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SUITABILITY OF ADAPTABLE STATE FEEDBACK FOR KEEPING PRESCRIBED CONTROL QUALITY UNDER CHANGING OPERATION CONDITIONS

VHODNOST ADAPTABILNÍ STAVOVÉ ZPĚTNÉ VAZBY PRO UDRŽENÍ PŘEDEPSANÉ KVALITY REGULACE ZA PROMĚNNÝCH PROVOZNÍCH PODMÍNEK

Abstract

A state variable feedback technique is usually used when controlled system dynamics is to be changed in a desired way. The results are very good provided that the linear state space representation used in the design is sufficiently valid in the whole operating range. In the paper, we present a case study of a tank cascade with changeable static and dynamic properties. When these changes are so significant that superior PI or PID controllers require resetting, it is advantageous to adapt the proportional state variable feedback. We assessed the impact of the feedback gain changes resulting from changes in the linear state model parameters used in the design of the gains.

Abstrakt

Stavová zpětná vazba je obvykle využívána k požadované změně dynamiky regulované soustavy. Výsledky bývají velmi dobré, pokud lineární stavová reprezentace má dostatečnou platnost v celém pracovním rozsahu. V příspěvku je prezentován případ kaskády nádrží, u níž dochází ke změnám statických i dynamických vlastností v závislosti na pracovním bodu. Jakmile jsou tyto změny natolik významné, že by vyžadovaly přestavení nadřazených PI nebo PID regulátorů, je výhodné provést přizpůsobení zesílení v použitých zpětných stavových vazbách. Byl vyhodnocován dopad změn zesílení ve zpětných vazbách vyplývající ze změn parametrů lineárního stavového modelu, který využíván k návrhu zesílení.

1 INTRODUCTION

In the design of advanced control algorithms, tests of the functionality of the control system need to be carried out with the help of simulation models that provide a sufficiently true representation of reality. A frequently occurring fault that impedes the implementation of these algorithms into practical applications is the use of intuitively suggested changes in the model parameters. These are easy to simulate, but they do not in fact correspond to the real mutually interacting changes bounded by the validity of physical laws. This is one reason why realistic mathematical models of some real devices or processes designed as laboratory setups have been proved and they are preferred for use in advanced algorithm testing. They make possible a sufficiently precise mathematical description by means of physical laws whose mathematical formulation is sufficiently exactly fulfilled under real conditions. One example can be a cascade of two or more tanks. This is often found in labs as a physical model, and its main advantage is the

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possibility to describe and quantify accumulation and flow processes relatively exactly with full respect to the valid physical laws providing a satisfactory measure of nonlinear properties.

This paper demonstrates both the procedure for creating such a model and the exploitation of the modelling in the design of a control algorithm, in this case specifically focused on investigating the adaptable state variable feedback. Some of the computer aided techniques based on the MATLAB program package, including the Simulink and Symbolic toolboxes, are briefly outlined.

2 DESCRIPTION OF THE TANK CASCADE MODEL

The cascade considered here consists of two mutually interconnected tanks. The first tank is supplied from a source which is idealized, assuming that delivery is independent from consumption. The second tank serves as an accumulator in further distribution characterized by variable consumption. The tanks can be interconnected with the pipeline either below or above the surface. In the mathematical model of the tank cascade this is taken into account by means of two binary variables s_1 and s_2. The level in the second tank is controlled by valve 1, placed in the inlet pipeline into the first tank, while valve 2 situated in the pipeline interconnecting the two tanks is beyond any manipulation. The random changes in the flow taken from the second tank are modelled by the changes in the opening of outlet valve 3. Models of the valves enable the user to define a level of nonlinearity in the opening characteristics by applying a special function using parameter v in conversion of a real stem stroke z_i into an imaginary stroke u_i. In the simulation model, the heights of the levels in tanks are generally not limited, and all the dimensions and physical constants are selected as if the tank cascade were a laboratory set-up and water was the liquid flowing through the cascade.

3 MATHEMATICAL MODEL

The notation of the volume flow rate balance for each of the tanks leads to the equations:

$$\begin{aligned} F_1\dot{h}_1(t) &= k_1 l_1(t)\sqrt{h_0 - s_1 h_1(t)} - k_2 l_2(t)\sqrt{h_1(t) - s_2 h_2(t)} \\ F_2\dot{h}_2(t) &= k_2 l_2(t)\sqrt{h_1(t) - s_2 h_2(t)} - k_3 l_3(t)\sqrt{h_2(t)} \end{aligned} \tag{1}$$

where F_i (i = 1, 2) are the cross sections of the tanks, k_i (i = 1, 2, 3) are the flow coefficients of the valves, the opening of which is measured by means of an imaginary stem stroke $l_i(t)$ (i = 1, 2, 3), i.e. after the conversion from real strokes, and then the square root dependence on the valve pressure difference is expressed by means of the level heights $h_i(t)$ (i = 1, 2). Numerically, the flow coefficient indicates the value of the water flow rate assuming full valve opening and unit size of the difference in level heights. Variables s_1 and s_2 work like switches, and they indicate how the tanks are interconnected; e.g. if s_1 = 1, the liquid is delivered into the first tank by a pipe issuing under a surface of level h_1, if s_1 = 0, the inflow is free (above the surface).The setting of the numerical parameter values is facilitated by using no dimensional expression by means of which absolute numerical values are converted into relative values. Doing this for the level heights, it is suitable to use as a reference value the imaginary level height in the source h_0; for the valve stem strokes, their maximum value l_{imax} is used. The flow rates can be expressed as relative values of the maximum flow rate Q_{max}. Numerically, Q_{max} is equal to the flow rate through the first valve free to the atmosphere, if it is fully opened and connected to the source. The notation of the model equation (1) in non-dimensional form is

$$F_1 h_0 \dot{x}_1(t) = Q_{max} u_1(t)\sqrt{1 - s_1 x_1(t)} - k_2 l_{2max}\sqrt{h_0}\, u_2(t)\sqrt{x_1(t) - s_2 x_2(t)} \tag{2}$$

$$F_2 h_0 \dot{x}_2(t) = k_2 l_{2max}\sqrt{h_0}\, u_2(t)\sqrt{x_1(t) - s_2 x_2(t)} - k_3 l_{3max}\sqrt{h_0}\, u_3(t)\sqrt{x_2(t)} \tag{3}$$

where $Q_{max} = k_1 l_{1max}\sqrt{h_0}$, $x_i(t) = \dfrac{h_i(t)}{h_0}$, $i = 1, 2$, $u_j(t) = \dfrac{l_j(t)}{l_{j\,max}}$, $j = 1, 2, 3$.

It is obvious that while Q_{max} expresses the maximum flow capacity (measured in volume flow rate units) of a fully opened valve one connected to the source without any further connection to a tank, Q_{jmax} denotes the flow capacities of valves two and three, assuming that they are separately connected to the source. Dividing Equation (2) and (3) by Q_{max}, time constants T_1, T_2 and non-dimensional flow constants q_2 a q_3 ($q_1 = 1$) can be introduced

$$Q_{jmax} = k_j l_{jmax} \sqrt{h_0}\ ,\ j = 2,3 \quad T_i = \frac{F_i h_0}{Q_{max}},\ i = 1,2\ , \quad q_j = \frac{k_j l_{jmax}\sqrt{h_0}}{Q_{max}} = \frac{Q_{jmax}}{Q_{max}},\ j = 2,3 \tag{4}$$

In non-dimensional notation, a nonlinear state model of a two-tank cascade is

$$\begin{aligned} \dot{x}_1(t) &= \frac{1}{T_1}\left\{u_1(t)\sqrt{1 - s_1 x_1(t)} - q_2 u_2(t)\sqrt{x_1(t) - s_2 x_2(t)}\right\} \\ \dot{x}_2(t) &= \frac{1}{T_2}\left\{q_2 u_2(t)\sqrt{x_1(t) - s_2 x_2(t)} - q_3 u_3(t)\sqrt{x_2(t)}\right\} \\ y(t) &= x_2(t) \end{aligned} \tag{5}$$

4 LINEAR STATE MODEL OF THE TANK CASCADE

The steady state from which all experiments start and in which the linearization will be carried out is denoted in variables symbols by index 0. The steady state values of these variables are mutually linked by the equalities

$$u_{1_0}\sqrt{1 - s_1 x_{1_0}} = q_2 u_{2_0}\sqrt{x_{1_0} - s_2 x_{2_0}} = q_3 u_{3_0}\sqrt{x_{2_0}} = q_0 \tag{6}$$

$$x_{1_0} = \frac{s_2 u_{1_0}^2 u_{2_0}^2 q_2^2 + u_{1_0}^2 u_{3_0}^2 q_3^2}{s_1 u_{1_0}^2 u_{3_0}^2 q_3^2 + u_{2_0}^2 q_2^2 u_{3_0}^2 q_3^2 + s_1 s_2 u_{1_0}^2 u_{2_0}^2 q_2^2} \quad x_{2_0} = \frac{u_{1_0}^2 u_{2_0}^2 q_2^2}{s_1 u_{1_0}^2 u_{3_0}^2 q_3^2 + u_{2_0}^2 q_2^2 u_{3_0}^2 q_3^2 + s_1 s_2 u_{1_0}^2 u_{2_0}^2 q_2^2} \tag{7}$$

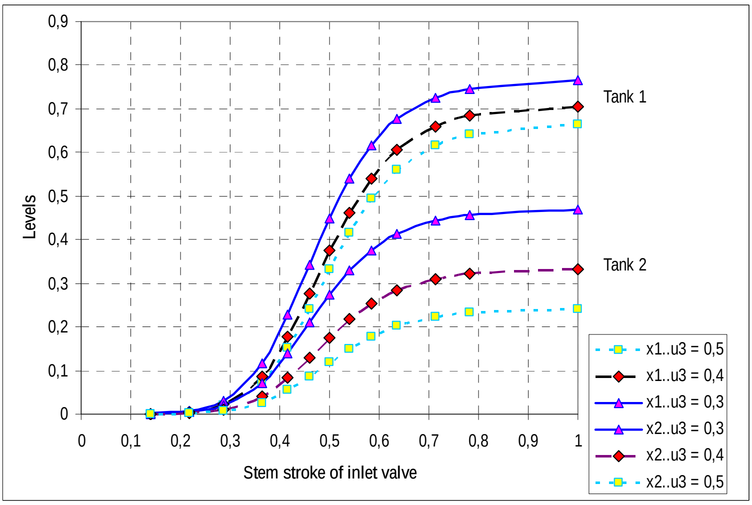


Fig. 1 Steady state characteristics in the numeric example

Mathematically expressed mutual relations (6) and (7) can be depicted graphically (Figure 1).

With help of the Symbolic Toolbox in the MATLAB program, partial derivation of the non-linear state space model (5) with respect to all variables can be obtained. This procedure can be seen from the results in Eq. (8) to (14), where new symbols for coefficients in the matrices of state formulation have been simultaneously introduced.

$$A_{11} = -\frac{q_2 u_{2_0}}{2T_1\sqrt{x_{1_0} - s_2 x_{2_0}}} - \frac{u_{1_0}}{2\sqrt{1 - s_1 x_{1_0}}} s_1 = -\frac{q_0}{2T_1(x_{1_0} - s_2 x_{2_0})} - \frac{1}{2T_1}\frac{q_0}{1 - s_1 x_{1_0}} s_1 = -\frac{1}{T_1}(a_{12} + a_1 s_1) \tag{8}$$

$$A_{12} = \frac{q_2 u_{2_0}}{2T_1\sqrt{x_{1_0} - s_2 x_{2_0}}} s_2 = \frac{q_0}{2T_1(x_{1_0} - s_2 x_{2_0})} s_2 = \frac{1}{T_1} a_{12} s_2 \tag{9}$$

$$A_{21} = \frac{q_2 u_{2_0}}{2T_2\sqrt{x_{1_0} - s_2 x_{2_0}}} = \frac{q_0}{2T_2(x_{1_0} - s_2 x_{2_0})} = \frac{1}{T_2} a_{12} \tag{10}$$

$$A_{22} = -\frac{q_2 u_{2_0}}{2T_2\sqrt{x_{1_0} - s_2 x_{2_0}}} s_2 - \frac{1}{T_2}\frac{q_3 u_{3_0}}{2\sqrt{x_{3_0}}} = -\frac{q_0}{2T_2(x_{1_0} - s_2 x_{2_0})} s_2 - \frac{1}{T_2}\frac{q_0}{2x_{3_0}} = -\frac{1}{T_2}(a_{12}s_2 + a_2) \tag{11}$$

$$B_{11} = \frac{\left(1 - \tanh^2\left((z_{1_0} - 0{,}5)v\right)\right)v}{2\tanh(0{,}5v)} \frac{\sqrt{1 - s_1 x_{1_0}}}{2T_1} = b_1 \tag{12}$$

$$B_{23} = -\frac{\left(1 - \tanh^2\left((z_{3_0} - 0{,}5)v\right)\right)v}{2\tanh(0{,}5v)} \frac{q_3\sqrt{x_{2_0}}}{2T_2} = b_3 \tag{13}$$

The linear state space formulation in the deviations of variables is represented by the equations

$$\begin{bmatrix} \Delta\dot{x}_1(t) \\ \Delta\dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}\begin{bmatrix} \Delta x_1(t) \\ \Delta x_2(t) \end{bmatrix} + \begin{bmatrix} B_{11} & 0 \\ 0 & B_{23} \end{bmatrix}\begin{bmatrix} \Delta z_1(t) \\ \Delta z_3(t) \end{bmatrix} \quad \Delta y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix}\begin{bmatrix} \Delta x_1(t) \\ \Delta x_2(t) \end{bmatrix} \tag{14}$$

5 DESIGN OF PROPORTIONAL STATE FEEDBACK

In order to modify the dynamics of the tank cascade control, to the manipulated variable $\Delta z_{1\mathrm{R}}(t)$ generated by a superior PI controller further changes $\Delta z_{1\mathrm{S}}(t)$ proportional to the state variable changes $\Delta x_1(t)$, $\Delta x_2(t)$ are added:

$$\Delta z_{1_S} = -\boldsymbol{K}^T \Delta\boldsymbol{x}(t) \tag{15}$$

After substituting (15) in (14) we get a new state formulation

$$\Delta\dot{\boldsymbol{x}}(t) = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}\begin{bmatrix} \Delta x_1(t) \\ \Delta x_2(t) \end{bmatrix} + \begin{bmatrix} B_{11} \\ 0 \end{bmatrix}\left(-\boldsymbol{K}^T\Delta\boldsymbol{x}(t) + \Delta z_{1_R}(t)\right) = (\boldsymbol{A} - \boldsymbol{B}\boldsymbol{K}^T)\Delta\boldsymbol{x}(t) + \boldsymbol{B}\Delta z_{1_R}(t) \tag{16}$$

(the influence of disturbance manipulation with $\Delta z_3(t)$ is not reflected) whose characteristic polynomial can be influenced by the matrix $\boldsymbol{K}$ of the gains in the state feedback. There are several options in defining them, but, in principle, quicker dynamics of the controlled system makes control by the superior controller easier and quicker. The quickest but still non-oscillating dynamics is achieved when the characteristic polynomial has multiple roots. In the case of a polynomial of the second degree, only one parameter must then be defined. This is time constant τ. The equality of the two polynomials is expressed by the formula

$$\det\left[s\boldsymbol{I} - \boldsymbol{A} - \boldsymbol{B}\boldsymbol{K}^T\right] = s^2 + 2s/\tau + 1/\tau^2 \tag{17}$$

A search for the values of gains k_1, k_2 representing elements of the matrix (vector) $\boldsymbol{K}^T$ was carried out with support from the Symbolic toolbox. The toolbox offers a function *poly* that creates characteristic polynomials to matrices. Using notation similar to standard symbolic declarations, the whole procedure can be described as follows

Initial assignments:

```
aa =sym('[A11 A12;A21 A22]')=[ A11, A12]
                             [ A21, A22]
K = [ k1, k2]
bb=sym('[B11;0]')= B11
                    0
```

Characteristic polynomials:

- designed

```
d =poly(aa-bb*K,'s')=
s^2-(A11+A22)*s+B11*k1*s-
B11*A22*k1+B11*A21*k2+A11*A22-A12*A21
```

- desired

```
d_ =poly2sym([1 2/tau 1/tau^2],'s')=
s^2+2*s/tau+1/tau^2
```

Comparison of coefficients at the same powers of *s* in polynomials *d* and *d_* leads to the notation

$$\begin{bmatrix} B_{11} & 0 \\ -B_{11}A_{22} & B_{11}A_{21} \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} 2/\tau \\ 1/\tau^2 \end{bmatrix} - \begin{bmatrix} -A_{11} - A_{22} \\ A_{11}A_{22} - A_{12}A_{21} \end{bmatrix} \tag{18}$$

Solution of Eq. (18):

```
ab=sym('[B11,0;-B11*A22,-B11*A21]')= [B11, 0]
                                      [-B11*A22, B11*A21]
ttau=sym('[2/tau;1/tau^2]')= 2/tau
                             1/tau^2
aa12 =sym('[-A11-A22;A11*A22-A21*A12]')= -A11-A22;
                                         A11*A12-A12*A21
k=inv(ab)*(ttau-aa12)= 1/B11*(2/tau+A11+A22)
                       1/B11/A21*(1/tau^2+A12*A21-A11*A22)+1/B11*A22/A21*(2/tau+A11+A22)
```

6 EXPERIMENTS WITH GAINS ADAPTABILITY AND KEEPING FIXED PI CONTROLLER SETTING

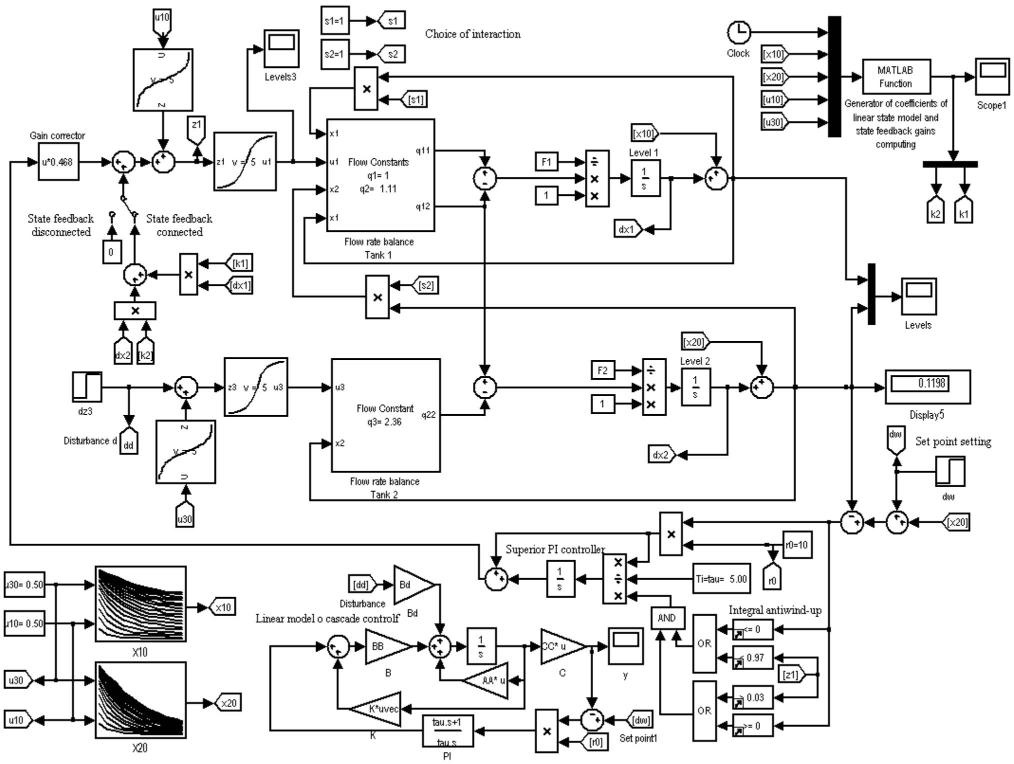
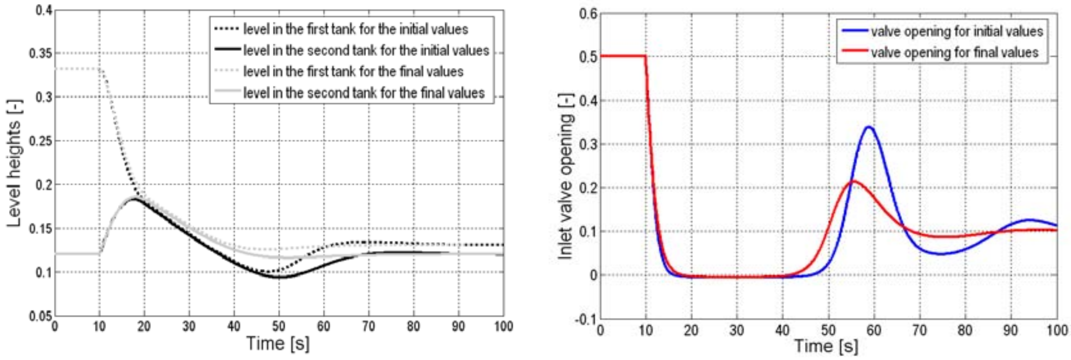


Fig. 2 Model of tank cascade control in Simulink, including auxiliary linear state model

The simulation model in Figure 2 consists of several parts. The two masked blocks in the middle perform computation of flows in the flow balance of each of the tanks. For comparison purposes, the block scheme contains a linear state model whose coefficients have been derived by linearization in the starting steady-state operating point. The state model shares all the input changes applied in the non-linear model and uses the same controller setting both for the gains blocks in the state feedback and the PI controller. The integral time constant of the PI controller is set to an optional time constant τ. With this setting, the controller cancels one of the double poles in the linearized model of the cascade whose dynamics was changed by the state feedback to achieve these



a) state variables b) manipulated variable

Fig. 3 Dependence of control responses on the state feedback gains according to the point of linearization that is used – in both cases the setting of the superior PI controller is the same properties designedly. The look-up table blocks are filled by data as a part of the initial operations performed by means of callbacks within the Simulink model properties definition.

Figure 3 demonstrates some impacts on the control results caused by the nonlinear properties of a real object. In the figure on the left, the solid and dotted black lines conform to a control process with values of the state feedback gains calculated for the initial steady state. The grey solid and dotted lines show the process with values of the state feedback parameters evaluated in the final steady state. On the right, the plot compares the course of changes in valve opening for both settings of the state feedback gains. By these plots suitability of on-line adaptation of the gains can be confirmed.

7 CONCLUSION

Results obtained till now have confirmed the importance of adaptability of state feedback gains, if the feedback is applied to a real device, in order to accelerate its dynamic behaviour and in such a way make the control task for the superior controller easier. If the nonlinear properties cannot be neglected, designing the state variable feedback on the basis of linear theory will cause significant problems when used without a mechanism that reflects the parameter changes linked with a change in operating point.

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