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MODELING TWO PHASES OF REFINING PROCESS OF ANIMAL FAT

MODELOVÁNÍ DVOU FAZÍ RAFINACE ŽIVOČIŠNÉHO TUKU

Abstract

This work describes the mathematical model of both phases of the refining process of animal fat – the phase of heating and the phase of melting. In order to prepare biodiesel using animal fat, it is necessary to heat the solid fat to the melting point, and then melt it until the whole fat becomes liquid. Points of interest are the temperature of fat depending on time during heating and the amount of changing the dimension of the fat portion during the melting subprocess. The first described model is valid only until the temperature of fat reaches the melting point. Afterwards, the second phase of refining process starts and it is modeled differently.

Abstrakt

Tato práce popisuje matematický model obou fází rafinace živočišného tuku – fázi ohřívání a fázi tání. Abychom mohli připravit biodiesel s použitím živočišného tuku, je nutné zahřát pevný tuk do bodu tání, a poté ho rozpouštět tak dlouho, dokud celý nezkapalní. Významnými body jsou jednak teplota tuku závisející na době ohřevu a množství střídání jeho objemu během tání. První popsaný model je platný pouze v případě, kdy teplota tuku dosáhne bodu tání. Poté nastává druhá fáze rafinace a ta se formuje různě.

1 INTRODUCTION

In the last few decades, because of the global industrial production, world is increasingly consuming energy. As the consumption is increasing, the energy production is reaching its limit. Most of the fuel used today originates from the limited fossil sources: oil, coal and gas. In order to enable the rapid increase of energy production, it is necessary to use nonfossil fuels as a main unlimited source for producing energy.

One of the main nonfossil fuels is biodiesel. Various different substances can be used for making biodiesel. Commercially available biodiesel is basically made of fresh vegetable oil, but it is also possible to produce biodiesel using animal fat or even used frying oil instead. The high amount of comprised energy is common to all the mentioned agents.

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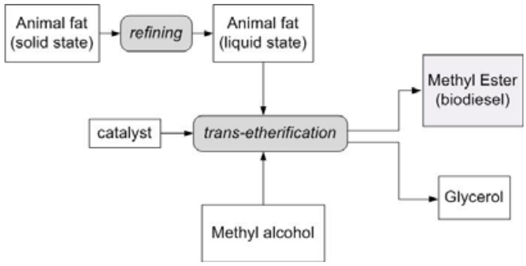

Fig. 1 Production of biodiesel

In order to prepare biodiesel using animal fat, it is necessary to heat the solid fat to the melting point, and then slowly melt it until the whole fat becomes liquid. Fat prepared this way with a certain amount of alcohol and some catalyst substance is ready for a chemical reaction of trans-etherification. As a result of it, biodiesel and glycerol are produced (figure 1).

In this contribution we focus on obtaining the mathematical models that can predict the exact temperature dependence during the heating of animal fat and also the dependence of dimension being decreased on time during melting.

2 FIRST PHASE - MODELING OF HEAT TRANSFER DURING HEATING

Conduction, convection and radiation are three distinct methods by which transfer of heat takes place. Only a process of conducting heat occurs in solid bodies; therefore, our model of heat transfer through the fragment of animal fat will consider conduction of heat only.

Equipment for the analysed heating process would be the simplest possible. In a chamber we would have a portion of animal fat, heater and a ventilator (figure 2). We would consider the chamber as an absolutely isolated from the ambience and portion of fat of the ideal mathematical shape. The intention of the ventilator is to make the temperature of the air uniform in the whole chamber.

Having all those assumptions prior to modeling we can discus models for different mathematical shapes of portions of animal fat.

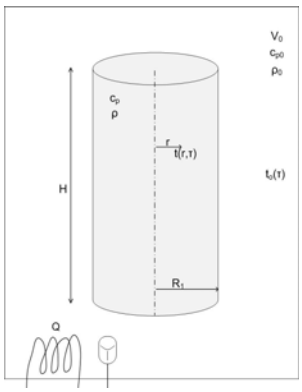

Fig. 2 Cylindrical portion of fat being heated

First, the cylindrical shape would be analysed. We would consider that the dimensions of the cylinder are such that the surface of the basis is much smaller than the surface of the side. In other words, we would consider that the heat is transferring only through the side of the cylinder.

Parameters used in the model:

R_1 – radius [m], H – height [m], ρ – density of the animal fat $\left[\frac{kg}{m^3}\right]$, c_p – heat capacity of the fat $\left[\frac{J}{kg\ °C}\right]$, V – volume of the cylinder $\left[m^3\right]$, V_0 – volume of the air in the chamber $\left[m^3\right]$, c_{p0} – heat capacity of air $\left[\frac{J}{kg\ °C}\right]$, ρ_0 – density of air $\left[\frac{kg}{m^3}\right]$, $\dot{Q}$ – power of the heater $\left[\frac{J}{s}\right]$, τ – time [s], $t(r,\tau)$ – temperature of fat in the point with coordinate r in the moment τ $[°C]$, $t_0(\tau)$ – temperature of the air in the chamber $[°C]$, λ – thermal conductivity of fat $\left[\frac{J}{s\,m\,°C}\right]$ and a – thermal diffusivity $\left[\frac{m^2}{s}\right]$.

$$\frac{\partial t(r,\tau)}{\partial \tau} = a\left[\frac{\partial^2 t(r,\tau)}{\partial r^2} + \frac{1}{r}\frac{\partial t(r,\tau)}{\partial r}\right], \tag{1}$$

$$\frac{\partial t(0,\tau)}{\partial r} = 0, \tag{2}$$

$$t(R_1,\tau) = t_0(\tau), \tag{3}$$

$$t(r,0) = t_0(0) = t_p, \tag{4}$$

$$\dot{Q} = V_0 \rho_0 c_{p0} \frac{dt_0(\tau)}{d\tau} + \lambda S \frac{\partial t(R_1,\tau)}{\partial r}, \quad S = 2\pi R_1 H. \tag{5}$$

This model is valid only under conditions: $0 < r < R_1$, $\tau > 0$, $t < t_m$; where t_m is the temperature of melting of fat. The first equation has been obtained from Fourier's law of heat conduction for the cylindrical coordinate system. Boundary condition (2) expresses the presumed axial symmetry of the portion of fat. Boundary condition (3) claims that the temperature of surrounding air is the same like the temperature of the surface of the cylinder. This equation can be obtained from Newton's law of linear cooling $\lambda \frac{\partial t(R_1,\tau)}{\partial r} + \alpha\big(t(R_1,\tau) - t_0(R_1,\tau)\big) = 0$ when the coefficient of surface heat transfer λ tends to infinity. Initial condition (4) is always accomplished if we leave the fat in the air surrounding long enough. Equation (5) comes from the law of conservation of energy.

Using the proposed model we want to obtain the dependence of temperature on time and coordinate. First, we introduce the dimensionless time $F_0 = \frac{a\tau}{R_1^2}$, coordinate $R = \frac{r}{R_1}$ and temperatures $T = \frac{t}{t_p}$ and $T_0 = \frac{t_0}{t_p}$. After employing the Laplace transformation we finally obtain the dependence of dimensionless temperature of fat on dimensionless coordinate and dimensionless time

$$T(R,F_0) = 1 + \frac{2F_0 P}{2A+1} + \frac{P \cdot R^2}{4A+2} - \frac{P \cdot (4A+1)}{4 \cdot (2A+1)^2} + Sum,$$
$$Sum = \sum_{n=1}^{\infty} \frac{4P \cdot J_0(R \cdot q_n) e^{-q_n^2 F_0}}{2A \cdot q_n^3 \cdot J_1(q_n) - q_n^2 \left(J_0(q_n) - J_2(q_n)\right) - 2q_n J_1(-q_n)} \tag{6}$$

where A and P are constants: $A = \dfrac{aV_0 c_{p0} \rho_0}{3\lambda V}$, $P = \dfrac{\dot{Q} R_1^2}{3Vt_p \lambda}$ and q_n are roots of transcendental equation $A \cdot J_0(q) + \dfrac{J_1(q)}{q} = 0$.

If we use the real physical parameters $\lambda = 0.35 \dfrac{W}{m\,K}$, $\rho = 900 \dfrac{kg}{m^3}$, $c_p = 2000 \dfrac{J}{kg\,°C}$, $a = \dfrac{\lambda}{c_p \rho} = 1.94 \cdot 10^{-7} \dfrac{m^2}{s}$, $t_p = 20\,°C$, $V = 0.1 m^3$ $\left(R_1 = 0.126 m,\, H = 2m\right)$, $V_0 = 1 m^3$, $\rho_0 = 1.2 \dfrac{kg}{m^3}$, $c_{p0} = 1005 \dfrac{J}{kg\,°C}$, $\dot{Q} = 300 \dfrac{J}{s}$ we will get the graph of dependence of temperature (figure 3).

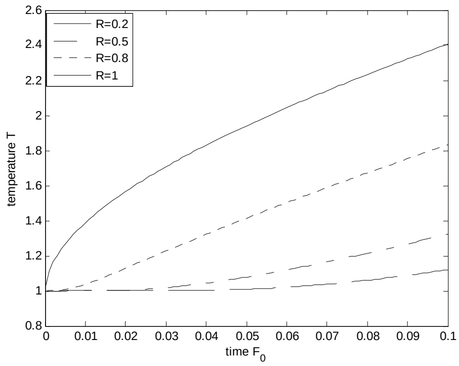

Fig. 3 Temperature of the cylinder-shaped fat depending on time F_0 and coordinate R

Melting point of fat is at about $40\,°C$; therefore, this model is valid when $T < 2$. According to the graph, melting starts at the moment $F_{01} = 0.0555$, that is $t = 4530\,s$ (about 75min). Temperature of the points which have coordinate $R = 0.8$ at the moment F_{01} is $T = 1.46$ which corresponds to the $29.2\,°C$. It means that 36% of the volume of fat was heated by at least $9.2\,°C$.

In order to examine how the shape of fat affects the temperature distribution inside the fat, we will analyse two more shapes – plane and sphere. The principles of proposing those models are similar to the already described model for the cylinder, so here would be presented only the final dependencies. In order to justly compare the temperature distribution in various shapes of portion of fat, we will choose the coordinates of plane and sphere which intersect the portion to the equal volumes like in the case of cylinder. The figures 4 and 5 show the temperature distribution for the cases of plane and sphere.

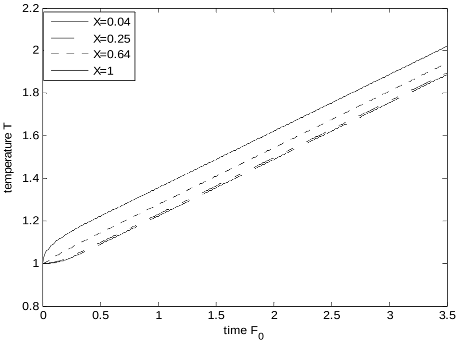

Fig. 4 Temperature of the plane-shaped fat depending on time F_0 and coordinate X

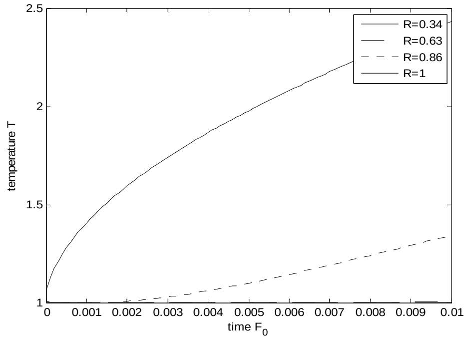

Fig. 5 Temperature of the sphere-shaped fat depending on time F_0 and coordinate R

Dependence of temperature distribution on the power of the heater is also worth discussion.

3 SECOND PHASE – MODEL OF MELTING

During this phase of the process the portion of fat changes its dimension. It will be assumed that it keeps the ideal mathematical shape and the model for the cylindrical shape (figure 6) would be developed.

New parameters used in the model:

L – heat of fusion for fat $\left[\frac{J}{kg}\right]$, y – current radius of cylinder, $t(y)$ - temperature of fat on the surface, S – surface of the cylinder side.

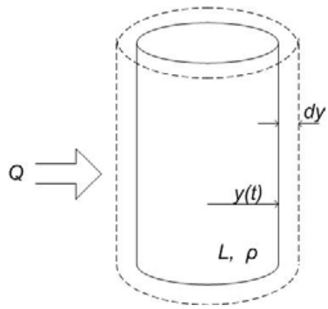

Fig. 6 Melting of the cylinder-shaped fat

Model of melting is presented by equations (1), (7) - (8). Equation (9) is equivalent to the equation (7) and it shows how the dimension of cylinder changes.

$$\dot{Q}dt = \lambda S \frac{\partial T(y(t))}{\partial y} dt + Sdy\rho L \,, \tag{7}$$

$$S = 2\pi H y(t) \,, \tag{8}$$

$$S\rho L \frac{dy}{dt} = \dot{Q} - \lambda S \frac{\partial T(y)}{\partial y} \,. \tag{9}$$

Equation (7) is obtained from the law of conservation of energy. The first clause on the right side represents the transfer of heat through the surface and the second one is the amount of energy used for melting the surface layer of fat.

It was not possible to obtain the solution (time dependence of the cylinder's radius), so the numerical method (discrete equations in time and programming them in Matlab) was applied. Figure 7 shows the dependence of radius on time.

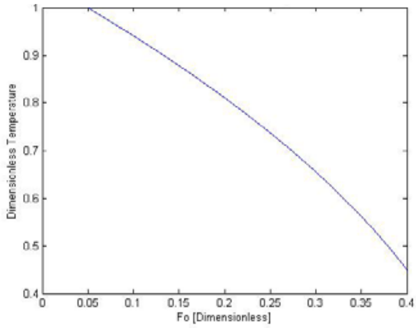

Fig. 7 Dependence of cylinder's radius on time

4 CONCLUSION

As can be seen, the temperature distribution drastically depends both on the shape of portion of fat and on the power of heater. For the power of 300 W, temperature of the whole solid increases almost linear in the case of a plane, so at the moment $F_0=3.425$ (about 180min) when the melting starts, the temperature of the whole plane is at least $T=1.8$ (about 36 ˚C). On the other hand, at the moment $F_0=3.425$ (about 37min) when the sphere fat starts to melt, the temperature of the most of the sphere is almost unchanged (stays at about 20 ˚C). Considering the laws of conduction and its dependence on the surface, these results are expected. Also dependence on the power load reveals that stronger the power – melting point is reached faster, but the temperature inside the portion cannot be increased faster because of the limited speed of heat transfer through the surface.

Models presented in this contribution are simplified and do not represent the reality reliably. Some factors which have influence are not taken into account; e.g. heat lack through the walls of the chamber and gravity influence on portion shape. But only this simple model can help us to easier choose the shape of the portion of fat which we will use for the trans-etherification process and shows us how different factors influence on temperature disturbance.

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