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STABILITY OF RIGID ROTOR IN JOURNAL BEARING

STABILITA TUHÉHO ROTORU V KLUZNÉM LOŽISKU

Abstract

The article is devoted to active increasing of a stability of a rigid rotor housed in a journal bearing on the supposition that the linear and nonlinear mathematical model is considered.

Abstrakt

Příspěvek je věnován aktivnímu zvýšení stability tuhého rotoru uloženého v kluzném ložisku za předpokladu, že je uvažován lineární a nelineární model.

1 INTRODUCTION

When the rotational speed is increased a rotor instability can arise for high-speed rotational machines with journal bearings. It is manifested by vibrations and whippings. This instability is not caused by external forces from the unbalanced masses or by other external periodical forces [Muszyńska, 2005; Muszyńska et al., 1996]. The main reason of the rotor instability arising in journal bearing is the oil film, which causes the lateral vibrations of the rotor for a certain rotational speed [Muszyńska et al., 1989; Muszyńska et al., 1996; Zapoměl, 2007].

2 LINEAR MATHEMATICAL MODEL OF PLANT „ROTOR – BEARING“

The simplified linear mathematical model of the rigid rotor housed in a journal bearing (Fig. 1), assuming that only its mass and behaviour of the oil film are considered in the motion equations, can have in stationary coordinates the form [Cheng et al., 2006; Li et al., 2003; Li et al., 2007]

$$\left.\begin{aligned} M\,\ddot{x}(t) + D\dot{x}(t) + Kx(t) &= -u_x(t) - D\lambda\Omega y(t) + mr\Omega^2\cos\Omega t \\ M\,\ddot{y}(t) + D\dot{y}(t) + Ky(t) &= u_y(t) + D\lambda\Omega x(t) - Mg + mr\Omega^2\sin\Omega t \end{aligned}\right\} \quad (1)$$

where M – the total rotor mass [kg], D – the generalized damping coefficient [Ns/m], K – the generalized stiffness coefficient [N/m], λ – the oil circumferential average velocity ratio [-], x – the horizontal displacement of the rotor centre [m], y – the vertical displacement of the rotor centre [m], Ω – the rotor angular velocity [rad/s], m – the unbalanced mass [kg], r – the radius for

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the unbalanced mass [m], $u_x(t)$ – the actuator horizontal force [N], $u_y(t)$ – the actuator vertical force [N], g – the acceleration of gravity [m/s^2].

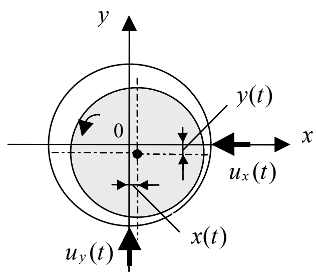

Fig. 1 Rotor in a journal bearing

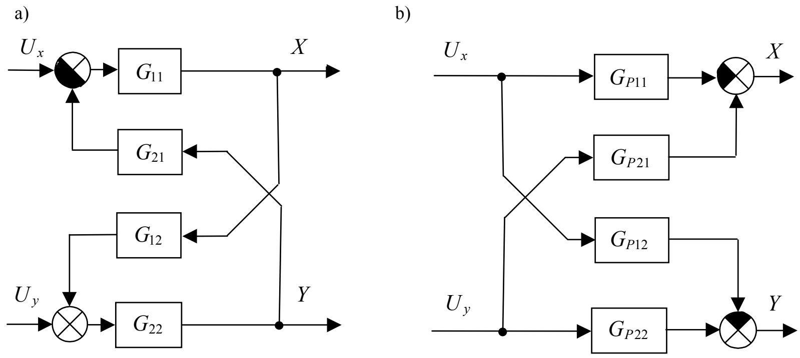

Fig. 2 Block diagram of plant „rotor – bearing“: a) V – structure, b) P – structure

The linear model of the plant „rotor – bearing“ without consideration of the unbalanced mass forces and the gravity is in Fig. 2. Fig. 2a shows the canonical V – structure of the linear model, where

$$\left.\begin{aligned} G_{11}(s) = G_{22}(s) &= \frac{1}{Ms^2 + Ds + K} \\ G_{12}(s) = G_{21}(s) &= D\lambda\Omega \end{aligned}\right\} \tag{2}$$

For stability checking of the linear model the canonical P – structure in Fig. 2b is suitable, where

$$\left.\begin{aligned} G_{P11}(s) = G_{P22}(s) &= \frac{Ms^2 + Ds + K}{(Ms^2 + Ds + K)^2 + D^2\lambda^2\Omega^2} \\ G_{P12}(s) = G_{P21}(s) &= \frac{D\lambda\Omega}{(Ms^2 + Ds + K)^2 + D^2\lambda^2\Omega^2} \end{aligned}\right\} \tag{3}$$

It is obvious that the characteristic polynomial of the linear model has the form

$$\begin{aligned} N_P(s) &= (Ms^2 + Ds + K)^2 + D^2\lambda^2\Omega^2 = \\ &= M^2s^4 + 2DMs^3 + (2KM + D^2)s^2 + 2DKs + D^2\lambda^2\Omega^2 + K^2 \end{aligned} \tag{4}$$

from which the stability condition

$$\Omega < \frac{1}{\lambda}\sqrt{\frac{K}{M}} \tag{5}$$

can be easily obtained.

The critical value of the angular rotor velocity

$$\Omega_{kr} = \frac{1}{\lambda}\sqrt{\frac{K}{M}} \tag{6}$$

expresses the so-called Bently – Muszyńska stability threshold.

3 CONTROL OF LINEAR PLANT „ROTOR – BEARING“

The stability threshold (6) considerably constrains the maximum rotor angular velocity that is why there is the need to increase the stability threshold.

One of the possibilities of the active increasing of the stability threshold is the use of the two-input two-output (TITO) control system with objective

$$\left.\begin{aligned} X(s) &\rightarrow X_w(s) = 0 \\ Y(s) &\rightarrow Y_w(s) = 0 \end{aligned}\right\} \tag{7}$$

which is shown in Fig. 3.

In accordance with Fig. 3 the relations can be written (for reasons of simplification and lucidity the complex variable s will be omitted if needed)

$$\left.\begin{aligned} X &= \frac{G_{11}}{1+G_xG_{11}}(G_xX_w + V_x - G_{21}Y) \\ Y &= \frac{G_{22}}{1+G_yG_{22}}(G_yX_w + V_y + G_{12}Y) \end{aligned}\right\} \tag{8}$$

where G_x and G_y – the controller transfer functions, V_x and V_y – the transforms of the disturbances [unbalanced mass forces and the gravity, see (1)]

$$\left.\begin{aligned} v_x(t) &= mr\Omega^2 \cos\Omega t \\ v_y(t) &= mr\Omega^2 \sin\Omega t - Mg \end{aligned}\right\} \tag{9}$$

By the modification of the relations (8) the relations

$$\left.\begin{aligned} X &= \frac{G_{11}}{(1+G_xG_{11})(1+G_yG_{22}) + G_{11}G_{12}G_{21}G_{22}} \cdot \\ &\cdot[(1+G_yG_{22})G_xX_w - G_{21}G_yG_{22}Y_w + (1+G_yG_{22})V_x - G_{21}G_{22}V_y] \\ Y &= \frac{G_{22}}{(1+G_xG_{11})(1+G_yG_{22}) + G_{11}G_{12}G_{21}G_{22}} \cdot \\ &\cdot[G_{12}G_xG_{11}X_w + (1+G_xG_{11})G_yY_w + G_{11}G_{12}V_x + (1+G_xG_{11})V_y] \end{aligned}\right\} \tag{10}$$

can be obtained.

The characteristic polynomial $N(s)$ of the TITO control system can be got by the modification of the one of the denominators of the relations (10).

Providing that the identical PD controllers are used (the use of the controllers with the integrating component for the analytical computation is very complicated) i. e.

$$G_x(s) = G_y(s) = k_P(1+T_Ds) \tag{11}$$

the characteristic polynomial has the form

$$N(s) = M^2 s^4 + 2M(D + k_P T_D)s^3 + [(D + k_P T_D)^2 + 2M(K + k_P)]s^2 + \\ + 2(D + k_P T_D)(K + k_P)s + (K + k_P)^2 + (D\lambda\Omega)^2 \tag{12}$$

where k_P – the controller gain, T_D – the controller derivative time.

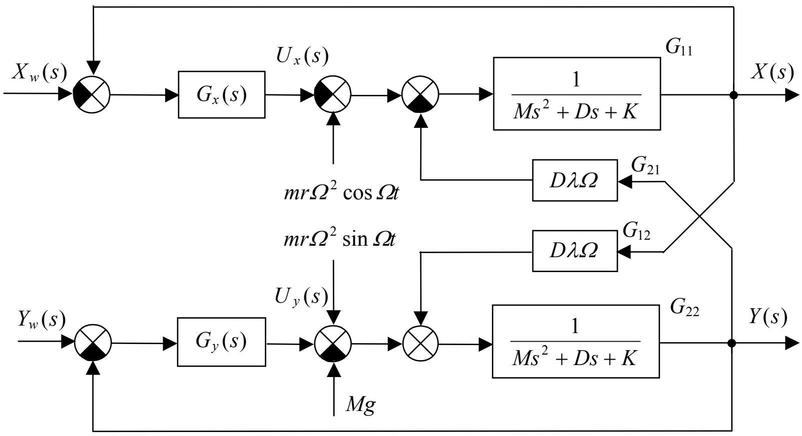

Fig. 3 TITO control system for increasing of stability threshold of the linear plant „rotor – bearing“

On the basis of the Hurwitz stability criterion the relation

$$\Omega < \frac{1}{\lambda}\sqrt{\frac{K + k_P}{M} + \frac{(K + k_P)(2D + k_P T_D)k_P T_D}{MD^2}} \tag{13}$$

can be obtained.

From the relation (13) it is obvious that using TITO control system with the PD controllers the increasing of the stability threshold (6) takes place. For $k_P = 0$ the stability condition (5) is obtained. Because the PD controllers (11) do not include the integrating component, the rotor centre displacement in respect of the bearing centre will not be removed, but for high enough values of the controllers gain k_P will be significantly lowered.

All conclusions hold for the linear model of the plant „rotor – bearing“ and they were confirmed by the digital simulation for the parameter values [Tůma et al, 2007; Tůma et al, 2008; ČSN ISO 1940-1]: $M = 1.6$ kg, $\lambda_0 = 0.475$, $K = 4\ 000$ N/m, $D = 1\ 000$ Ns/m.

4 NONLINEAR MATHEMATICAL MODEL OF PLANT „ROTOR – BEARING“

It is supposed that the nonlinear mathematical model of the plant „rotor – bearing“ has the same form as (1), but the generalized coefficients of damping D and stiffness K and the oil circumferential average velocity ratio λ are generally nonlinear functions of the relative radial eccentricity, which is defined by the relation

$$e = \frac{1}{c}\sqrt{x^2 + y^2} \tag{14}$$

where c – the radial clearance [m].

Most frequently the following relations are used [Li et al., 2007; Li et al., 2003; Cheng et al., 2006; Ding et al., 2002] :

$$K = K_0(1-e^2)^{-n}, \quad D = D_0(1-e^2)^{-n}, \quad n = 0.5 \div 3 \tag{15}$$

$$\lambda = \lambda_0 /(1-e)^b, \quad 0 < b < 1 \tag{16}$$

where K_0 – the initial value of the generalized stiffness coefficient [N/m], D_0 – the initial value of the generalized damping coefficient [Ns/m], λ_0 – the initial value of the oil circumferential average velocity ratio [-].

5 SIMULATION

For the verification of the nonlinear mathematical model of the plant „rotor – bearing“ the digital simulation for the following parameter values was realized [Tůma et al, 2007; Tůma et al, 2008; ČSN ISO 1940-1; Li et al., 2007; Li et al., 2003; Cheng et al., 2006; Ding et al., 2002]: M = 1.6 kg, λ_0 = 0.475, K_0 = 4 000 N/m, D_0 = 1 000 Ns/m, c = 0.0002 m, mr = 0.00001 kg m, b = 0.1, n = 2.

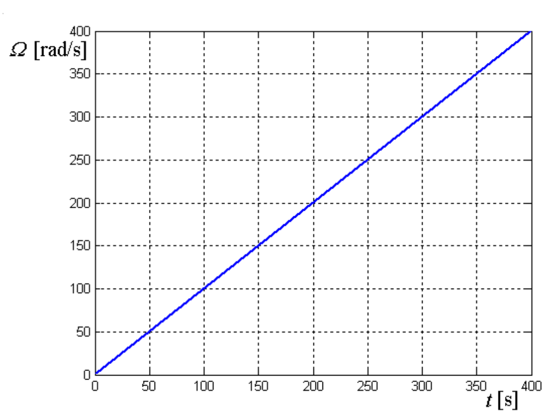

Fig. 4 Linear growth of the rotor angular velocity

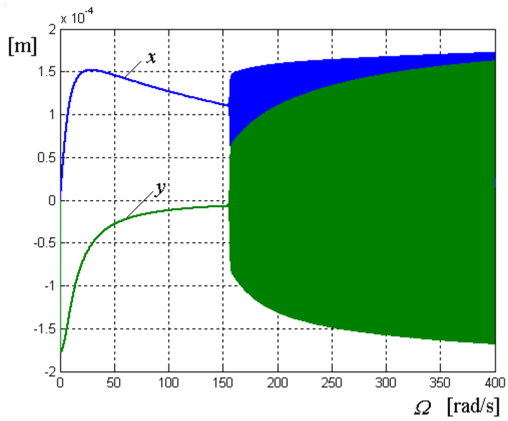

Fig. 5 Rotor instability rise for nonlinear mathematical model (without control)

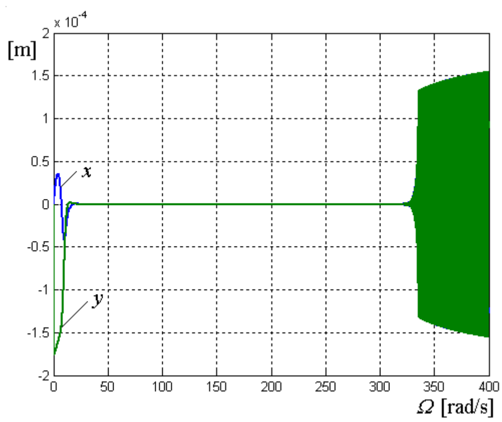

Fig. 6 Stability threshold increasing using TITO control system for $k_p = 10000$, $T_I = 1\,\text{s}$, $T_D = 0.05\,\text{s}$

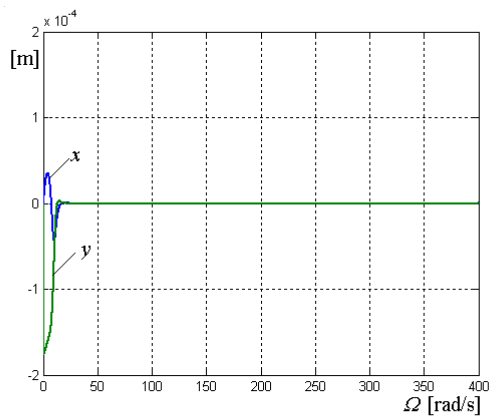

Fig. 7 Additional stability threshold increasing using TITO control system for $k_p = 10000$, $T_I = 1\,\text{s}$, $T_D = 0.2\,\text{s}$

In the digital simulation the linear growth of the rotor angular velocity was supposed (Fig. 4).

Fig. 5 shows the rise of a rotor instability in the journal bearing for the nonlinear mathematical model of the plant „rotor – bearing“ without use of the control. From the courses of the displacements x and y in the dependency on the rotor angular velocity Ω results, that for the nonlinear model the stability threshold increases. For the linear model (1) in accordance with relation (6) $\Omega_{kr} = 105.3$ rad/s is obtained.

Using the TITO control system with the PID controllers the stability threshold is significantly increased, which is shown in Figs 6 and 7.

6 CONCLUSIONS

In the article it is shown that the force actuating the Bently – Muszyńska stability threshold of the rotor can be significantly increased. For the linear mathematical model of the plant „rotor – bearing“ the results obtained by the analytical way were confirmed by digital simulation entirely.

For the nonlinear mathematical model of the plant „rotor – bearing“, it was shown by digital simulation that the nonlinear dependency of the generalized coefficients of damping and stiffness and the oil circumferential average velocity ratio have a positive effect on a rotor stability. The nonlinear mathematical model will be further improved on the basis of the preparation of a real model of the rotor housed in journal bearing. The results obtained by theoretical and simulation methods will be verified on the real model.

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