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Martin BLATOŇ*

THE MATHEMATICAL MODEL OF ASSIGNING VEHICLES TO TRANSPORT LINES MODIFICATION

MODIFIKACE MATEMATICKÉHO MODELU PŘIDĚLOVÁNÍ VOZIDEL LINKÁM

Abstract

This paper deals with the problem of allocating vehicles to public transport links. This article contains mathematical model to minimize CO and numerical experiment. Numerical experiment was conducted in fictitious traffic network and demonstrated the functionality of the proposed model.

Abstrakt

Článek se zabývá problémem přidělování vozidel na linky MHD. Článek obsahuje matematický model minimalizující emise CO a numerický experiment. Numerický experiment byl prováděn ve fiktivní dopravní síti a prokázal funkčnost navrženého modelu.

INTRODUCTION

One of the basic problems of traffic practice is creation of the system of urban public transport. In practice this system is set up mostly on the basis of previous experience but there are more approaches to use. A linear mathematical model is one of them. The linear mathematical model is an optimization model compound of two parts – binding limits and objective function. Binding limits specify a set of feasible solutions, objective functions define the specific feasible solutions. From the set of acceptable solutions we choose the most suitable (according to minimisation or maximisation of the objective function). The objective function of public transport operation is an instrumental towards optimization of diverse criteria – maximize minimal ratio between the numbers of seats offered and needed on specific section of traffic network, minimize the numbers of vehicles or minimize the impacts of mass transportation on environment.

There is a linear mathematical model for the minimizing emissions produced by vehicles presented in this paper.

THE PROBLEM AND CURRENT SITUATION SETUP

Currently the mathematical models are used to project transportation lines. Those models follow model of prof. RNDr Jan Černy, DrSc., Dr.h.c. [1]. PRIVOL is the model of one-criterion optimization. It's aim is to maximize the minimal ratio between the numbers of seats offered and demands of passengers that are calculated for the particular section of traffic network. It is a very versatile model. It is possible to modify it in order to optimize various criteria. In literature [2] some modifications of original model are mentioned.

Further we describe the modification of basic model that minimize the total volume of emissions produced by vehicles operating on the line.

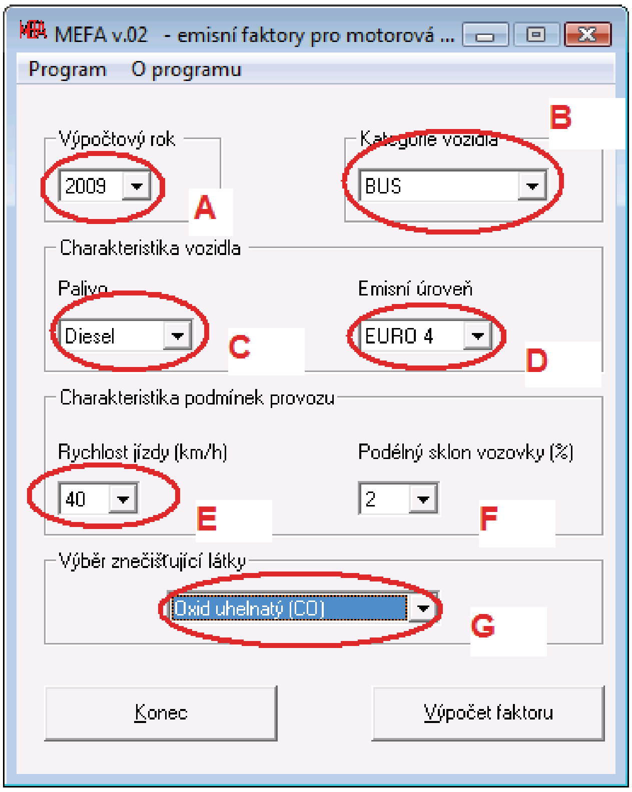
The volume of emissions depends on various factors. The emissions produced by a vehicle can be calculated for example from these criteria:

* Ing., Institute of Transport, Faculty of Mechanical Engineering,VŠB-Technical University of Ostrava, 17. listopadu 15, 708 33 Ostrava-Poruba, tel. (+420) 59 732 4113, e-mail: blatonm@centrum.cz

- the category of vehicle,
- fuel,
- the level of emission,
- speed,
- average gradient of a line,
- pollutants.

The volume of emissions produced with regard to these factors can be calculated with the assistance of MEFA v.02 program [3]. This program makes it possible to calculate the volume of various pollutants such as CO, C_XH_Y, NO_x (such as NO_2), SO_2, Methan, Benzen and dust particles.

List of parameters used to calculate the emissions can be seen in the Fig. 1.



A – design year,
B – vehicle category,
C – characteristics of the vehicle,
D – emission level,
E – average running speed,
F – longitudinal slope of road,
G – selection of pollutants.

Fig. 1 Working enviroment of MEFA v.02 software

Let's define the problem. We have a set of lines L_0. For each of them we have a time of one circuit o_l, where $l \in L_0$. The number of variables that act in the model will depend on the number of lines and the total number of vehicles in all types of transport. It will be the product of the number of lines and the total number of types.

Various vehicles can be used, the set of vehicles is I. For each sort of vehicle $i \in I$ we have a set of types of vehicles J_i that differs in capacity and we know their numbers. Various sorts of vehicles and their types differ in volume of emissions produced.

On each connecting line $h \in H$ between two vertices of traffic network we know the intensity of passengers q_h per hour. Let´s mark K_{ij} the number of vehicles of i-sort j-type and k_{ij} their capacity.

We will present a model that will minimize the volume of CO. In the mathematical model the variables will determine the number of vehicles of certain $i \in I$, type of vehicle $j \in J_i$ operating on the line $l \in L_0$. The variables are called x_{lij}. This variable will be non-negative, integral. There is a condition to be satisfied that on each line $l \in L_0$ only one sort of vehicle $i \in I$ is operating and so we establish a variable z_{li}. This variable will be bivalent, taking the values 0 and 1. When $z_{li} = 0$, the sort of vehicle $i \in I$ will not be operating on the line $l \in L_0$. When $z_{li} = 1$, the sort of vehicle $i \in I$ will be operating on the line $l \in L_0$. Any vehicle $i \in I$ can be operating on a line. A symbol e_{lij} stands for the volume of CO that is produced by the vehicle of i-type and j-sort on the line *l*.

Mathematical model requirements:

- sufficient number of the passenger places per hour on every section must be offered,
- maximal number of vehicles in vehicle fleet must be respected,
- for every line can be used only one type of transport mean.

MATHEMATICAL MODEL

Mathematic model for CO capacity minimization is defined by following formula:

Minimize

$$f(x) = \sum_{l \in L_0} \sum_{i \in I} \sum_{j \in J_i} N_l \cdot e_{lij} \cdot x_{lij} \qquad (1)$$

Subject to:

$$\frac{\sum_{l \in L_H} \sum_{i \in I} \sum_{j \in J_i} N_l \cdot x_{lij} \cdot k_{ij}}{q_h} \geq 1 \qquad for \quad h \in H \qquad (2)$$

$$\sum_{i \in I} z_{li} \leq 1 \qquad for \quad l \in L_0 \qquad (3)$$

$$\sum_{j \in J_i} x_{lij} \leq z_{li} \cdot T \qquad for \quad i \in I \qquad (4)$$

$$for \quad l \in L_0$$

$$\sum_{l \in L_0} x_{lij} \leq K_{ij} \qquad for \quad i \in I$$

$$for \quad j \in J_i \qquad (5)$$

$$z_{li} \in \{0,1\}$$
$$x_{lij} \in Z^+ \cup \{0\} \qquad (6)$$

where:

x_{lij} ... variable determining the numbers of vehicles of i-sort and j-type of vehicle operating on the line l [-],

e_{lij}...... an average volume of emissions produced by the vehicle of i-sort and j-type during one circuit l [g/circuit],

N_l.... numbers of circuits of vehicle on the line $l \in L_0$ [h^{-1}], we assume that number of circuits per hour is the same for all sorts of vehicles,

k_{ij}.... the capacity of a vehicle type $j \in J_i$ sort $i \in I$,

K_{ij} ... number of vehicles of i-type, j-sort,

q_h.... the intensity of transport stream on section $h \in H$ [seats.h^{-1}],

T..... prohibitiv constant. If the condition did not prohibitive constant, given the fact that the line could not be assigned more than one vehicle allocated to the type of vehicle.

Statement (1) is objective function, the constraint (2) ensure at minimum the number of seats, that are needed on the connecting lines, the constraints (3) and (4) assure, that if the line is under operation, there is exactly one sort of vehicle operating, the constraint (5) assure that we do not distribute more vehicles than those at our disposal. The constraints (6) are obligatory.

NUMERICAL EXPERIMENT

Designed mathematical model will be used now. The urban traffic lines network is defined (see Fig 2.) There are 13 sections and 5 lines in the network. Routes run shuttle. Two types of transport means are available and their capacities are known. The section rating $h \in H$ means intensity of the passengers per hour q_h.

Turn-round periods of lines:

t_{o1} = 55 min

t_{o2} = 25 min

t_{o3} = 35 min

t_{o4} = 40 min

t_{o5} = 30 min

Vehicle fleet composition:

LPG buses:...

- k_{11} = 40 seats, K_{11} = 10 vehicles
- k_{12} = 70 seats, $K1_2$ = 10 vehicles

Diesel engine buses:

- k_{21} = 60 seats, K_{21} = 12 vehicles
- k_{22} = 80 seats, K_{22} = 12 vehicles

Rating edge represents the intensity of passengers going in the direction of more loaded.

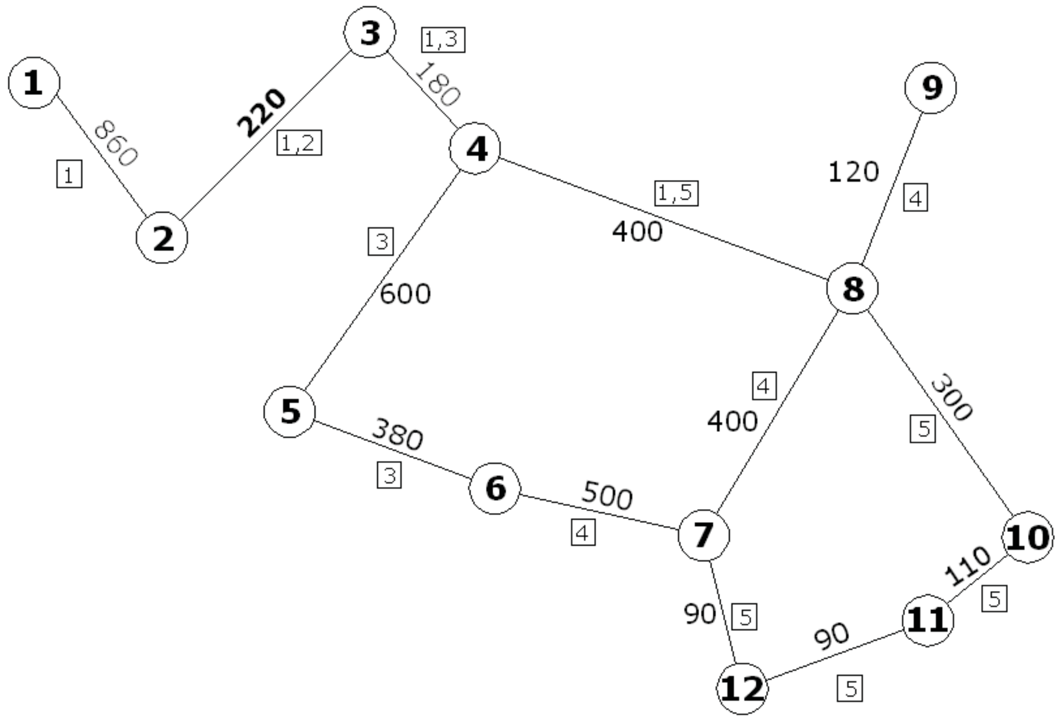


Fig. 2 Defined traffic network

The volume of CO produced by vehicles is depicted in Table 1. The values of CO are shown in gram per circuit.

Table 1: One turn round CO capacity

Line No.	LPG		Diesel	
	Type 1 $k_{11} = 40$	Type 2 $k_{12} = 70$	Type 1 $k_{21} = 60$	Type 2 $k_{22} = 80$
1	11,7	14,6	69,7	87,2
2	5,3	6,7	31,7	39,6
3	7,5	9,3	44,4	55,5
4	8,5	10,7	50,7	63,4
5	6,4	8,0	38,0	47,5

The volumes of emissions can be calculated with the assistance of the MEFA v.02 program. The analysis of the mathematical problem was carried out by Xpress – IVE software [4]. Working enviroment of optimalization software is depicted in the Fig. 3.

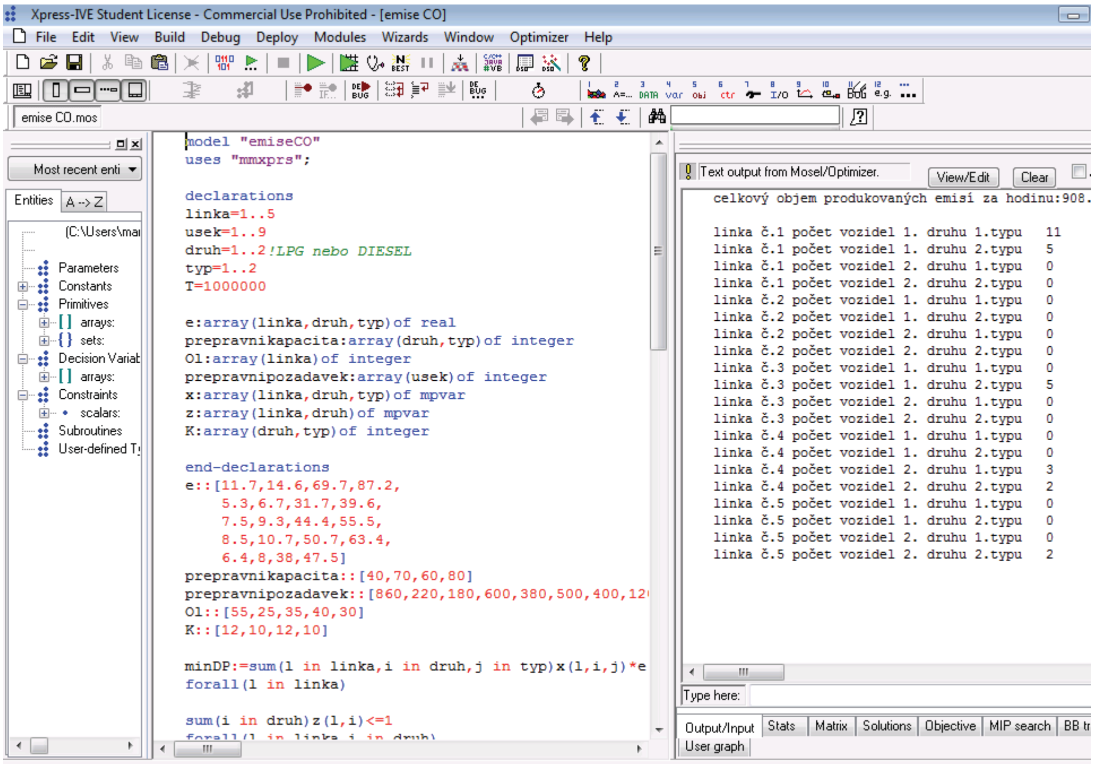


Fig. 3 Working enviroment of optimalization software Xpress – IVE

After solving the problem we have received the following output results (Table 2).

Table 2: Numbers of vehicles in actual operation

Line No.	LPG		Diesel	
	Type 1 $k_{11} = 40$	Type 2 $k_{12} = 70$	Type 1 $k_{21} = 60$	Type 2 $k_{22} = 80$
1	0	0	5	9
2	0	0	0	0
3	0	5	0	0
4	0	5	0	0
5	6	0	0	0

The above mentioned algorithm was run on the personal computer equipped with the Intel (R) Core (TM)2 Duo T7500 processor with parameters 2,2 GHz and 2GB RAM.

Time: 0,2 seconds.

CONCLUSIONS

This article deals with the problem of vehicle allocation for the public city transport lines. When distributing the vehicles on the lines we took account of the volume of emissions produced by the vehicles of mass transportation. Mathematical model and numerical experiment in this paper has been presented. The numerical experiment was applied on the virtual traffic network. The numerical experiment results confirm functionality of the used model.

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[3] MEFA v.02 software: www.env.cz

[4] Xpress - IVE software: www.dashoptimization.com