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ROBUST DECENTRALIZED PID CONTROLLER: A CASE STUDY

NÁVRH ROBUSTNÉHO DECENTRALIZOVANÉHO PID REGULÁTORA: PRÍPADOVÁ ŠTÚDIA

Abstract

Robust stability is an important aspect in control of real world systems, since uncertainties have to be considered in dynamic system model. This paper studies the robust decentralized controller design for case study: quadruple tank process, [3,4]. Several important aspects of system analysis are shown to choose appropriate pairing and assess stabilizability via decentralized control structure; the main contribution is in decentralized discrete-time controller design. Simulation results illustrate the obtained results and their qualities.

Abstrakt

Robustná stabilita je dôležitou stránkou pri návrhu riadenia reálnych systémov. Tento príspevok sa zaoberá návrhom robustného decentralizovaného regulátora pre prípadovú štúdiu: systém štyroch nádrží, [3,4]. Príspevok ilustruje niektoré dôležité aspekty analýzy systému potrebné na správny výber párovania vstupov a výstupov umožňujúci stabilizáciu systému decentralizovaným riadením; hlavným prínosom je návrh decentralizovaného diskrétneho algoritmu riadenia. Výsledky simulácie ilustrujú vlastnosti získaných regulátorov.

1 INTRODUCTION

The basic required quality of the system is its stability in the whole uncertainty domain – this quality is called robust stability. Both in time and frequency domains, various approaches to robust stability have been developed. In this paper we use results based on small gain theorem to controller design in frequency domain and polytopic description of uncertain system which is appropriate for using LMI approach to robust control design.

The quadruple-tank process presented recently in [3,4] provides possibility to study the multivariable dynamics (two inputs and two outputs) both for minimum and nonminimum-phase configurations. In this paper the decentralized controller is designed for a model of a quadruple-tank process in frequency domain and in time domain. Results of both approaches are compared and simulation results are presented to illustrate system qualities obtained via adopted approaches.

The paper is organized as follows. In section 2, the quadruple tank process model is given and pairing and structural stabilizability are studied to provide for decentralized control design scheme. In section 3 approaches used in frequency and time domain to design robust discrete-time PID controller (denoted sometimes as PSD) are presented. The obtained results are shown in section 4 together with simulation results to verify the controller design. The proposed controllers based on linearized model has been applied to nonlinear model to check the robust stability. Section 5 is devoted to conclusions.

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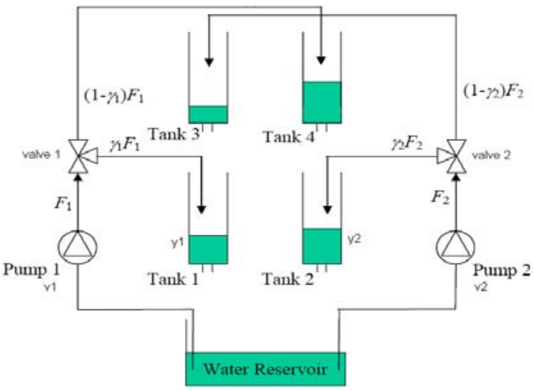
2 PRELIMINARIES AND PROBLEM FORMULATION

In this section the quadruple tank process is presented and several aspects concerning its decentralized control design, such as pairing and structural stabilizability are studied. Robust decentralized control design problem is formulated.

2.1 Model of quadruple-tank process

The quadruple-tank process shown in Fig.1 has been introduced in [3,4] to provide a case study to analyse qualities of both minimum and nonminimum phase system on the same plant. The aim is to control the level in the lower two tanks with two pumps. The inputs v_1 and v_{21} are pump 1 and 2 flows, the controlled outputs y_1 and y_2 are levels in lower tanks 1 and 2 respectively. The plant can be shifted from minimum to nonminimum phase configuration and vice versa simply by changing a valve controlling the flow ratios γ_1 and γ_2 between lower and upper tanks. The minimum-phase configuration corresponds to $1 < \gamma_1 + \gamma_2 < 2$ and a nonminimum-phase one to $0 < \gamma_1 + \gamma_2 < 1$.



the nonlinear model:

$$\frac{dh_1}{dt} = -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_1 k_1}{A_1}v_1$$
$$\frac{dh_2}{dt} = -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_2 k_2}{A_2}v_2$$
$$\frac{dh_3}{dt} = -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{(1-\gamma_2)k_2}{A_3}v_2$$
$$\frac{dh_4}{dt} = -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\gamma_1)k_1}{A_4}v_1 \qquad (1)$$

Fig. 1 Quadruple tank process scheme and nonlinear model (1)

where A_i is cross-section and a_i is cross-section of the outlet hole of tank i, h_i is water level in tank i, g is acceleration of gravity, the flow corresponding to pump i is $k_i v_i$. Parameter γ_1 denotes position of the valve dividing the pump 1 flow into the tanks 1 and 4, analogically γ_2 serves for pump 2 and tanks 2, 3. The flow to tank 1 is $\gamma_1 k_1 v_1$ and to tank 4 it is $(1-\gamma_1)k_1 v_1$, similarly for tanks 2 and 3.

The nonlinear model can be linearized around the working point given by the water levels in tanks $h_{10}, h_{20}, h_{30}, h_{40}$ using Taylor series. To obtain state space equations, state variables are defined as differences $x_i = h_i - h_{i0}$, the respective control variables are $u_i = v_i - v_{i0}$. The linearized model is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_3 \\ \dot{x}_2 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} \frac{-1}{T_1} & \frac{A_3}{T_3 A_1} & 0 & 0 \\ 0 & \frac{-1}{T_3} & 0 & 0 \\ 0 & 0 & \frac{-1}{T_2} & \frac{A_4}{T_4 A_2} \\ 0 & 0 & 0 & \frac{-1}{T_4} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_3 \\ x_2 \\ x_4 \end{bmatrix} + \begin{bmatrix} \frac{\gamma_1 k_1}{A_1} & 0 \\ 0 & \frac{(1-\gamma_2)k_2}{A_3} \\ 0 & \frac{\gamma_2 k_2}{A_1} \\ \frac{(1-\gamma_1)k_1}{A_4} & 0 \end{bmatrix} \cdot \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad \text{where } T_i = \frac{A_i}{a_i}\sqrt{\frac{2h_{i0}}{g}}, \; i = 1,\dots,4 \qquad (2)$$

.The argument t has been omitted; the state variables corresponding to levels in tanks 2 and 3 has been interchanged in state vector so that subsystems respective to input u_1 from pump 1 (tanks 1 and 3) and u_2 from pump 2 (tanks 2 and 4) are more apparent. This decomposition into two subsystems is used later in decentralized control design.

The respective transfer function matrix having inputs v_1 and v_2 and outputs y_1 and y_2 is

$$G(s) = \begin{bmatrix} \frac{c_1\gamma_1}{T_1 s+1} & \frac{c_1(1-\gamma_2)}{(T_3 s+1)(T_1 s+1)} \\ \frac{c_2(1-\gamma_1)}{(T_4 s+1)(T_2 s+1)} & \frac{c_2\gamma_2}{T_2 s+1} \end{bmatrix} \quad \text{where } c_i = \frac{T_i k_i}{A_i}\sqrt{\frac{2h_{i0}}{g}}, \; i = 1,2 . \tag{3}$$

Plant zeros for (3) can be obtained from det($G(s)$)=0:

$$\det(G(s)) = \frac{c_1 c_2 \gamma_1 \gamma_2}{(T_1 s+1)(T_2 s+1)(T_3 s+1)(T_4 s+1)}\left[(T_3 s+1)(T_4 s+1) - \frac{(1-\gamma_1)(1-\gamma_2)}{\gamma_1\gamma_2}\right] \tag{4}$$

using denotation $\eta = \frac{(1-\gamma_1)(1-\gamma_2)}{\gamma_1\gamma_2}$, we have nonminimum phase system (with zero in right half plane) for $\eta > 1$, i.e. for $0 < \gamma_1 + \gamma_2 < 1$ and minimum phase for $\eta < 1$, i.e. for $1 < \gamma_1 + \gamma_2 < 2$.

2.2 Decentralized control of quadruple tank

The control aim for quadruple tank is to reach the given level in the lower two tanks, i.e. prescribed values of y_1 and y_2 by controlling input flow v_1 and v_2 delivered by two pumps. To achieve this aim, the decentralized control structure should be employed, with two control loops respective to output values y_1 and y_2; therefore the appropriate input-output pairing has to be chosen for both configurations. Using steady-state RGA index, for $1<\gamma_1+\gamma_2<2$ (minimum phase system) the indicated pairing is $v_1 - y_1$, $v_2 - y_2$; for $0<\gamma_1+\gamma_2<1$ (nonminimum phase system), the opposite pairing $v_1 - y_2$, $v_2 - y_1$ is indicated. This result is approved by Niederlinski index:

Our control design aim is to find robust decentralized PID controller for respective pairing for both minimum and nonminimum phase configurations appropriate for specified uncertainty region.

3 ROBUST DECENTRALIZED DISCRETE-TIME PID CONTROLLER DESIGN

The robust decentralized PID controller is designed in this section for quadruple tank process linearized model both in frequency and time domains. In frequency domain, small gain theorem based approach is adopted to formulate the test for stability robustness. In time domain, the LMI approach is used to incorporate the uncertainties into controller design procedure. In this case, polytopic uncertainty domain is considered, which yields from considered different working points.

3.1 Model of control system

In state space, the uncertain closed-loop polytopic system is considered in general form

$$x(k+1) = A_C(\alpha)x(k) \tag{5}$$

where

$$A_C(\alpha) \in \left\{\sum_{i=1}^{N}\alpha_i A_{Ci}, \quad \sum_{i=1}^{N}\alpha_i = 1, \quad \alpha_i \geq 0\right\}, \qquad A_{Ci} = A_i + B_i FC. \tag{6}$$

with decentralized feedback control law $\quad u(k) = FCx(k) \quad$ (7)

where F is a block diagonal matrix conforming to the structure of B and C.

A discrete-time PID (PSD) controller is

$$u(k) = k_P e(k) + k_I \sum_{i=0}^{k} e(k) + k_D[e(k) - e(k-1)] \tag{8}$$

where $u(k)$ is control variable, $y(k)$ is controlled (output) variable, $e(k)$ is control error $e(k) = w - y(k)$, w is reference value; k_P, k_I, k_D are controller parameters to be designed.

The respective description in state space is

$$z(k+1) = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} z(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} e(k) = A_R z(k) + B_R e(k)$$
$$u(k) = \begin{bmatrix} k_D & k_I - k_D \end{bmatrix} z(k) + (k_P + k_I + k_D) e(k) \tag{9}$$

Combining (5) and (9) the augmented closed loop system is received as

$$\begin{bmatrix} x(k+1) \\ z(k+1) \end{bmatrix} = \begin{bmatrix} A + \delta A & 0 \\ -B_R C & A_R \end{bmatrix} \begin{bmatrix} x(k) \\ z(k) \end{bmatrix} + \begin{bmatrix} B + \delta B \\ 0 \end{bmatrix} \begin{bmatrix} -K_2 & K_1 \end{bmatrix} \begin{bmatrix} C & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} x(k) \\ z(k) \end{bmatrix} \tag{10}$$

where $K_2 = (k_P + k_I + k_D)$, $K_1 = \begin{bmatrix} k_D & k_I - k_D \end{bmatrix}$.

When decentralized controller is considered, the state space model has block structure respective to individual inputs and outputs with diagonal blocks structured according to (10).

3.2 Robust control design in frequency domain

In frequency domain, the robust PID controller is designed via common approach, based on small gain theorem applied on uncertain system, the robust stability condition is

$$\left\| W G_D^{-1} \right\| < \frac{1}{l \left\| G_M \right\|} \tag{11}$$

where nominal system matrix has been divided into its diagonal and off-diagonal part: $G_0 = G_D + G_M$; $l = \max_k \sigma_M (G_0 - G_k)$, G_k is system matrix in working point k.

The approach described above is based on Nyquist stability criterion which has analogous formulation for discrete-time systems, formally same formulas can be derived for robust discrete-time controller. The inverse dynamics approach is used to design controllers in individual loops, [8].

3.3 Robust control design in state space

We consider the state space system model with polytopic uncertainties (5), (6). In the development of robust discrete time controller in state space the Lyapunov stability condition (12) for closed loop system (5) is employed using parameter dependent Lyapunov function (13).

$$A_C^T(\alpha) P(\alpha) A_C(\alpha) - P(\alpha) < 0 \tag{12}$$

$$P(\alpha) = \sum_{i=1}^{N} \alpha_i P_i \qquad \text{where } P_i = P_i^T > 0 \tag{13}$$

The proposed PSD controller design scheme is based on results of [1] and [2]. The static output feedback controller (SOF) is obtained solving LMI (14)-(16) for unknown matrices F, M, G and P_i of appropriate dimensions, the P_i being symmetric, M, G are block diagonal with block dimensions conforming to subsystem dimensions (for quadruple tank system model has 4 states, two subsystems are 2x2).

$$\begin{bmatrix} -P_i & A_i G + B_i K C \\ G^T A_i^T + C^T K^T B_i^T & -G - G^T + P_i \end{bmatrix} < 0, \quad i = 1, \dots, N \tag{14}$$

$$MC = CG \tag{15}$$

Compute the corresponding output feedback gain matrix

$$F = KM^{-1} \qquad \text{where } F = blockdiag\{[-(k_{Pi} + k_{Ii}) \quad k_{Ii} - k_{Di}]\} \tag{16}$$

The algorithm above is quite simple and often provides reasonable results. Another possibility for SOF design has been employed: the iterative solution of robust stability condition (17) alternatively for unknown P_i , G and F, G (see [6]).

$$\begin{bmatrix} -P_i & A_i G + B_i FCG \\ G^T A_i^T + G^T C^T F^T B_i^T & -G - G^T + P_i \end{bmatrix} < 0, \quad i = 1,\ldots,N \tag{17}$$

4 DECENTRALIZED DISCRETE-TIME PID CONTROLLER FOR QUADRUPLE TANK

The robust decentralized PID controller for quadruple tank process linearized model has been designed in this section using robust conrol approaches described in sections 3.2 and 3.3, the results have been verified by simulation on nonlinear model. The relevant part is after water levels reach the nominal steady state values. The following changes around the working point test the closed loop performance qualities (in dashed circles).

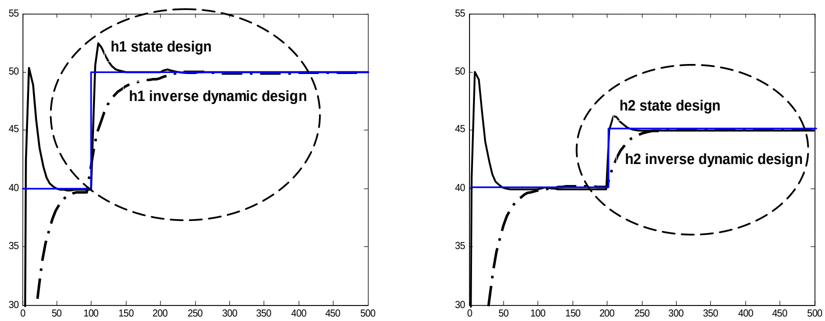


Fig. 2 Comparison of step responses for nonlinear model with designed controllers (inverse dynamics design, robust state-space design); minimum-phase case

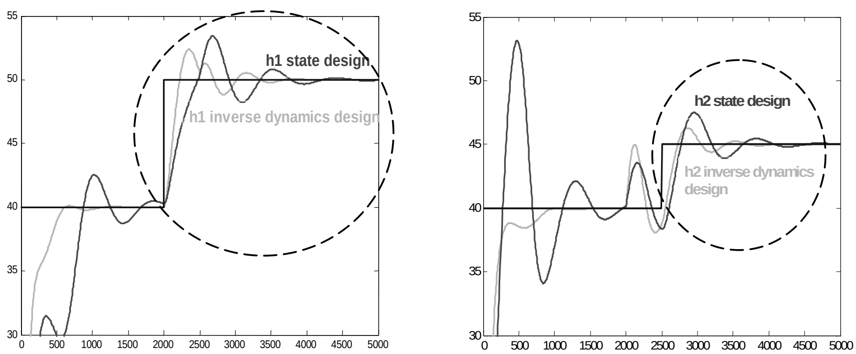


Fig. 3 Comparison of step responses for nonlinear model with designed controllers (inverse dynamics design, robust state-space design); nonminimum-phase case

As shown in Fig.2 and 3, state space design provides quicker response, however with overshoot. Inverse dynamics yields significantly slower response.

The situation changes rapidly for nonminimum-phase configuration, which is much more difficult for controller design. In this case, inverse dynamics is superior to state space approach, as far as maximal overshoot and settling time are considered.

5 CONCLUSION

The robust decentralized PS controller has been designed both in frequency and time domain for quadruple-tank process model. The LMI based design of static output feedback controller provides good results for minimum-phase configuration, verified on nonlinear process. The nonminimum-phase case prefers inverse dynamics approach, state space design yields in this case too big integration constant, therefore oscillating response. The question of appropriate SOF design procedure for this case remains open. The provided results in frequency domain will be used also in teaching complex systems control in degree course.

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