir}} = -f_{\mathrm{In}}(\delta) - \beta_{\mathrm{I}} \frac{\mathrm{d}\, f_{\mathrm{In}}}{\mathrm{d}\,\delta}\dot{\delta} && \text{for} \quad e_{\mathrm{I}} > c_{\mathrm{I}} && \text{and} \quad -f_{\mathrm{In}}(\delta) - \beta_{\mathrm{I}} \frac{\mathrm{d}\, f_{\mathrm{In}}}{\mathrm{d}\,\delta}\dot{\delta} < 0\,, \\
& f_{\mathrm{Ir}} = 0 && \text{for} \quad e_{\mathrm{I}} \le c_{\mathrm{I}} && \text{and} \quad -f_{\mathrm{In}}(\delta) - \beta_{\mathrm{I}} \frac{\mathrm{d}\, f_{\mathrm{In}}}{\mathrm{d}\,\delta}\dot{\delta} \ge 0\,, \\
& \delta = e_{\mathrm{I}} - c_{\mathrm{I}}\,, \; c_{\mathrm{I}} = r_{\mathrm{S}} - r_{\mathrm{D}}\,,
\end{aligned}
\tag{4}
$$

where f_{In} is the normal component of the impact force, δ is the penetration, $\dot{\delta}$ is the penetration velocity of the disc into the stationary part, e_{I} is the radial displacement of the shaft centre in the place of disc, c_{I} is the radial clearance between the disc and the stationary part, r_{D} is the radius of the disc, r_{S} is the radius of cylindrical hole in the stationary part and β_{I} is the coefficient of viscous contact damping. The normal impact force depends on the penetration, the material properties and the geometry of the colliding bodies. In the cases when a Hertz theory [3] can not be applied the normal component of the impact force can be calculated by the help of finite element method.

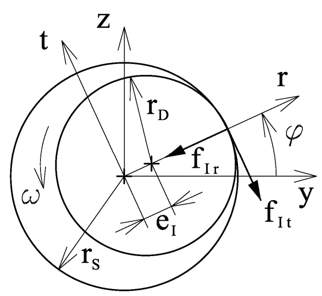

Fig. 1 Scheme of the impact between the disc and the stationary part

The components of the impact force acting in the contact point can be expressed by radial f_{Ir} and tangential f_{It} components in the rotor-fixed coordinate system. The magnitude of the impact force in the tangential direction f_{It} can be calculated as follows

$$
f_{\mathrm{It}} = \mu\, f_{\mathrm{I}r}\,, \tag{5}
$$

where μ is the coefficient of friction between the disc and the stationary part. The friction model is defined employing Coulomb friction with a constant friction coefficient.

The impact force included in the rotor system motion equation (1) is expressed in the stationary coordinate system. Its horizontal f_{Iy} and vertical f_{Iz} components can be written as

$$
\begin{aligned}
f_{\mathrm{Iy}} &= -f_{\mathrm{Ir}}\cos(\varphi) + f_{\mathrm{It}}\sin(\varphi)\,, \\
f_{\mathrm{Iz}} &= -f_{\mathrm{Ir}}\sin(\varphi) - f_{\mathrm{It}}\cos(\varphi)\,,
\end{aligned}
\tag{6}
$$

where φ is the position angle of disc centre and of the stationary part centre.

4 NUMERICAL SIMULATION RESULTS

Presented computational procedure was built up in the computer system MATLAB and tested by means of computer simulations on the test rotor system. The rotor system (Fig. 2) is consist of the shaft (SH) driven by electromotor (EL) over the clutch (CL) and two discs (D1, D2) fixed on cantilever. Disc D2 is situated in the cylindrical hole of the stationary part (ST). The outer surface of the disc D2 is shaped by spherical surface and due to large radius of the cylindrical hole its surface is considered as planar. The radial clearance between the disc D2 and the stationary part ST is small and both parts are made of steel. The connection of the rotor with bed plate (BP) is realized by two radial AMBs (MB1 and MB2).

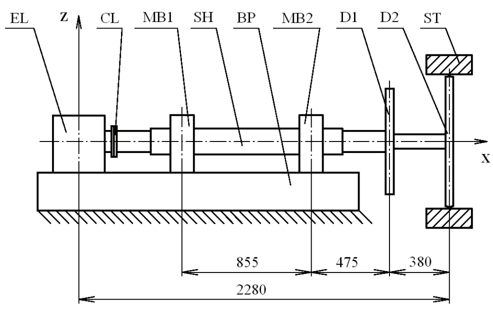

Fig. 2 Scheme of the rotor system

The task was to compute response of a rotor system on excitation by centrifugal forces of discs unbalances and analyze lateral vibration of the rotor during impacts between the disc D2 and the stationary part ST. The rotor is running by constant angular speed 314.159rad·s^{-1} and is loaded by self-weight and by centrifugal forces caused by unbalance of the discs (D1 and D2). The shaft in the computational model is represented by a beam-like body that is discretized into twenty four finite elements of equal length (95mm), more detailed data are given in [2].

Hertz theory [3] is used to determine the normal impact force between the disc and the stationary part assuming that the impacts occur between the sphere-plane surfaces. The coefficient of viscous contact damping, the radial clearance and the coefficient of friction between the disc D2 and the stationary part ST was taken to be $1 \cdot 10^{-6}$s, 1.0260mm and 0.2 respectively.

In the presented parameter study the Rayleigh coefficient of external damping, α, was increased from 2s^{-1} to 10s^{-1} with step value 1s^{-1}. The zero vector of initial conditions was used in all simulations. The simulation was stopped after 1000 periods of the shaft rotation when the steady-state orbit is achieved. The bifurcation diagram (Fig. 3) is drawn from the points corresponding to the multipliers of basic revolution period and for one value of Rayleigh coefficient of external damping is built up from one numerical simulation. In the range of Rayleigh coefficient of external damping ($2s^{-1} \leq \alpha \leq 4s^{-1}$) it is based on the bifurcation diagram, steady-state orbits of the shaft centre (Fig. 4 and Fig. 5) and Fourier spectra (Fig. 7) obvious that the rotor system response has chaotic character. By increasing of Rayleigh coefficient of external damping changes the vibration character into transient or intermittent chaotic ($4s^{-1} < \alpha < 8s^{-1}$) and further into quasi-periodic vibration ($8s^{-1} \leq \alpha \leq 10s^{-1}$).

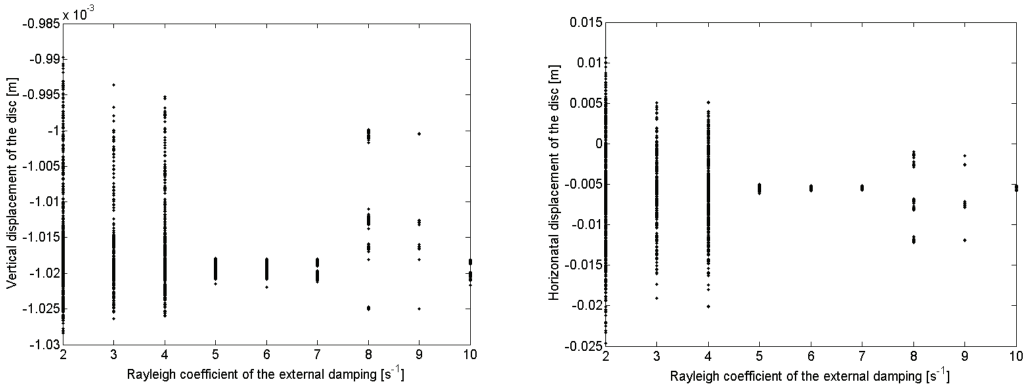

Fig. 3 Bifurcation diagram based on vertical (left) and horizontal (right) displacement of the centre of the disc D2

The shape of steady-state orbit of the shaft centre in the location of the disc D2 and MB2 for the magnitude of Rayleigh coefficient of external damping of α=2s^{-1} and α=8s^{-1} is drawn in Fig. 4 and Fig. 5. It is obvious that for a low damping value (left Fig. 4 and left Fig. 5) the orbit almost fills the part of bearing clearance, the orbit is an open curve and due to this fact is the character of vibration chaotic. In the contrary, for the value of external damping of α=8s^{-1} (right Fig. 4 and right Fig. 5) the orbit does not fill the whole space between contour lines and seems to be a complicated open curve and hence the vibration character is quasi-periodic.

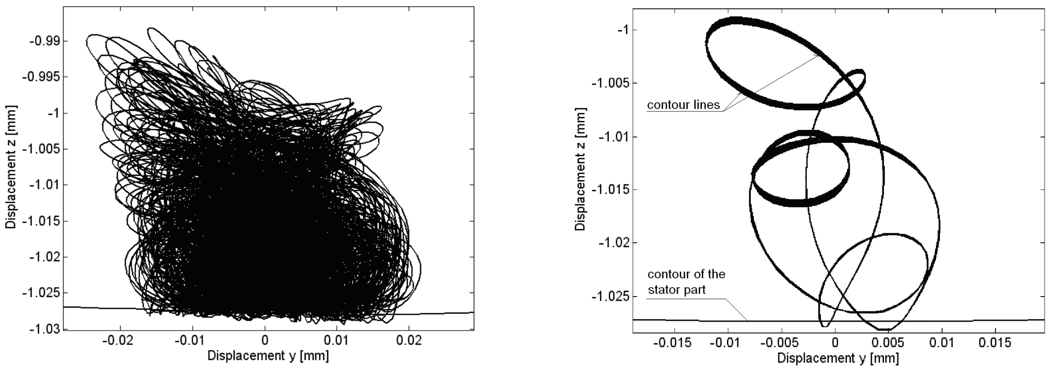

Fig. 4 Steady-state orbit of the centre of the disc D2 for magnitude of Rayleigh coefficient of external damping α=2s^{-1} (left) and α=8s^{-1} (right)

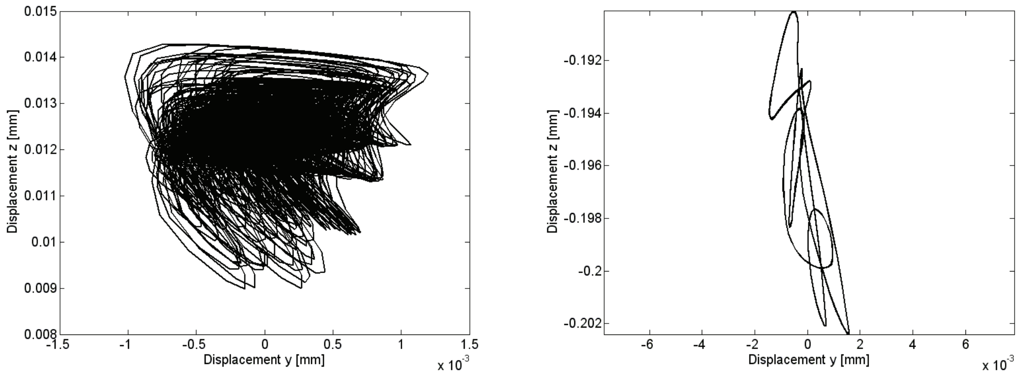

Fig. 5 Steady-state orbit of the centre of the magnetic bearing MB2 for magnitude of Rayleigh coefficient of external damping α=2s^{-1} (left) and α=8s^{-1} (right)

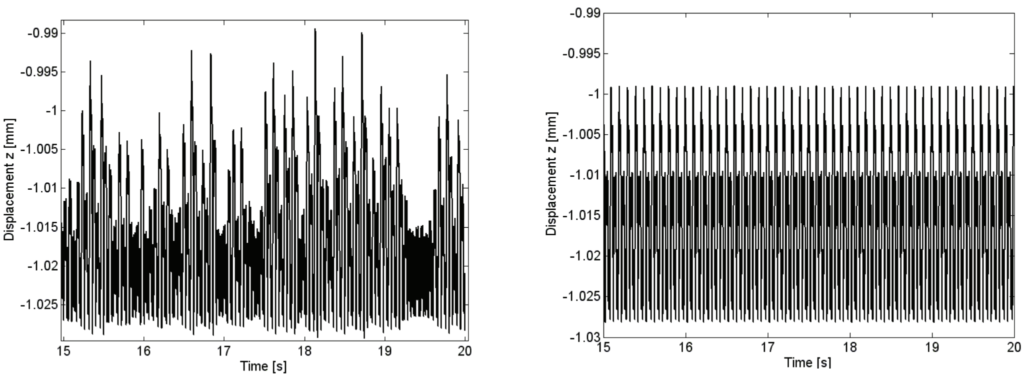

Fig. 6 The time history vertical displacement of the centre of the disc D2 in the steady-state of the rotor system for magnitude of Rayleigh coefficient of external damping α=2s^{-1} (left) and α=8s^{-1} (right)

The steady-state time history of the displacements in vertical direction of the vibration in the location of the disc D2 for two values of external damping is stated in Fig. 6. It can be seen that the time history has for a lower value of Rayleigh coefficient of external damping of α=2s^{-1} (left Fig. 6) completely irregular character and the rotor vibration is chaotic. In the contrary, for the magnitude of

Rayleigh coefficient of external damping α=8s^{-1} (right Fig. 6) is the rotor vibration only "slightly" quasi-periodic.

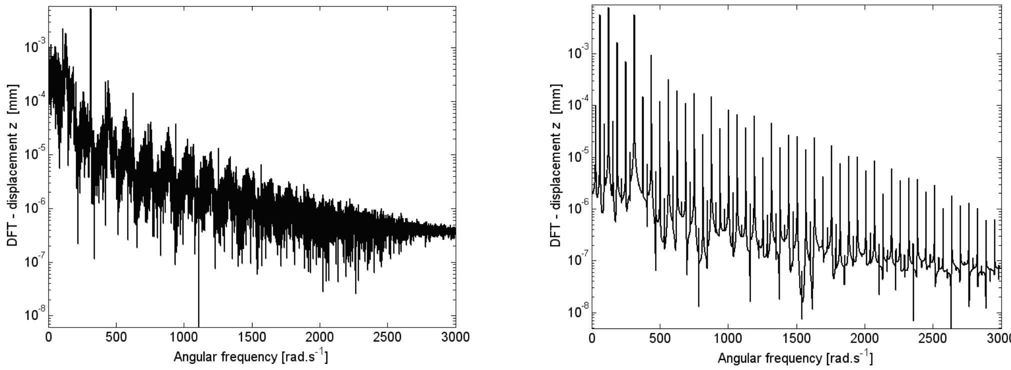

Fig. 7 Fourier spectra of the steady-state response for magnitude of Rayleigh coefficient of external damping α=2s^{-1} (left) and α=8s^{-1} (right)

The Fourier spectra displayed in Fig. 7 are determined from the displacements in the vertical direction of the vibration. For magnitude of Rayleigh coefficient of external damping of α=2s^{-1} has spectrum a broadband character which is typical for a chaotic vibration. This broadband character of the spectrum is disappearing with increasing magnitude of Rayleigh coefficient of external damping. The frequency spectrum for higher values of Rayleigh coefficient of external damping has a typical line character with the small frequency bands at the sides rising in the surrounding of dominant frequencies (typical for quasi-periodic vibration). In case of studied rotor system the dominant frequency in the spectra is equal to the basic multiple of the angular speed 314.159rad·s^{-1} of a rotor (excitation frequency due to unbalance force). Furthermore the steady-state response is predominantly formed by frequencies corresponding to $\omega \pm i{\cdot}\omega/5$ and $i{\cdot}\omega/2$, i=1, 2, 3, …, see Fig. 7.

Fig. 8 show time history probably chaotic vibration of normal component of the impact force during the corresponding period of steady-state response computed for magnitude of Rayleigh coefficient of external damping α=2s^{-1}.

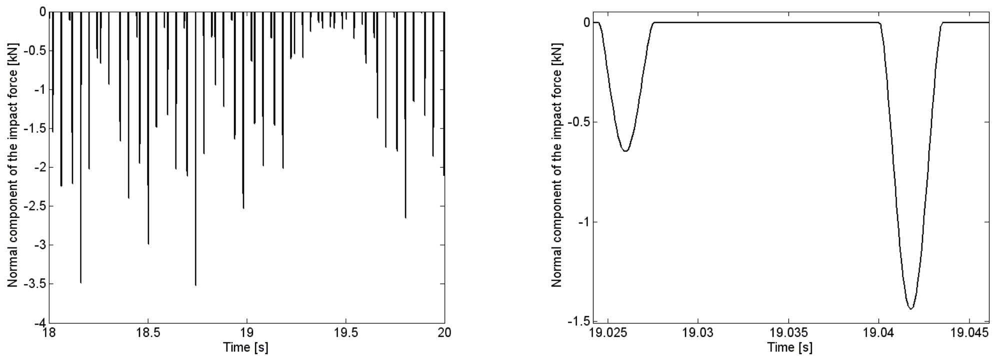

Fig. 8 Normal component of the impact force (left) and the detail its time history (right)

5 CONCLUSIONS

The presented algorithm provides a numerical method for investigation of the response on the unbalance force excitation and impacts between the disc and the stationary part, the shaft journal and the auxiliary bearing, etc. of a rotor system supported by radial AMBs. The rotor was modelled as a flexible body by using the beam finite elements based on Bernoulli-Euler theory. In the mathematical model are the radial AMBs and impacts incorporated by means of nonlinear force coupling. To the direct determination contact stiffness of the radial component of impact forces a Hertz theory is applied. The tangential component of impact forces assumed to consider a Coulomb friction model.

In the most articles the mathematical models of a rotor system supported by radial AMBs are formulated as low degree of freedom systems. The novel contribution of developed approach is in implementation of both nonlinear vibration sources (radial AMB and impacts) into the mathematical model of the rotor system discretized with finite elements. Accordingly, the flexibility of shaft and its viscous material damping is taken into account in the mathematical model of a rotor system.

The applicability of proposed algorithm has been verified on the test rotor system. Using the developed model, the effects of Rayleigh coefficient of external damping on characteristics of the different vibrations of the test rotor system were discussed. A Runge-Kutta formula was applied to solve the response on unbalance force excitation. To investigate of Rayleigh coefficient of external damping effect on the steady-state response with impacts between the disc and the stationary part of a rotor system were used the bifurcation diagrams, frequency spectrums and time histories of kinematic parameters and orbits of the rotor centre. The numerical simulations have shown that the developed approach is the numerically efficient and stable. The computed results of steady-state response on unbalance force excitation with impacts found out that magnitude of Rayleigh coefficient of external damping has considerable influence on characteristic vibration. The rich forms of periodic, quasi-periodic and chaotic vibrations were observed. The results can be useful to diagnose the impacts in this kind of rotor systems.

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