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Renata WAGNEROVÁ*

SLIDING MODE CONTROL AND ITS MODIFICATION

KLOUZAVÉ ŘÍZENÍ A JEHO MODIFIKACE

Abstract

The paper describes design of sliding mode control. The advantage of this control algorithm is a high robustness, simplicity, and the fact that it does not need knowledge of a mathematical model, only the control system's order and the ability to measure control disturbances. The disadvantage is a high control activity, which is not useful for all types of control elements. Further there is description of the primary approach to sliding control design, which consists in the extension of non-linear sliding control of an integration element. For the stabilization task this algorithm will significantly lower the activity of a control variable while maintaining its robustness. In case of following the state trajectory the activity of a control variable will also lower, if the change in the required trajectory is not to fast.

Abstrakt

Příspěvek popisuje návrh klouzavého řízení. Výhodou tohoto algoritmu řízení je jednoduchost a robustnost, není nutná přesná znalost matematického modelu, pouze musíme znát řád řízeného podsystému a regulační odchylky musí být měřitelné. Nevýhodou jsou rychlé změny akční veličiny. Dále je zde uveden původní přístup k návrhu klouzavého řízení rozšířením o integrační složku. V případě stabilizace se výrazně sníží aktivita akční veličiny oproti nespojitému klouzavému řízení. Pokud změny požadované trajektorie nejsou rychlé, výrazně se sníží aktivita řízení i pro úlohu sledování stavové trajektorie.

1 SLIDING MODE CONTROL

The quality of close control systems without knowledge of mathematical model or measurable disturbances can be also ensured by using sliding mode control. Sliding mode control means discontinuous control, where according to value switching function control has marginal value [2]. The control is written by equation

$$\boldsymbol{u}^{sl} = \left[u_1^{sl}, u_2^{sl}, \ldots, u_m^{sl}\right]^T \tag{1}$$

$$u_j^{sl} = \begin{cases} u_j^+ & \text{for} \quad m_j > 0 \\ u_j^- & \text{for} \quad m_j < 0 \end{cases} \tag{2}$$

where:

u_j^+, u_j^- - marginal value of control,

m_j – element of switching function.

* Ing., Ph.D., Department of Control Systems and Instrumentation, Faculty of Mechanical Engineering, VŠB-Technical University of Ostrava, 17. listopadu 15, 708 33 Ostrava-Poruba, tel. (+420) 59 732 4113, e-mail: renata.wagnerova@vsb.cz

The form of switching function m_j can come out from state variables aggregation method [X]. Then it is describes

$$\boldsymbol{u}^{sl} = \boldsymbol{U}^{m}\mathbf{sgn}(\boldsymbol{m}) \tag{3}$$

$$\boldsymbol{m} = \boldsymbol{D}(\boldsymbol{e} - \boldsymbol{e}_0) + \boldsymbol{T}^{-1}\boldsymbol{D}\int_0^t \boldsymbol{e}\, d\tau \tag{4}$$

$$\boldsymbol{U}^{m} = \operatorname{diag}\left[u_1^m, u_2^m, \ldots, u_m^m\right] \tag{5}$$

$$\mathbf{sgn}(\boldsymbol{m}) = \left[\operatorname{sgn}(m_1), \operatorname{sgn}(m_2), \ldots, \operatorname{sgn}(m_m)\right]^T \tag{6}$$

where:

$\boldsymbol{U}^m$ - diagonal matrix, whose elements u_j^m are marginal values of control variables,

sgn – sign function.

Sliding mode control is discontinuous, robust and simple, but its disadvantage is control high activities; it means quick switching between marginal values. It can be removed by using continuous approximation of sign function instead of sign function. Another possibility is extending discontinuous sliding control of integral element. The property of integration element is decreasing higher frequency which can be see on integration calculation of harmonic function

$$\int \sin(\omega)t dt = -\frac{1}{\omega}\cos(\omega t) \tag{7}$$

It is equivalent for cosine function.

The most simple possibility of sliding mode control with integration element will be supposed, it means the linear combination of two expression, so that the control algorithm is described by equation

$$\boldsymbol{u}^{sl} = \boldsymbol{U}^{m}\mathbf{sgn}(\boldsymbol{m}) + \boldsymbol{K}\int_0^t \boldsymbol{m}\, d\tau \tag{8}$$

where:

$\boldsymbol{K}$, $\boldsymbol{U}^{\boldsymbol{m}}$ – diagonal matrices with constant elements and dimension m.

The switching function has the same form as in (4), elements of matrix $\boldsymbol{K}$ have same value and sign as element of matrix $\boldsymbol{U}^{\boldsymbol{m}}$ otherwise the control algorithm will be unstable.

2 SLIDING MODE CONTROL APPLICATION

The sliding mode controls (3), (8) were applied to the levitation in magnetic field. The movement of levitating subject has one degree of freedom. The behavior of this kind of system can be written by equations [1, 3]

$$m\ddot{x} = mg + \frac{1}{2}i^2\frac{\partial L(x)}{\partial x} \tag{9}$$

$$u = Ri + \frac{d}{dt}\left[L(x)i\right] \tag{10}$$

$$L(x) = \frac{Q}{X_\infty + x} + L_\infty \tag{11}$$

where:

m - a weight of a steel bar [kg],

X - distance between magnetic circuit and bar [m],

I - current [A],

U - electrical voltage [V],

$L(x)$ - inductance of coil [H],

R - electrical resistance [Ω],

Q, L_∞, X_∞ - constants values of magnetic stand [H.m, H, m].

By editing equations (9) - (11) and implementing state variables $x_1 = x,\ x_2 = \dot{x},\ \ x_3 = \ddot{x}$ the mathematical model of levitation task on state representation is obtained

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= f_3(\boldsymbol{x}) + g_3(\boldsymbol{x})u \end{aligned} \tag{12}$$

where:

$f_3(\mathbf{x})$, $g_3(\mathbf{x})$ - common nonlinear functions of state variables with a representation

$$f_3(\boldsymbol{x}) = \left[\frac{2R}{\dfrac{Q}{X_\infty + x_1} + L_\infty} + \frac{2x_2}{(X_\infty + x_1)} - \frac{2Qx_2}{(X_\infty + x_1)^2\left(\dfrac{Q}{X_\infty + x_1} + L_\infty\right)}\right](g - x_3)$$

$$g_3(\boldsymbol{x}) = -\sqrt{\frac{2Q}{m}}\frac{\sqrt{g - x_3}}{\left(\dfrac{Q}{X_\infty + x_1} + L_\infty\right)(X_\infty + x_1)}$$

It is evident that the system is strongly nonlinear, third order that is why the aggregation matrix $\boldsymbol{D}$ and matrix of time constants $\boldsymbol{T}$ have presentation [4]

$$\boldsymbol{D} = \boldsymbol{d}^T = \begin{bmatrix} \dfrac{1}{T_0^2} & \dfrac{2\xi_0}{T_0} & 1 \end{bmatrix}, \boldsymbol{T} = T_3 \tag{13}$$

where:

T_i – time constants choosing according to the required closed-loop system behavior.

The switching function is described by equation

$$m = \left[\frac{1}{T_0^2 T_3}\int_0^t e_1 d\tau + \left(\frac{1}{T_0^2} + \frac{2\xi_0}{T_0 T_3}\right)(e_1 - e_{10}) + \left(\frac{2\xi_0}{T_0} + \frac{1}{T_3}\right)(e_2 - e_{20}) + e_3\right] \tag{14}$$

where:

e_1 – difference between required and real position of levitating object,

$e_2 = \dot{e}_1$, $e_3 = \dot{e}_2$.

The sliding mode controls according to the (3), (8) have presentation

$$u^{sl} = u^m \operatorname{sgn}(m) \tag{15}$$

$$u^{sl} = u^m \operatorname{sgn}(m) + k \int_0^t m \, d\tau \qquad (16)$$

The designed control algorithms were verified by computer simulation using simulation program MATLAB/SIMULINK. The values of switching function parameters were same for both control algorithms (T_3=0,05 s, T_0=0,05 s, ξ_0=1, u^m=-85, k=-85). The course of levitating object position and both controls are shown on Fig. 1 – 6. On all figures with position it can be see courses of three variables: variable **x** is position for control algorithm (15), variable **xw** is required position, and variable **x1** is position with control (16). Under figure with position there are situated two courses of control, the upper shows control (15), and the lower is control (16).

From the Fig. 1 – 4 it is obvious that the sliding mode control with integration element (16) achieved the same control quality as the sliding control (15) but the activity is significantly lower for both tasks stabilization and also tracking. When the required state trajectory changes quickly then the activity of both sliding mode control is the same (Fig. 5 – 6).

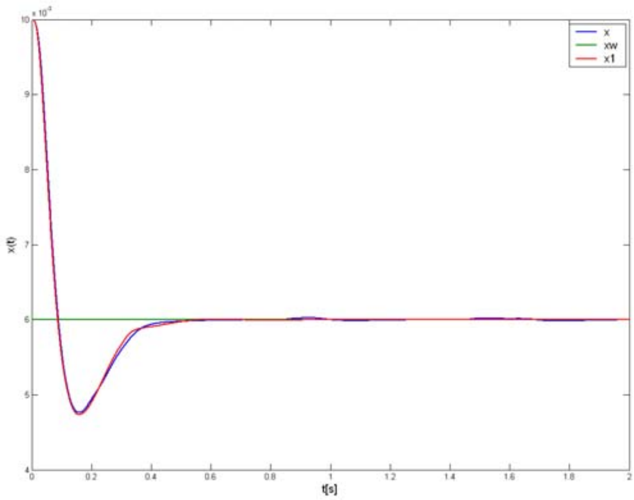

Fig. 1 The course of position for stabilization task

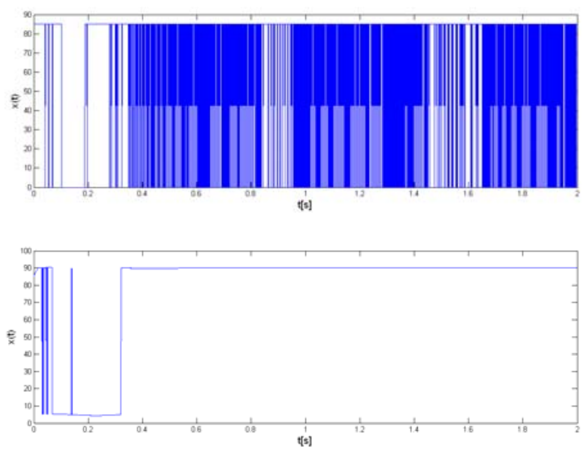

Fig. 2 The course of controls for stabilization task

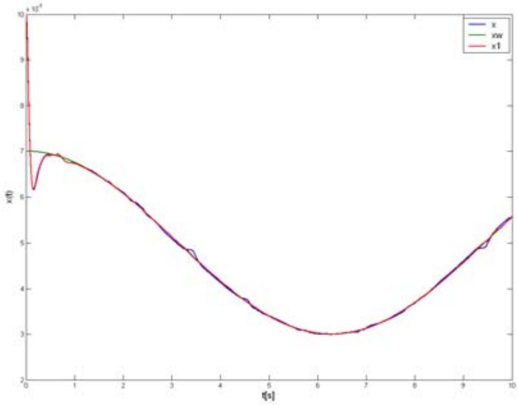

Fig. 3 The course of position for tracking the state trajectory

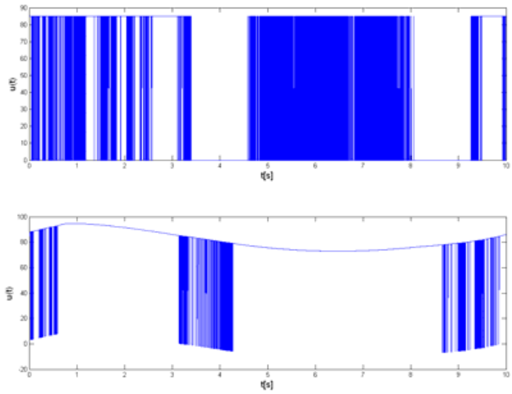

Fig. 4 The course of controls for tracking the state trajectory

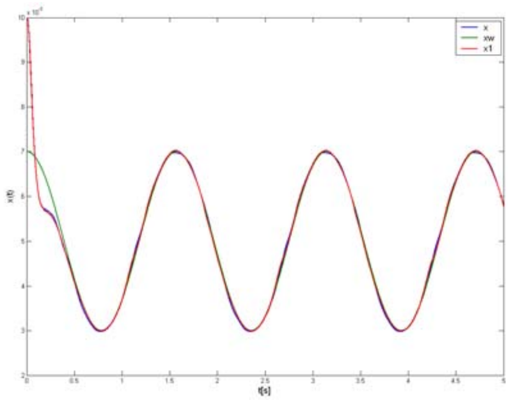

Fig. 5 The course of positron for tracking the state trajectory

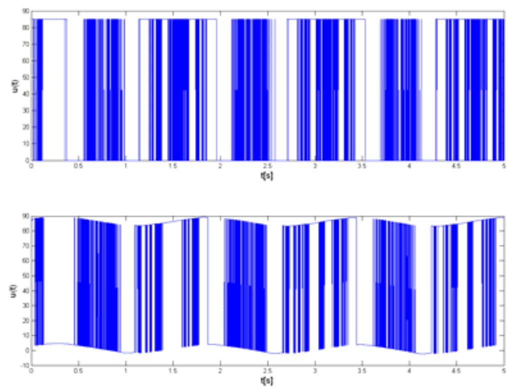

Fig. 6 The course of controls for tracking the state trajectory

3 CONCLUSION

The contribution presents sliding mode control design. There are described properties of two types of sliding mode control: discontinuous and with integral element. The both were used to position control of levitating object in magnetic field and ensured reaching the required trajectory.

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Reviewers:

prof. Ing. Štefan Kozák, PhD., Slovak University of Technology in Bratislava

prof. Ing. Vladimír Vašek, CSc., Tomas Bata University in Zlín