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NEW METHOD OF SIMULTANEOUS TRANSMISSION ANY TYPE OF MEASUREMENT  
SENSORS SIGNALS IN THE FOUR CHANNEL TRANSMULTIPLEXER STRUCTURE

NOVÉ METODY MĚŘENÍ SIGNÁLŮ PRO PŘENOS DIGITÁLNÍCH PARALELNĚ  
SOUBĚŽNÝCH DAT Z MĚŘICÍCH SENZORŮ

**Abstract**

The signal transmultiplexation system implemented to simultaneous digital data transmission from measurement sensors is presented in the paper. In general, those can be data from any type of sensor generating digital data on location, force, acceleration, pressure, etc. The issue dealt with in the paper is important as devices manufactured at present contain ever-growing number of sensors. Increasing the volume of sensor wiring is a non-optimal solution. Instead, multiplexers are used, which alternately place elements of successive signal samples in successive time slots. The proposed solution based on the use of transmultiplexers involves combining measurement signal samples into one signal that is transmitted over a single path. If properly designed, FIR type filters make it possible to obtain the signal that is scattered both in time and frequency domains. Coefficients of filter banks of the order  $I=16$  were determined numerically with the accuracy  $Q_{\min}=2,962 \cdot 10^{-6}$  for 4-input system. Frequency characteristics of filters are shown in Fig. 3. The characteristics do not resemble those of known filters (low pass, high pass) as they are distributed in the same frequency band. If properly designed, they make it possible to almost ideally reconstruct measurement signals from a complex signal. The latter combines all independent input signals thus making one entity. Each sample of the complex signal contains a fragment of the information of all input signals. It is preliminarily encoded and thus resistant to individual transmission errors. The filter order is virtually arbitrary, with the restriction that it should at least be equal to the number of input signals. It was noted that for real measurement signals, transmultiplexation in which filter coefficients were determined numerically and rounded to 4 significant places turned out to be sufficient.

**Abstrakt**

V příspěvku je prezentována implementace signálového transmultiplexního systému pro přenos digitálních paralelně souběžných dat z měřicích senzorů. Obecně se může jednat o jakýkoliv typ senzoru generující digitální výstup, např. síla, zrychlení, tlak atd. Řešení navrženým systémem prezentovaným v tomto příspěvku je důležité především v současnosti pro zařízení s neustále rostoucím počtem senzorů.

## 1 INTRODUCTION

The term of transmultiplexation is primarily associated with data conversion from time domain to frequency domain (TDM→FDM) and, in reverse order, from frequency domain to time domain (TDM→FDM→TDM). Transmultiplexation involves the addition of a certain number of filtered and oversampled input signals. The complex signal formed in this way is transmitted over a single communications channel. In the receiver, filtration is repeated and undersampling – reconstruction of individual source signals takes place. The quality of reconstruction depends on sets

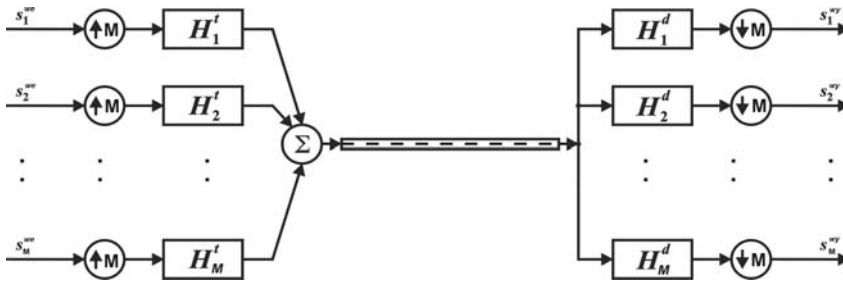
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of transmultiplexation/ detransmultiplexation filters. Initially, band pass filters of theoretically ideal frequency characteristics were used. In practice, however, such a structure generated crosstalk and distorted signal amplitude and phase. Conditions of perfect reconstruction established by Vetterli made it possible to design a set of band pass filters whose characteristics partly overlapped, and which completely eliminated crosstalk. In practice, the determination of filter coefficients was not free from rounding errors and numerical procedures precision errors. Yet the reconstruction errors were 3-5 orders smaller. In the present work, the transmultiplexer system with all-pass filters was used. Here, the term all-pass indicates that all frequencies in the range  $f = 0 - f_s/2$  are amplified or attenuated - without clear band structure. Coefficients of individual filters were determined numerically on the basis of the solution to bilinear equation derived from the analysis of transmultiplexer mathematical model.

## 2 TRANSMULTIPLEXER STRUCTURE

The diagram of M-channel transmultiplexer is shown in Fig. 1.

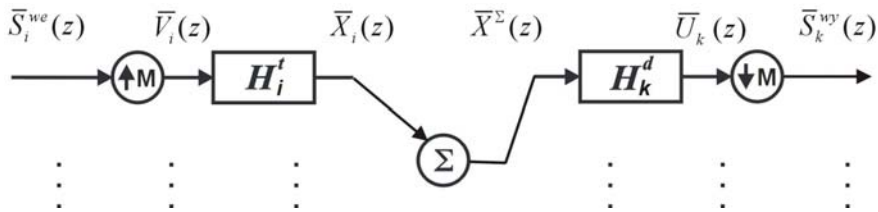


**Fig. 1** M-channel transmultiplexer

The independent input signals are oversampled. That involves inserting samples of zero value between every two neighbouring samples of the input signal. Oversampling of this kind does not change the energy or periodicity of input signals. If the number of added zero samples equals the number of input signals diminished by one, the term critical oversampling is used. Then all oversampled signals are filtered and summed up making one complex signal. In the receiver, the complex signal undergoes filtration in parallel with undersampling. The latter involves the removal of a certain number of samples, which is specified by the undersampling constant. The most important components of the system in Fig. 1 are sets of analysis and synthesis filters. If properly designed, they satisfy the conditions of perfect reconstruction.

## 3 PERFECT RECONSTRUCTION AND MATHEMATICAL MODEL

Because of linear components of transmultiplexer, the mathematical model can be analysed in Z-domain. Fig. 2 present successive transformations of the  $i$ -th input signal.



**Fig. 2** Successive transformations of the  $i$ -th input signal

The synthesising part can be expressed by the following equation

$$\bar{X}^\Sigma(z) = \sum_{i \in M} \bar{X}_i(z), \quad (1)$$

where

$$\bar{X}_i(z) = \bar{H}_i^t(z) \bar{V}_i(z) = \bar{H}_i^t(z) \bar{S}_i^{we}(z^M). \quad (2)$$

for  $i \in \{1, 2, \dots, M\}$ .

The undersampled and filtered signal in the receiver has the form

$$\bar{S}_k^{wy}(z) = \frac{1}{M} \sum_{m=0}^{M-1} \bar{U}_k(w_M^m z^{1/M}), \quad (3)$$

where

$$\bar{U}_k(z) = \bar{H}_k^d(z) \bar{X}^\Sigma(z), \quad (4)$$

and  $w_M = e^{-j2\pi/M}$ ,  $k \in \{1, 2, \dots, M\}$ .

For the sake of clarity, the transmitter signals were denoted by the  $i$ -th index, whereas those of the receiver by  $k$ -th. Substituting (4), (1), (2) into equation (3) we receive

$$\bar{S}_k^{wy}(z) = \sum_{p=-\infty}^{\infty} u_k(p) \frac{1}{M} \sum_{m=0}^{M-1} e^{-j2\pi pm/M} z^{-p/M} = \frac{1}{M} \sum_{m=0}^{M-1} \sum_{p=-\infty}^{\infty} u_k(p) e^{-j2\pi pm/M} z^{-p/M} = \frac{1}{M} \sum_{m=0}^{M-1} \bar{U}_k(z^{1/M} w_M^m). \quad (5)$$

The equation (5) determines the dependence of an arbitrary output signal on all input signals of the transmultiplexer. For well designed transmultiplexers, the output signal  $s_k^{wy}$  is an exact copy of the input signal  $s_i^{we}$ . Transmultiplexer achieves perfect reconstruction if  $s_k^{wy}$  is a delayed and amplified version of  $s_i^{we}$ , strictly speaking there exist a nonzero  $c_i$  and positive integer  $\tau_i$  such that

$$s_k^{wy}(n) = c_i s_i^{we}(n - \tau). \quad (6)$$

Binding conditions of the perfect reconstruction (5) with (6) we receive

$$\frac{1}{M} \sum_{i=1}^M \bar{S}_i^{we}(z) \left[ \sum_{m=0}^{M-1} \bar{H}_k^d(w_M^m z^{1/M}) \bar{H}_i^t(w_M^m z^{1/M}) \right] = c_i s_i^{we} \delta_{k,i}, \quad (7)$$

where  $\delta_{k,i}$  is a Kronecker function. Using FIR type filters of the order  $I$

$$\bar{H}(z) = \sum_{i=0}^I h(i) z^{-i}, \quad (8)$$

the product of transmultiplexer filters takes on the form

$$\bar{H}^d(z) \bar{H}^t(z) = \sum_{p=0}^I \sum_{q=0}^I h^d(p) h^t(q) z^{-p-q}. \quad (9)$$

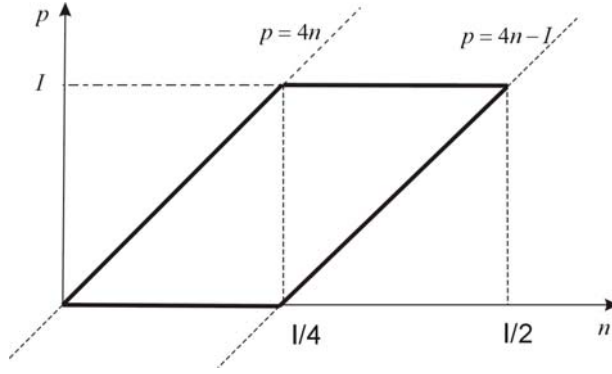
Taking into account (7) and (9), we receive

$$\frac{1}{M} \left[ \sum_{m=0}^{M-1} \bar{H}_k^d(w_M^m z^{1/M}) \bar{H}_i^t(w_M^m z^{1/M}) \right] = M \sum_{n=0}^{2I/M} z^{-n} \left[ \sum_{\substack{p=0 \\ 0 \leq Mn-p \leq I}}^I h_k^d(p) h_i^t(Mn-p) \right]. \quad (10)$$

Assuming coefficients at the same powers of the variable  $z$ , we receive

$$c \cdot \delta_{n,\tau} \cdot \delta_{k,i} = \sum_{\substack{p=0 \\ 0 \leq Mn-p \leq I}}^I h_k^d(p) h_i^t(Mn-p), \quad (11)$$

with the limitations  $n \leq (I+p)/M$  i  $n \geq p/M$ . The system of equations (11) does not provide a direct solution in the form of the values of individual filter coefficients, but binds them in the system of bilinear equations. Fig. 2 presents the summing area for a 4-input transmultiplexer.



**Fig. 2** Summing area for a 4-input transmultiplexer

The exemplary formula (11) for  $I=4$ ,  $M=2$ ,  $\tau=1$  corresponds to the system of bilinear equations below

$$\begin{cases}
 h_1'(0) \cdot h_1^d(0) = 0 \\
 h_1'(0) \cdot h_2^d(0) = 0 \\
 h_2'(0) \cdot h_1^d(0) = 0 \\
 h_2'(0) \cdot h_2^d(0) = 0 \\
 \\
 h_1'(2) \cdot h_1^d(0) + h_1'(1) \cdot h_1^d(1) + h_1'(0) \cdot h_1^d(2) = 1 \\
 h_1'(2) \cdot h_2^d(0) + h_1'(1) \cdot h_2^d(1) + h_1'(0) \cdot h_2^d(2) = 0 \\
 h_2'(2) \cdot h_1^d(0) + h_2'(1) \cdot h_1^d(1) + h_2'(0) \cdot h_1^d(2) = 0 \\
 h_2'(2) \cdot h_2^d(0) + h_2'(1) \cdot h_2^d(1) + h_2'(0) \cdot h_2^d(2) = 1 \\
 \\
 h_1'(4) \cdot h_1^d(0) + h_1'(3) \cdot h_1^d(1) + h_2'(2) \cdot h_1^d(2) + h_1'(1) \cdot h_1^d(3) + h_1'(0) \cdot h_1^d(4) = 0 \\
 h_1'(4) \cdot h_2^d(0) + h_1'(3) \cdot h_2^d(1) + h_1'(2) \cdot h_2^d(2) + h_1'(1) \cdot h_2^d(3) + h_1'(0) \cdot h_2^d(4) = 0 \\
 h_2'(4) \cdot h_1^d(0) + h_2'(3) \cdot h_1^d(1) + h_2'(2) \cdot h_1^d(2) + h_2'(1) \cdot h_1^d(3) + h_2'(0) \cdot h_1^d(4) = 0 \\
 h_2'(4) \cdot h_2^d(0) + h_2'(3) \cdot h_2^d(1) + h_2'(2) \cdot h_2^d(2) + h_2'(1) \cdot h_2^d(3) + h_2'(0) \cdot h_2^d(4) = 0 \\
 \\
 h_1'(4) \cdot h_1^d(2) + h_1'(3) \cdot h_1^d(3) + h_1'(2) \cdot h_1^d(4) = 0 \\
 h_1'(4) \cdot h_2^d(2) + h_1'(3) \cdot h_2^d(3) + h_1'(2) \cdot h_2^d(4) = 0 \\
 h_2'(4) \cdot h_1^d(2) + h_2'(3) \cdot h_1^d(3) + h_2'(2) \cdot h_1^d(4) = 0 \\
 h_2'(4) \cdot h_2^d(2) + h_2'(3) \cdot h_2^d(3) + h_2'(2) \cdot h_2^d(4) = 0 \\
 \\
 h_1'(4) \cdot h_1^d(4) = 0 \\
 h_1'(4) \cdot h_2^d(4) = 0 \\
 h_2'(4) \cdot h_1^d(4) = 0 \\
 h_2'(4) \cdot h_2^d(4) = 0.
 \end{cases} \tag{12}$$

#### 4 MINIMIZATION OF QUALITY COEFFICIENT

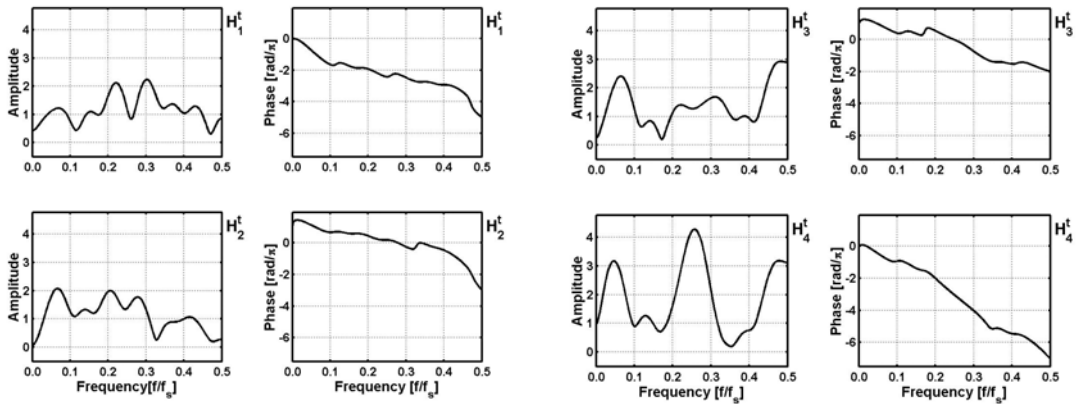
In practice, in order to determine appropriate parameters, the minimization of an indicated quality coefficient is usually used. The most popular one is the quadratic mean criterion. On the basis of this criterion, the values of transmultiplexation filter coefficients were determined numerically

$$Q_{\min} = \left( c \cdot \delta_{n,\tau} \cdot \delta_{k,i} - \sum_{\substack{p=0 \\ 0 \leq Mn-p \leq l}}^l h_k^d(p) h_i^d(Mn-p) \right). \quad (13)$$

The solution that was obtained is only an approximation of the exact solution, which results from a limited accuracy of computation and the rounding of the numerical minimising procedure. Exemplary coefficients of 4 channel transmultiplexer filters are shown in Tab.1. Some frequency characteristics for the coefficients in Tab. 1 are presented in Fig. 3.

**Tab.1** Filters coefficients

No.	$H_1^t$	$H_2^t$	$H_3^t$	$H_4^t$	$H_1^d$	$H_2^d$	$H_3^d$	$H_4^d$
0	-0,0001	-0,0000	0,0000	-0,0002	0,7954	0,2245	0,1064	1,4454
1	0,5366	-0,5092	0,6371	-0,1696	-0,1085	0,4690	-0,6502	0,6920
2	-0,5973	-0,8602	-0,0268	0,4638	-0,1157	-0,6436	0,1978	0,8242
3	-0,5422	0,2351	-0,9266	0,3295	0,8135	-0,4714	-0,5452	0,1416
4	0,5477	0,4279	0,5214	0,2864	0,5376	0,9457	1,0682	-0,0164
5	0,1678	0,0809	-0,1619	0,5096	0,5745	0,5714	0,6064	0,6416
6	0,1534	0,0857	-0,0521	0,7164	0,6567	0,4762	0,4931	0,6633
7	0,1162	0,0633	-0,0736	0,6680	0,4178	0,3305	0,3062	0,7058
8	0,0995	0,4569	0,8215	-1,0513	0,1734	0,4878	0,7207	-0,4905
9	0,1415	0,0613	0,0164	0,2895	0,3721	0,6403	0,6947	0,0320
10	0,1787	0,1224	0,0969	0,3423	0,3437	0,6288	0,7326	-0,0574
11	0,2015	0,1711	0,1962	0,3859	0,2758	0,5780	0,5874	0,0268
12	-0,2509	-0,1443	0,2024	-1,1828	0,0096	0,0265	0,0397	-0,0301
13	0,0444	0,0374	0,0334	0,0921	0,1178	0,3442	0,5316	-0,3916
14	0,0165	0,0078	0,0113	0,0273	0,1025	0,3051	0,4602	-0,3417
15	-0,0235	-0,0191	-0,0189	-0,0498	0,1068	0,2849	0,4415	-0,3295



**Fig. 3** Frequency characteristics of filters for 4-input transmultiplexer

## 5 CONCLUSIONS

Transmultiplexer systems can transmit a lot of independent measurement signals over a single transmission line. The signal that is obtained is a complex one, in which each arbitrary digital sample contains information that originates in all input signals. An almost ideal reconstruction of source signals can be obtained. Minimal errors result from rounding and the accuracy level of the numerical procedure. It was observed that no high order filters were necessary to construct a system that would be capable of ensuring accurate reconstruction. It would be sufficient for the filter order to be equal to the number of input signals. Apart from undeniable speed of action, the reconstructed signals are only affected by unit delay. High order filters have got complicated impulse response characteristics. When filter coefficients are determined numerically, the question arises how many decimal places must be assumed so that the computation complexity and thus resultant errors would be the smallest. The answer can be seen in many experiments, which indicates that for real signals, transmultiplexation filters of coefficients rounded to 4 places seem sufficient.

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