Sborník vědeckých prací Vysoké školy báňské - Technické univerzity Ostrava
číslo 1, rok 2008, ročník LIV, řada strojní
článek č. 1606

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THE INSTABLE PARAMETRIC OSCILLATION DURING THE ROPE TRACK TRANSPORT

NESTABILNÍ PARAMETRICKÉ KMITÁNÍ PŘI DOPRAVĚ NA ZÁVĚSNÉ LANOVÉ DRÁZE

Abstract

The parametric oscillation appears if some parameter of the oscillating system, e.g. stiffness, changes its value in the process of oscillation. The two different frequencies have influence on the resulting motion. The natural frequency of the system and frequency of the stiffness change. The analytical solution for the harmonic change was performed by Mathieu. The resulting motion can have stable or instable form, depending on the value of two dimensionless constants. First represents the ratio between natural frequency a frequency of the stiffness change, second represents ratio between the variation and average value of stiffness. During the transport on the rope track in the horizontal mine opening two forms of the parametric oscillations can appear. The diagonal swinging in the horizontal direction (mathematical pendulum) and vertical oscillations due to rope stiffness are analyzed and the conditions of the parametric oscillations appearance are determined.

Abstrakt

Parametrické kmitání se objevuje jestliže některý z parametrů, např. tuhost, mění svou hodnotu. Výsledný pohyb je ovlivněn dvěma odlišnými frekvencemi. Vlastní frekvence kmitání a frekvence změny tuhosti. Analytické řešení pro harmonicky proměnný parametr provedl Mathieu. Výsledný pohyb může mít stabilní nebo nestabilní podobu v závislosti na hodnotě dvou bezrozměrných konstant. První představuje poměr vlastní frekvence a frekvence změny tuhosti, druhá poměr změny tuhosti k její průměrné hodnotě. Při dopravě na lanové dráze ve vodorovném důlním díle se parametrické kmitání může objevit ve dvou podobách. Příčné kývání ve vodorovném směru (matematické kyvadlo) a svislé kmitání, dané poddajností lana, jsou analyzována a jsou určeny podmínky rozvoje parametrického kmitání.

1 INTRODUCTION

The rope track transport in the horizontal mine opening is the useful kind of personal transport. A person is sitting on the seat, hanging on the moving rope. The path can change the direction both in horizontal (left, right) and vertical (upward, downward) direction. The transport can be accompanied by the oscillations and swinging. In normal situation the swinging should not be too dangerous. But appearance of the parametric oscillations could dramatically increase this phenomenon.

2 THE PARAMETRIC OSCILLATIONS - THE THEORETICAL BACKGROUND

Suppose the oscillating system with mass m and stiffness k.

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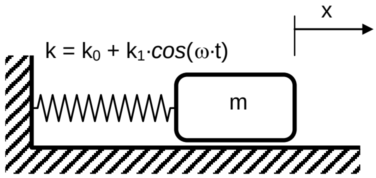
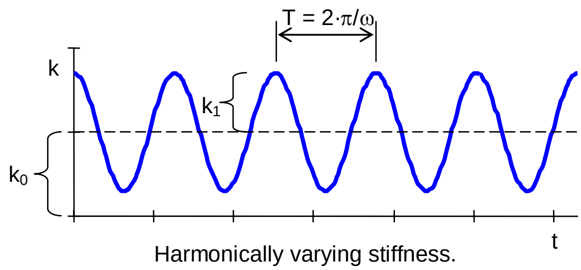


The oscillating system with harmonically varying stiffness.

Suppose then harmonically varying stiffness :

$$k_{(t)} = k_0 + k_1 \cdot \cos(\omega \cdot t)$$

where k_0 is the average value and k_1 is fluctuation of the stiffness, ω is the circular frequency of the stiffness change, finally t is time.



Harmonically varying stiffness.

The equation of motion is :

$$m \cdot \ddot{x} + k_{(t)} \cdot x = m \cdot \ddot{x} + (k_0 + k_1 \cdot \cos(\omega \cdot t)) \cdot x = 0$$

After substitutions :

$$\tau = \omega \cdot t \quad \lambda = \frac{k_0}{m \cdot \omega^2} = \frac{\Omega^2}{\omega^2} \qquad \gamma = \frac{k_1}{m \cdot \omega^2} = \frac{k_1}{k_0} \cdot \lambda = \frac{k_1}{k_0} \cdot \frac{\Omega^2}{\omega^2} \qquad \Omega = \sqrt{\frac{k_0}{m}}$$

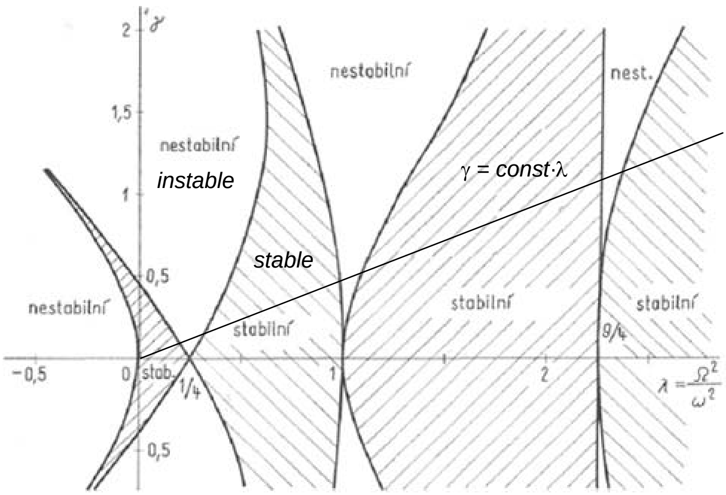
the equation of motion then has the form of so called Mathieu equation :

$$\frac{d^2x}{d\tau^2} + (\lambda + \gamma \cdot \cos\tau) \cdot x = 0$$

The solution is :

$$x = A \cdot e^{\mu \cdot \tau} \cdot p_{1(\tau)} + B \cdot e^{-\mu \cdot \tau} \cdot p_{2(\tau)}$$

Because μ is complex number, the solution has the harmonic part (imaginary part of μ - stable solution) and exponential part (real part of μ, instable), depending on the values of λ and γ. The areas of instability are shown in the γ-λ diagram.



The stable and instable regions - graphical interpretation.

With increasing ω the λ parameter decreases (ω=0 leads to λ→∞, vice versa high ω imply λ→0). However the γ/λ ratio stays constant. The varying ω (with unchanging stiffness parameters) means that the γ-λ points lay on the straight line $\gamma = const \cdot \lambda$.

3 THE DIAGONAL SWINGING IN THE HORIZONTAL DIRECTION

The mathematical pendulum - the theoretical solution.

ϕ - the swinging angle,

ω - the angular velocity,

ε - the angular acceleration,

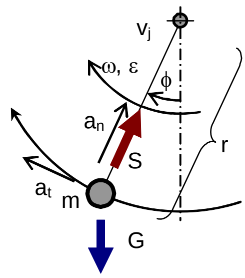
r - the pendulum length,

G - the gravitational force,

S - the tension force,

a_t - the tangential acceleration,

a_n - the normal acceleration.



The mathematical pendulum.

The equations of motion are :

The tangential direction :

$$m \cdot a_t = -G \cdot \sin\phi$$

$$m \cdot r \cdot \varepsilon + m \cdot g \cdot \sin\phi = 0$$

The normal direction :

$$m \cdot a_n = S - G \cdot \cos\phi$$

$$S = m \cdot r \cdot \omega^2 + G \cdot \cos\phi$$

If ϕ < 15° the simplification : *sin*ϕ ≅ ϕ, *cos*ϕ ≅ 1 can be accepted. The equation of motion then has the form of differential equation of the II. order, linear, homogenous with constant coefficients.

$$r \cdot \ddot{\phi} + g \cdot \phi = 0$$

The solution is :

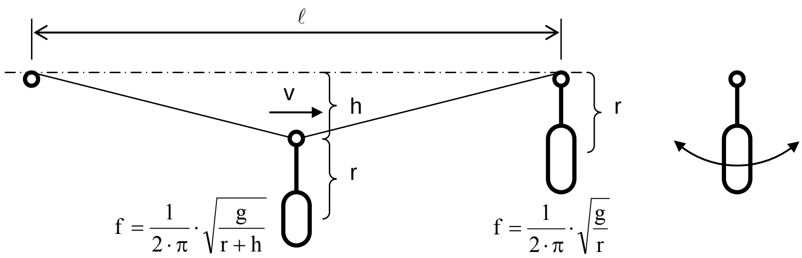
$$\phi_{(t)} = \phi_{max} \cdot \sin(\Omega \cdot t + \gamma_0)$$

where the natural circular frequency is :

$$\Omega = \sqrt{\frac{g}{r}}$$

The amplitude ϕ_{max} and the phase angle γ_0 depends on the initial conditions.

On the rope track the pendulum length vary between r and r+h, where r is the length of the hanging stool, h is the rope suspension due to gravitational force.



The varying pendulum length.

The squares of the natural circular frequencies are :

$$\Omega_{min}{}^2 = \frac{g}{r+h} \qquad \Omega_{max}{}^2 = \frac{g}{r}$$

These values can be interpreted as the limit values of the stiffness with the unit mass.

$$m \cdot \ddot{x} + k_{(t)} \cdot x = 0$$

$$\Omega^2 = \frac{k}{m}$$

if m = 1 kg, then numerically $\Omega^2 = k$. Then $k_{min} = \Omega_{min}{}^2$ and $k_{max} = \Omega_{max}{}^2$.

After that :

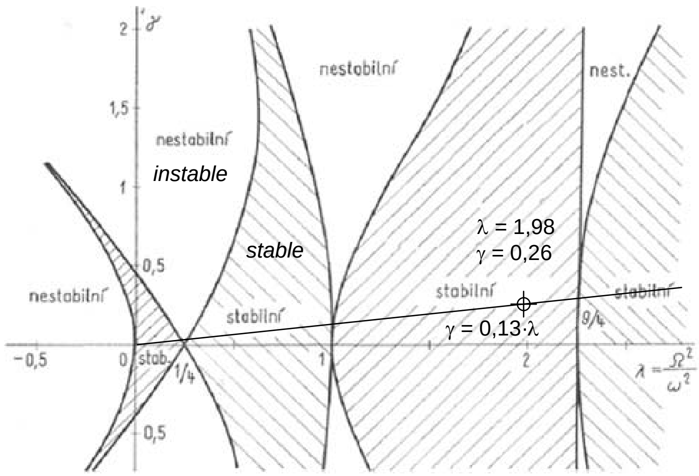
$$k_0 = \frac{k_{max} + k_{min}}{2} \qquad k_1 = \frac{k_{max} - k_{min}}{2}$$

and finally :

$$\Omega = \sqrt{\frac{k_0}{m}} \qquad \lambda = \frac{\Omega^2}{\omega^2} \qquad \gamma = \frac{k_1}{k_0} \cdot \frac{\Omega^2}{\omega^2}$$

The period of the stiffness varying comes from the distance between rope supports ℓ and transporting velocity v.

$$T = \frac{\ell}{v} \qquad \omega = \frac{2 \cdot \pi}{T} = \frac{2 \cdot \pi \cdot v}{\ell}$$



The parametric oscilation of the pendulum - stable and instable intervals.

The rate γ/λ is constant and represents the line in the γ-λ graph. (With increasing velocity the γ-λ point moves from the right end to the left one.)

$$\frac{\gamma}{\lambda} = \frac{k_1}{k_0} = \text{constant} \qquad\qquad \gamma = \text{constant} \cdot \lambda$$

In the diagram three instable points appear.

$\lambda = 2{,}25$ $\qquad$ $\lambda = 1$ $\qquad$ $\lambda = 0{,}23 \div 0{,}27$

the circular frequency : $\omega = \sqrt{\dfrac{\Omega^2}{\lambda}}$

$\omega = 1{,}96\ s^{-1}$ $\qquad$ $\omega = 2{,}95\ s^{-1}$ $\qquad$ $\omega = 5{,}67 \div 6{,}14\ s^{-1}$

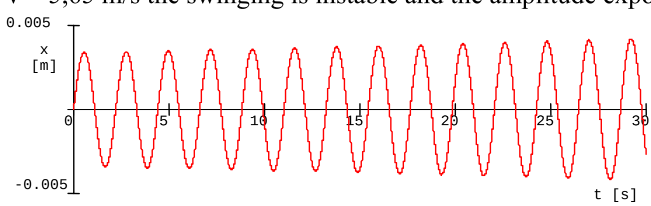
and finally critical velocity : $v_{crit} = \dfrac{\omega \cdot \ell}{2 \cdot \pi}$

$v_{crit} = 1{,}88$ m/s $\qquad$ $v_{crit} = 2{,}81$ m/s $\qquad$ $v_{crit} = 5{,}41 \div 5{,}87$ m/s

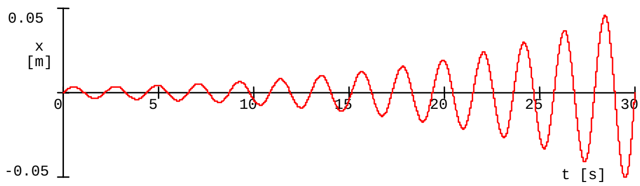
At these critical velocities the instable swinging can appear. This means that after negligible impulse the diagonal swinging appears and increases to the high amplitude. Thanks to the low value of γ the critical velocity is in the discrete value or in the very narrow range.

But if the vertical rope suspension h increases, the value of k_1 and the γ value increases and subsequently the critical velocities appear in the wider range.

The stable and instable swinging was simulated via numerical integration. While at the velocity v = 5,4 m/s the swinging is stable and the steady amplitude depends only on the initial conditions, at the velocity v = 5,65 m/s the swinging is instable and the amplitude exponentially increases.



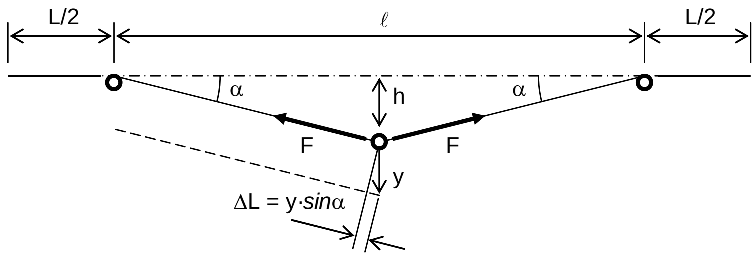
The swinging at the velocity v = 5,4 m/s.



The swinging at the velocity v_j = 5,65 m/s.

4 THE VERTICAL OSCILLATIONS

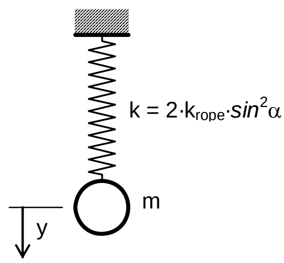
The vertical stiffness depends on the rope parameters, such as the Young modulus E and the cross section area S, on the rope total length L, and finally on the vertical suspension due to gravitational force h.



The elongation of the carrying rope due to vertical oscillation.

If vertical displacement is y, then the rope elongation is :

$$\Delta L = y \cdot \sin \alpha$$



The oscillating system.

where α is angle of the rope to the horizontal direction.

Suppose the half length L/2 on both sides of the hanging stool, the rope stiffness is then :

$$k_{rope} = \frac{E \cdot S}{\frac{1}{2} \cdot L}$$

and resulting vertical stiffness is :

$$k_{min} = 2 \cdot k_{rope} \cdot \sin^2 \alpha = 4 \cdot \frac{E_{rope} \cdot S}{L} \cdot \sin^2 \alpha$$

Here :

E_{rope}. the Young modulus of the rope (not equal to the Young modulus of the full material),

S ..... the square area of the rope,

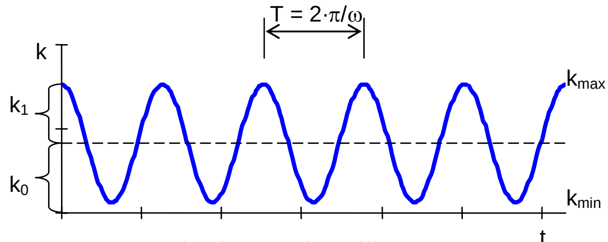
L ..... total length of the rope,

α ..... the angle of the suspended rope to the horizontal direction.

This value represents the minimum stiffness, when the hanging point is in the middle between two supports of the rope track. The maximum value, corresponding to the hanging point under support, reaches extremely high values (the stiffness of the supporting steel structure).

$$k_{max} >> k_{min}$$

The time curve of the changing stiffness vary between a certain value (k_{min}) and extremely higher value (k_{max}). The average value of the stiffness is only little bit higher then ½ of k_{max} ($k_0 > k_{max}/2$ but near), the fluctuation of stiffness is only little bit lower then ½ of k_{max} ($k_1 < k_{max}/2$ but near).

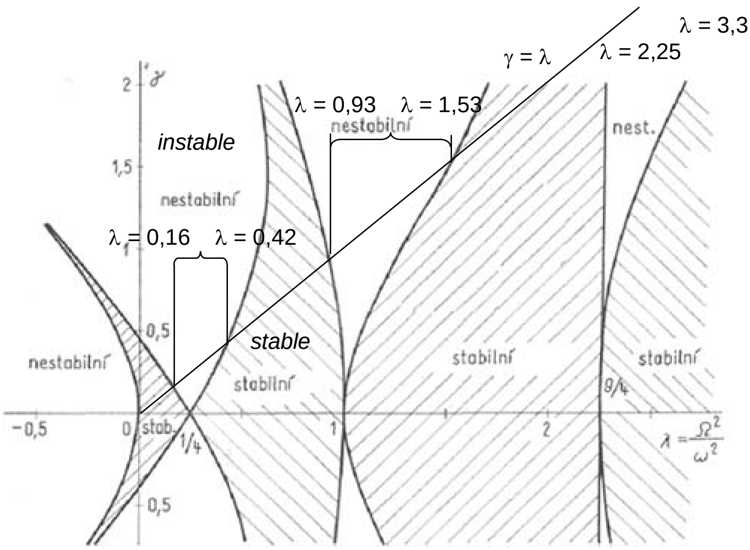


The time-varying stiffness.

When parameters $k_0 \cong k_1$, their ratio is then :

$$\frac{\gamma}{\lambda} = \frac{k_1}{k_0} \cong 1$$

The γ-λ line has 45º to the λ axis.



The vertical parametric oscillation - stable and instable intervals.

The areas of instability are then :

= 0,16÷0,42	λ = 0,93÷1,53	λ = 2,25÷3,3
$\omega = 24{,}8 \div 30\ s^{-1}$	$\omega = 36{,}4 \div 46{,}7\ s^{-1}$	$\omega = 69{,}4 \div 113\ s^{-1}$
v = 23,7÷28,6 m/s	v = 34,7÷44,6 m/s	v = 66,3÷107 m/s

Where λ is dimensionless constant, ω is the circular frequency of the stiffness change, finally v is the velocity. That is clear that the transporting velocity will never reach the critical values.

5 CONCLUSION

At the transport on the rope track the instable, so called parametric oscillations can appear. This will take effect as increasing oscillations without visible cause. Such an oscillation can appear both as a diagonal swinging in horizontal direction and as vertical oscillation.

The condition, in which the instable oscillation appears, depends on stiffness and force parameters of the track and cannot be determined “once forever”. Regardless some general conclusions can be phrased.

The instable transversal swinging can appear at certain critical transport velocities. Exactly there are ranges of critical velocity values. But in typical conditions the range is very narrow, practically a single value. The probability that the instable swinging appears is extremely low. But the range of the critical velocity values became wider with larger vertical suspension of the rope due to gravity.

In case of vertical oscillations the range of critical velocity values is larger. The values depend on the stiffness of the rope support and probably will be much higher then the typical transport velocity.

Acknowledgement

The work was done with support of the grant project No. 49/2006 “Požadavky na dopravní zařízení při dopravě osob v úklonných trasách“, BU 3006011.

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