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RELAY TEST FOR ANISOCHONIC MODELS – TIME DOMAIN SOLUTION

RELÉOVÝ TEST PRO ANIZOCHRONNÍ MODELY – ŘEŠENÍ V ČASOVÉ OBLASTI

Abstract

Anisochronic models are characteristic for containing state delays. These models have some practical and attractive features; e.g. they enable to fit the dynamics of systems with very high order. This contribution utilizes an idea of anisochronic models identification based on limit cycle information obtained from relay feedback test is investigated. Unlike conventional approaches connected with the frequency analysis of a plant transfer function, the proposed alternative methodology is based on computation with functional differential equation only, i.e. in time domain. Plant parameters to be identified are obtained analytically. In addition, parameter estimation is also improved using autotune variable (ATV^+) technique which required an additional delay element. An illustrative example where parameters of a tenth order system are approximated by a first order anisochronic model is presented.

Abstrakt

Anizochronní modely jsou význačné tím, že obsahují zpoždění stavových veličin. Tyto modely mají některé zajímavé vlastnosti – například umožňují vystihnout dynamiku konvenčních soustav vyšších řádů a dále matematické modely mnoha procesů vedou právě na anisochronní modely. V tomto příspěvku je prezentována myšlenka identifikace těchto modelů pomocí reléového experimentu. Narozdíl od tradičního pojetí je zde uveden postup pro odhad parametrů modelu přímo z diferenciální rovnice, tedy v časové oblasti. Reléový test je dále vylepšen pomocí metody ATV^+ využívající umělého zpoždění. Ilustrační příklad, kde je systém 10. řádu aproximován anizochronním modelem 1. řádu, demonstruje uvedenou metodiku.

1 INTRODUCTION

In recent years there has been a growing interest in studying a class of systems containing both input and state delays. This endeavor is natural in the light of the fact that many industrial processes, e.g. in chemical processes [1], heat exchange networks [2] or in internal combustion engines with catalytic converter [3], etc have this feature. For such systems the notion of *anisochronic* systems was introduced, which expresses the non-synchronous effect of state variables. Unlike models containing input delays only, which are described in the form of ordinary differential equations, anisochronic models are expressed by the functional differential equations (FDEs). The essential feature of these models is the fact that their dynamics is described by both accumulations (integrations) and delays.

Anisochronic models are useful even in the case of absence of state delays in a system. Since these models have an infinite number of poles of the particular transfer function, they can be successfully utilized for estimating of “very high” order dynamics of an original system. For the same plant, an anisochronic model needs a lower number of state variables than a conventional model.

Despite the fact that the systems with both input and state delays are quite frequent, there is still a lack of practical and engineer admissible identification procedures, [1]. On the other hand,

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control designs cover broader spectrum of principles, see e.g. [1], [4]. The emphasis of this paper is placed on the identification of anisochronic systems. Utilization of successive integrations in the identification procedure is described in [5]. A brief description of the identification procedure based on a relay experiment and settled limit cycles is in [6]. The traditional evaluation of the limit cycles was based on the frequency domain description of an open loop. In contrast to this procedure, the method proposed in this contribution employs a plant description in time domain. It is well known that only one point of the Nyquist curve from a standard relay test can be identified. In case when a model has more than two unknown parameters, additional points of the Nyquist curve need to be identified. This is made possible by introducing a delay element between the relay and the process and performing other relay tests. This identification procedure was labeled as ATV+ (Auto-Tune Variations), see [7] – [8].

This contribution deals with a simple anisochronic model of four unknown parameters. Thus, two relay tests (with and without an additional delay) are to be performed for the parameter estimation. The final conditions are in the form of the set of four non-linear algebraic equations. These equations can be solved numerically, which requires a suitable initial estimation of the solution. The initial parameter values are done from the first relay test; however, this primary estimation is not based on identification of points of the Nyquist curve.

The obtained analytical and numerical results are verified by an illustrative example in which parameters of a tenth order system are approximated by a first order anisochronic model.

2 ANISOCHRONIC MODELS

Anisochronic models of so-called retarded type can be described in a single-input single output (SISO) [9] in the summation form

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{A}_0\mathbf{x}(t) + \sum_{i=1}^{l}\mathbf{A}_i\mathbf{x}(t-\vartheta_i) + \mathbf{b}_0u(t) + \sum_{j=1}^{k}\mathbf{b}_ju(t-\tau_j);\;\; y(t) = \mathbf{C}\mathbf{x}(t) \tag{1}$$

where:

$\vartheta_i > 0$, $\tau_j > 0$ – lumped delays,

$u(t)$, $x(t)$ – input and state variables, respectively.

The advantage of these models is the possibility to express a model in the form of transfer function:

$$G(s) = \frac{N(s)}{M(s)} = \frac{Y(s)}{U(s)} = \frac{\mathbf{C}\mathrm{adj}\left[s\mathbf{I} - \mathbf{A}_0 - \sum_{i=1}^{l}\mathbf{A}_i\exp(-s\vartheta_i)\right]\left[\mathbf{b}_0 + \sum_{j=1}^{k}\mathbf{b}_j\exp(-s\tau_j)\right]}{\det\left[s\mathbf{I} - \mathbf{A}_0 - \sum_{i=1}^{l}\mathbf{A}_i\exp(-s\vartheta_i)\right]} \tag{2}$$

where adj[] designates adjoint matrix.

Both, the numerator and denominator of the transfer function contain exponential terms. On account of this fact, $N(s)$ and $M(s)$ are *quasipolynomials* instead of polynomials. The transcendental character of quasipolynomial $M(s)$ ensures infinitely many poles, which can be used while endeavor to fit the dynamics of high-order plants.

Let us study the following particular plant:

$$G(s) = \frac{N(s)}{M(s)} = \frac{Y(s)}{U(s)} = \frac{K\exp(-\tau s)}{Ts + \exp(-\vartheta s)} \;\Leftrightarrow\; Ty'(t) + y(t-\vartheta) = Ku(t-\tau) \tag{3}$$

The time constant, T, estimates the maximum slope of the step response, τ approximate dead time and ϑ the inflex point position. With respect to the first order of the model, it is not suitable for fitting weakly damped oscillatory systems.

3 FEEDBACK RELAY TEST IDENTIFICATION

The relay feedback identification test, yielding the limit cycle oscillations, is widely used and in practice a well applicable technique; see e.g. [10] – [11]. The classical feedback loop scheme is depicted in Fig. 1. The goal of the test is to indicate the critical point in the Nyquist curve of a process. When the oscillations are settled, the amplitude A of error $e(t)$ equals the amplitude of output $y(t)$ and the phase shift between $e(t)$ and $y(t)$ is $-\pi$, see Fig. 2. Thus the following condition holds:

$$\left|R(A)G(\mathrm{j}\omega_u)\right| = 1; \quad \arg\left[R(A)G(\mathrm{j}\omega_u) =\right] - \pi \tag{4}$$

where:

$\omega_u = 2\pi / T_u$	– the ultimate frequency
T_u	– the period of the oscillations,
$R(A)$	– the equivalent transfer function of a relay
$G(\mathrm{j}\omega_u)$	– the frequency transfer function of a plant.

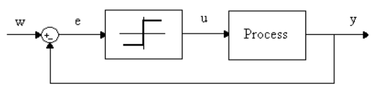

Fig. 1 Standard relay feedback test

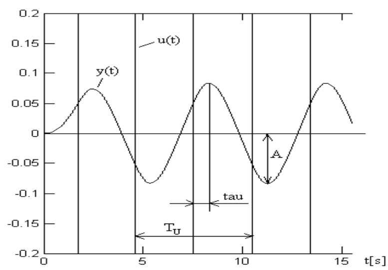

Fig. 2 Relay test settled oscillations

A relay is a non-linear element and it can be linearized for linear theory approach. The linearization is done via Fourier series approximation when upper harmonic components of the signal are neglected. Static characteristic of the on-off relay without hystheresis is in Fig. 3

If a harmonic signal of amplitude A enters the relay, the equivalent transfer function (gain) is

$$R(A) = (\pi A)^{-1} 4B \tag{5}$$

Standard test enables to estimate only two of unknown parameters K, T, τ, ϑ of model (3).

The static gain K can easily be estimated from the step response or using a biased relay [6] as

$$K = \left[\int_0^{T_u} u(t)dt\right]^{-1} \int_0^{T_u} y(t)dt\,;\ T_u = (\omega_u)^{-1} 2\pi \tag{6}$$

The value of a dead time τ can be deduced directly from steady state oscillations, see Fig. 2. Remaining model parameters, T and ϑ, can be obtained by direct calculation on (4) for model (3).

3.1 Alternative parameter identification from limit cycles

The following approach utilizes a FDE description in the time domain instead of transfer function. Rectangular waves on a plant input can be approximated by sinus waves using linearization (5)

$$u(t) = u_0 \sin(t\omega_u) = \pi^{-1} 4B \sin(t\omega_u)\,; \quad B = 0.5(B^+ + B^-). \tag{7}$$

Since a biased relay (5) does not evoke a phase shift, a plant output has a phase shift $-\pi$, i.e.

$$y(t) = y_0 \sin(t\omega_u) = -A \sin(t\omega_u) \tag{8}$$

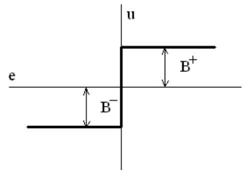

Fig. 3 On-off relay without hystheresis

Obviously, $\mathrm{sgn}(u_0) \neq \mathrm{sgn}(y_0)$. Hence, FDE (3) with respect to (7) and (8) reads

$$Ty_0\omega_u\cos(t\omega_u) + y_0\sin[(t-\vartheta)\omega_0] = Ku_0\sin[(t-\tau)\omega_0] \tag{9}$$

Now placing the appropriate time values into (9), the relations for T and ϑ can be derived:

Step 1: Let $t\omega_u = k2\pi;\ k = 0,1,2...$; i.e. $t = \omega_u^{-1}(2k\pi)$ and k be chosen so that $t > \max\{\tau, \vartheta\}$ and the limit cycle is stable. Then (9) gives

$$Ty_0\omega_u - y_0\sin(\vartheta\omega_u) + Ku_0\sin(\tau\omega_u) = 0 \quad \Rightarrow \quad T = \omega_u^{-1}\left[\sin(\vartheta\omega_u) - y_0^{-1}Ku_0\sin(\tau\omega_u)\right] \tag{10}$$

Step 2: Let $t\omega_u = \frac{\pi}{2} + k2\pi;\ k = 0,1,2...;\ t > \max\{\tau, \vartheta\}$ and the limit cycles are settled. Then

$$-y_0\cos(\vartheta\omega_u) + Ku_0\cos(\tau\omega_u) = 0 \quad \Rightarrow \quad \vartheta = \omega_u^{-1}\left[\arccos\left[\pi + y_0^{-1}Ku_0\cos(\tau\omega_u)\right]\right] \tag{11}$$

Computation of parameters estimation described above in time domain using FDE (3) is somewhat easier than solution of amplitude and phase shift conditions in frequency domain. The proposed methodology for parameters estimation of model (3) is generally applicable for other, more complex, anisochronic models as well, e. g. for remarkably oscillatory processes.

3.2 Modified relay test using additional delay element

Estimation of the static gain K and a dead time τ described previously are not based on the primary relay test information, i.e. ω_u and A. Utilization of knowledge of the ultimate frequency ω_u and amplitude A for the estimation of other model parameters requires a special technique. One of the possibilities is to use the ATV$^+$ technique [7] – [8]. The first step of the ATV$^+$ procedure is a standard relay test. The second step introduces a delay τ^+ between the relay and the process. The overall phase shift is $-\pi$, however only a part of this is attributed to the process, as τ^+ is characterized by the phase leg $\phi_D = \tilde{\omega}_u\tau^+$ where $\tilde{\omega}_u$ is the new ultimate frequency. The new amplitude $\tilde{A}$ of the output can be read as well. Every next setting of τ^+ determines one point of the Nyquist curve. In [7] it is suggested that $\tau^+ = 5\pi(12\omega_u)^{-1}$. Now the ATV$^+$ method can be used together with the technique described in section 3.1. Insert delay element τ^+ and let (7) holds with $\tilde{\omega}_u$ instead of ω_u and

$$y(t) = \tilde{y}_0\sin(t\tilde{\omega}_u + \phi_D) = \tilde{A}\sin(t\tilde{\omega}_u - \pi + \phi_D),\ \ \mathrm{sgn}(\tilde{u}_0) \neq \mathrm{sgn}(\tilde{y}_0) \tag{12}$$

Inserting the previous equations into (3), it is obtained:

$$T\tilde{y}_0\tilde{\omega}_u\cos(t\tilde{\omega}_u + \phi_D) + \tilde{y}_0\sin[(t-\vartheta)\tilde{\omega}_u + \phi_D] = Ku_0\sin[(t-\tau)\tilde{\omega}_u] \tag{13}$$

Using relay test without delay element, it is possible to estimate two unknown parameters of the model, i.e. T and ϑ. Thus it is demanded to run one additional test with an artificial delay for identifying K and τ. In the same way as in section 3.1., we can obtain from ATV$^+$ test the following:

$$T\tilde{y}_0\tilde{\omega}_u\cos(\phi_D) + \tilde{y}_0\sin(\phi_D - \vartheta\tilde{\omega}_u) + Ku_0\sin(\tau\tilde{\omega}_u) = -T\tilde{y}_0\tilde{\omega}_u\sin(\phi_D) + \tilde{y}_0\cos(\phi_D - \vartheta\tilde{\omega}_u) - Ku_0\cos(\tau\tilde{\omega}_u) = 0 \tag{14}$$

Hence, initial estimation of K and τ according to (6) and Fig. 2 and estimation of T and ϑ using (10) and (11) can be put more precisely by solution of the set of equations (10), (11) and (14). This estimation is obtained using limit cycle information, i.e. ultimate gain and ultimate frequency, only.

4 NUMERICAL SOLUTION

There are indeed many possibilities how to solve the sets of nonlinear equations. Due to the limit space, there is only a short remark about numerical solution of the set of equations (10), (11) and (14) here.

Traditional, Newton method, cannot be used due to the fact that Jacobi matrix of the set of equations is ill-conditioned. Thus we utilized Regula Falsi method (with a minor modification) and gradient method which minimizes the sum of squares of left hand sides of the equations. Principles of used method can be found e.g. in [12].

5 ILLUSTRATIVE EXAMPLE

Suppose a tenth order plant

$$G_S(s) = \frac{1}{(2s+1)^{10}} \tag{15}$$

which can be identified as a model (3) by a relay feedback experiment.

The first relay test is made without an additional delay. The relay with parameters $B^+ = 0.5$, $B^- = 0.2$ (i.e. $u_0 = 0.382$) gives the limit cycles values $T_u = 38.43$, i.e. $\omega_u = 0.164$, and $y_0 = -0.231$. Then the static gain is given by (6) as K=1. The value of a dead time was assed from Fig. 2 as $\tau = 11.21$. The calculation of T and ϑ according to (10) and (11) gives $T = 15.3$ and $\vartheta = 6.89$.

The second step inserts a delay element between a relay and a plant. The delay is calculated as $\tau^+ = 8$. Stable oscillations then afford $\widetilde{\omega}_u = 0.115$, and $\widetilde{y}_0 = -0.284$. The results of solution of (10), (11) and (14) are as follows: Regula Falsi: $\tau = 11.14$, $T = 15.34$ and $\vartheta = 6.65$. Gradient method: $\tau = 11.82$, $T = 14.04$ and $\vartheta = 5.28$. Graphical comparative results are in Fig. 4.

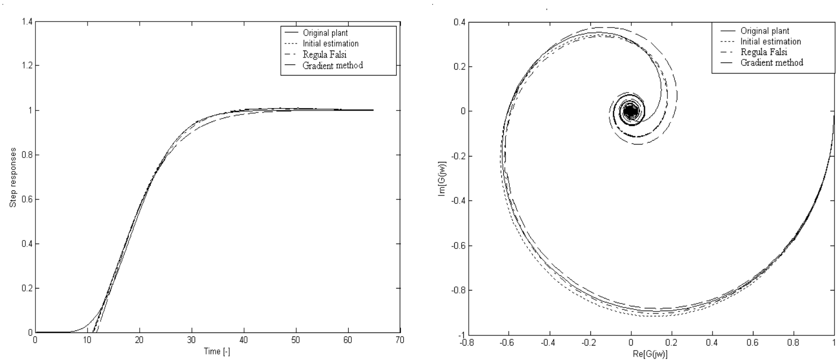

Fig 4 Comparison of step responses and Nyquist plots – the original plant and models

The simulations demonstrate the ability of the proposed method to estimate parameters of an anisochronic model and to fit the dynamics of a conventional "high order" system. Regula Falsi results in better approximation in time domain. However, ATV^+ did not improve the initial estimation in frequency domain. Recall that the main advantage of ATV^+ rests in calculation of model parameters using ultimate gain and ultimate frequency only. It is also usable in case of more complicated models.

6 CONCLUSIONS

This contribution offers an identification procedure for anisochronic systems based on a feedback relay test. Anisochronic models can describe then dynamics of conventional high order systems. In order to simplify the idea explanation, a first order system is discussed. By contrast to a traditional procedure, parameters estimation stems from the knowledge of the time domain model description and the limit cycle information. The identification technique is then extended using ATV^+ methodology which introduces another relay test and enables to use ultimate data only; no other information is needed. The limit cycle information from the second test is utilized for identification of model parameters and refines on the initial parameter estimation.

An illustrative example demonstrates the usability of the proposed method. The identification methodology is applicable to other anisochronic models as well. The future research should improve the identification philosophy and apply it to a real process.

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