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PARAMETERIZATION-BASED CONTROL OF ANISOCHRONIC SYSTEMS USING RMS RING

ŘÍZENÍ ANISOCHRONNÍCH SYSTÉMŮ ZALOŽENÉ NA PARAMETRIZACI S VYUŽITÍM OKRUHU RMS

Abstract

The contribution brings an engineering acceptable control design approach for so-called anisochronic systems. The method is based on algebraic tools in the ring of RQ-meromorphic functions and it was developed for a wide class of delayed systems. Anisochronic systems (models) include delays also in their dynamics and can be delayed in the conventional input-output sense as well. The control synthesis consists in the solution of the Bèzout identity and Youla-Kučera parameterization resulting in the Smith-like control structure. A final controller can be tuned by a suitable choice of a scalar real parameter. Among many others tuning methods, the equalization method is adopted. Anisochronic models and presented approach is suitable also for high order dynamics approximation and autotuning procedures. First order stable and unstable simulation examples are presented.

Abstrakt

Příspěvek se zabývá inženýrsky přívětivým návrhem regulátorů pro tzv. „anizochronní“ systémy. Metoda je založena na popisu v okruhu RQ-meromorfních funkcí (R_{MS}) a je použitelná pro širokou škálu systémů se zpožděním. Anisochronní systémy či modely obsahují zpoždění jako součást jejich dynamiky, avšak mohou být zpožděny i v tradičním vstupně-výstupním smyslu. Návrh regulátoru sestává z řešení diofantické rovnice (Bèzoutovy identity) a Youla-Kučerovy parametrizace. Výsledná řídicí část regulačního obvodu má pak charakter Smithova prediktoru a může být laděna pomocí skalárního parametru; tomto příspěvku je použita modifikace tzv. vyvážené metody nastavení. Anisochronní modely a uvedený návrh regulátoru jsou vhodné i pro systémy vyšších řádů a metody autotuningu. Uvedeny jsou simulační příklady jak pro stabilní, tak nestabilní systémy.

1 INTRODUCTION

A family of systems with delays also in internal feedback loops constitutes an attractive and interesting branch for theory as well as for industrial application. Traditionally delays are modelled in the sense of input-output relations. This approach is not suitable for state delays, i.e. on the left side of a differential equation [1]-[2]. However, different use of delay relations as one of the primary elements of model structure can be considered – *anisochronic* system description comprehending delay terms also in the state variables can cover a wide class of time delay systems. The models have transcendental structure, i.e. with infinite spectrum. There are several ways how to identify a plant with an anisochronic model, e.g. via relay test [3] or successive integrations [4]. The algebraic description of anisochronic systems requires a simultaneous use of both differential and delay operators. The Laplace-transform description of this class of systems results in the transfer functions

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that are ratios of the so-called *quasipolynomials*. Once a ring of quasipolynomials is established a set of *retarded quasipolynomial meromorphic functions* (R_{MS}) can be introduced as a ratio of two quasipolynomials where the denominator is stable. Any transfer function is then described as a ratio of two elements from R_{MS} and this representation is suitable for algebraic controller design.

Algebraic approaches play a significant role in modern control theory. Recently, there have been done some attempts to implement algebraic controller design algorithms over the R_{MS} ring [2], [5]-[6]. Two ideas predominate: functional extension of the internal model control principle using affine parameterization and solution based on the Diophantic equations and Youla-Kučera parameterization. With the only exception [2] the approaches cover only stable controlled systems. This contribution depicts the second mentioned approach which covers also controller design for unstable plants. The methodology brings a scalar parameter m_0 which can be used for controller tuning. The question how to choose the "right" value of this parameter has not been solved, one attempt is suggested in [7] where dominant closed loop poles shifting method was utilized. This contribution brings another choice employing an analogy with the *equalization method* [8]. The paper also demonstrates some simulation examples to verify the proposed algebraic controller design approach for various plant models.

2 DESCRIPTION OF DELAYED SYSTEMS USING MEROMORPHIC FUNCTIONS

A transfer function is obviously assumed as a ratio of two polynomials in the Laplace transform. A time delay in a system with the input-output delay where the dynamics is expressed by accumulations only is then expressed by an exponential

$$G(s)=\frac{Y(s)}{U(s)}=\frac{b_m s^m+...+b_1 s+b_0}{s^n+...+a_1 s+a_0}e^{-\tau s}=\frac{b(s)}{a(s)}e^{-\tau s}; \qquad m<n, \tau>0 \tag{1}$$

The innovative approach can be found e.g. in [1], [9] utilizing delays also in dynamics, i.e. in a denominator of a transfer function, providing a tighter model for a wide class of plants. Due to the infinite spectrum of the models they can be used for tracing of higher order systems dynamics. For the modelling, both inductive (e.g. in [3]-[4]) and deductive (e.g. in [1]) procedures can be utilized. Using Laplace transform, transfer functions results in a ratio of quasipolynomials. Quasipolynomials contain also exponentials terms in contrast to polynomials which consists of weighted sums of s-powers only. As an example, the first order anisochronic system can be depicted as

$$\tilde{G}(s)=\frac{b_0 e^{-\tau s}}{s+a_0 e^{-\vartheta s}} \tag{2}$$

Modern algebraic approaches utilize various rings and tools connected with their properties. Assume a ring of stable and proper retarded quasipolynomial (RQ) meromorphic functions (R_{MS}) rather than quasipolynomials. This representation allows convenient using of parameterization providing fulfil the specific control conditions. Considering transfer function (2), the plant description in R_{MS} is as follows

$$\tilde{G}(s)=\frac{\dfrac{b_0 e^{-\tau s}}{s+m_0 e^{-\vartheta s}}}{\dfrac{s+a_0 e^{-\vartheta s}}{s+m_0 e^{-\vartheta s}}}, \tag{3}$$

where analogously for higher order plants a common denominator is stable polynomial or quasipolynomial of appropriate order. Using of quasipolynomial rather the polynomials is necessary for unstable delayed plants [10]. Quasipolynomial stability can be easily tested by applying Mikhailov criterion, proved applicable for anisochronic systems in [11].

The issue of properness in R_{MS} ring is as the natural requirement as in the rational descriptions ensuring feasibility of both the plant and the controller. A term in R_{MS} is proper if the highest s-power in the denominator (polynomial or quasipolynomial), s^n, is higher or the same as the highest s-power in numerator, s^l, e.g. $n \geq l$.

3 PARAMETERIZATION OF STABILIZING CONTROLLERS

The above introduced transfer function description of delayed systems over R_{MS} ring is suitable especially for algebraic control synthesis. Let a single-input single-output plant be estimated by an anisochronic model in the form

$$\widetilde{G}(s) = \frac{B(s)}{A(s)} = \frac{\frac{b(s)}{m(s)}}{\frac{a(s)}{m(s)}}, \tag{4}$$

where (quasi)polynomials $b(s)$ and $a(s)$ represent input-output plant behaviour and dynamics and a selected stable (quasi)polynomial $m(s)$ makes $A(s)$ and $B(s)$ to be in R_{MS}, moreover, $A(s)$ and $B(s)$ are coprime - details about divisibility in R_{MS} can be found in [2]. Thus, consider the control loop as simple feedback system depicted in Fig. 1.

Let a transfer function of the controller be $G_R(s) = Q(s)/P(s)$. The aim of the control synthesis is to stabilize a feedback control system, obtain asymptotic tracking and attenuate load disturbance $d(t)$. As first, the stabilization of the feedback loop is guaranteed by solution of the Diophantine equation

$$A(s)P_0(s) + B(s)Q_0(s) = 1, \tag{5}$$

where $P_0(s)$ a $Q_0(s)$ is a particular solution.

Then, all stabilizing controllers can be expressed in a parametric form

$$\frac{Q(s)}{P(s)} = \frac{Q_0(s) + A(s)Z(s)}{P_0(s) - B(s)Z(s)}, \quad P_0(s) - B(s)Z(s) \neq 0, \tag{6}$$

where $Z(s)$ is an arbitrary element of R_{MS}.

The special choice of this element is capable to ensure additional control conditions. Details and proofs for general ring can be found e.g. in [12]-[13].

Secondly, conditions for asymptotic tracking and disturbance attenuation result from expression for $E(s)$ which reads

$$E(s) = P(s)\left[A(s)W(s) - B(s)D(s)\right], \tag{7}$$

where Laplace forms of reference and disturbance signals are $W(s) = H_W(s)/F_W(s)$ and $D(s) = H_D(s)/F_D(s)$, respectively, and all numerators and denominators in both $W(s)$ and $D(s)$ are over R_{MS}.

It is required that $E(s)$ must belong to R_{MS}, thus it is demanded that all unstable factors in both $F_w(s)$ and $F_D(s)$ divides $P(s)$. In practice, the most frequent case is both $w(t)$ and $d(t)$ are step functions then the condition (8) for the absolute term in numerator of $P(s)$ is demanded.

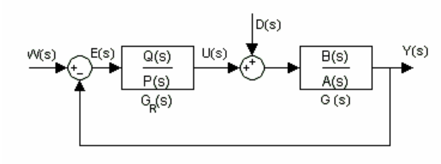


Fig. 1 Control feedback structure

$$\lambda\left(1-e^{-\tau s}\right) \tag{8}$$

This condition indeed contains at least one zero root. A free real parameter $\lambda \in \Re$ can be chosen in the form $\lambda = m_0^n$ (where n is the order of the plant) so that $m(s)$ has a multiple real pole $-m_0$. Condition (8) is ensured by the suitable choice of $Z(s)$ in (6). If $w(t)$ or $d(t)$ are another functions, divisibility conditions can be more complex.

4 CONTROLLER TUNING

The whole system behaviour is influenced by the choice of (quasi)polynomial $m(s)$ which arises as a factor in the characteristic (quasi)polynomial. In order to simplify, $m(s)$ it is usually chosen as the polynomial with multiple real stable pole $-m_0$, i.e. $m(s) = (s + m_0)^n$. However, from mentioned above facts follows that for unstable plants it is necessary to use a quasipolynomial instead of polynomials. Just and comprehensive solution of this problem has not been known yet. In [5], "a pole placement principle" is interpreted as a shifting of dominant poles. In this paper, "an equalization method" is adopted (e.g. in [8]). This method was originally derived for input-output delayed systems, nevertheless, in some limit approximation it seems to be useful also for anisochronic systems. According to this tuning approach, for the PI controller together with a first order plan it is postulated

$$K_P = \frac{1}{K}\frac{1+(1-\sigma)^2}{2} \qquad T_I = (T+\tau)\frac{1+(1-\sigma)^2}{2}, \tag{9}$$

where K_P and T_I are a gain and a integral time constant, respectively, K and T are a gain and a time constant of the plant, τ is time delay and $\sigma = \tau/(T+\tau)$. To fulfil simultaneously both conditions in (9) is impossible, however satisfaction of K_P gives better simulation results.

5 ILLUSTRATIVE EXAMPLES

5.1 First order stable plants

Let the plant be expressed by the anisochronic delayed first order model

$$G_S(s) = \frac{Ke^{-\tau s}}{Ts + e^{-\theta s}} = \frac{\dfrac{Ke^{-\tau s}}{s+m_0}}{\dfrac{Ts + e^{-\theta s}}{s+m_0}} = \frac{B(s)}{A(s)} \tag{10}$$

Parameters K, T, τ and θ can be acquired by relay experiment or via successive integrations [4]. Using (5) the following particular solution is obtained

$$P_0(s) = \frac{s + m_0 - Ke^{-\tau s}}{Ts + e^{-\theta s}};\ Q_0 = 1 \tag{11}$$

and if $Z(s)$ is chosen as in (12) then a controller satisfying both a reference tracking and a disturbance rejection has the transfer function

$$Z(s) = \frac{\kappa(s+m_0)}{Ts + e^{-\theta s}};\ \kappa = \frac{m_0}{K} - 1 \tag{12}$$

$$G_R(s) = \frac{Q(s)}{P(s)} = \frac{m_0}{K}\frac{Ts + e^{-\theta s}}{s + m_0\left(1 - e^{-\tau s}\right)} \tag{13}$$

The controller structure (13) can be easily compared with the well-known Smith predictor, see in [14]. For low frequencies, $s \to 0$, controller (13) works as PI (proportionally-integrative) one. Assuming steady-state behaviour of the controller the equalization principle (9) may be taking in account. This idea results in a choice for m_0 as

$$m_0 = \frac{1+(1-\sigma)^2}{2T} \tag{14}$$

Hence, for example let a plant be modelled as K = 4.5, T = 7, τ = 3, θ = 0.5 and the ideal agreement of a model and a plant is presumed. The presented approach (5) – (8) with respect to (14), m_0 = 0.036, results in the controller

$$G_R(s) = 8\cdot 10^{-3}\,\frac{7s+e^{-0.5s}}{s+0.036\left(1-e^{-3s}\right)} \tag{15}$$

To compare the influence of m_0 and the efficiency of the "equalization method", simulation results for various m_0 values are depicted in Fig. 2.

The following conditions were set for all simulations in this section: step reference value $w(t)$ = 1 for $t \in < 0; T_{SIM}/3)$, $w(t)$ = 2 for $t \in < T_{SIM}/3; T_{SIM})$, step load disturbance $d(t)$ = -0.1 for $t \geq 2T_{SIM}/3$, where T_{SIM} is simulation time.

Obviously, as can be seen from the figures, the higher m_0 value implies faster and more "aggressive" behaviour which can spoil an actuator and made the controller less robust; however, the equalization method results in too slow response.

5.2 First order unstable plants

Although many researches have implemented the R_{MS} ring for the controller design, only stable plants were accentuated. Hereinafter, the controller design approach above presented is utilized also for an unstable first order plant. Hence, let the plant is modelled as

$$G_S(s) = \frac{Ke^{-\tau s}}{Ts - e^{-\vartheta s}} = \frac{\dfrac{Ke^{-\tau s}}{r_0 Ke^{-\tau s} + Ts - e^{-\theta s}}}{\dfrac{Ts - e^{-\theta s}}{r_0 Ke^{-\tau s} + Ts - e^{-\theta s}}} \tag{16}$$

where a scalar real parameter r_0 stabilize a common quasipolynomial denominator in (16), see details in [15]. The stability can be checked by the Mikhailov criterion, see in [11].

The solution of the Diophantine equation (6) is clearly $Q_0 = r_0$, P_0 = 1. Selecting the parameterization factor

$$Z(s) = \frac{m_0\left[r_0 Ke^{-\tau s} + Ts - e^{-\theta s}\right]}{K\left(s+m_0\right)} \tag{17}$$

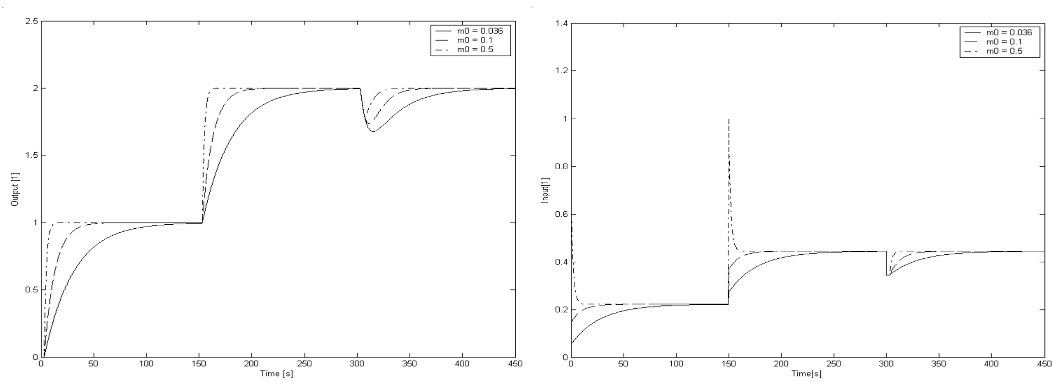


Fig. 2 First order stable plant - output and input for various m_0 values

leads to the controller $G_R(s)$ of the structure

$$G_R(s) = \frac{(r_0 K + m_0 T)s + m_0\left[r_0 K - e^{-\theta s}\right]}{K\left[s + m_0\left(1 - e^{-\tau s}\right)\right]} \tag{18}$$

Assuming steady-state approximation of (18) again, the PI structure is obtained and "equalization method" can be used. However, any attempt to fulfil the condition for K_P according to (9) leads to negative m_0, which is not allowed due to the stability requirements. Thus, the condition for T_I instead of K_P must be taken in account, which results in

$$m_0 = \frac{r_0 K(\delta - 1) - \delta}{T}$$
$$\delta = \frac{(T+\tau)^2 + T^2}{2(T+\tau)} \tag{19}$$

Consider the following plant parameters: $K = 3$, $T = 5$, $\tau = 4$, $\theta = 0.8$. Parameter r_0 can be taken as $r_0 = 0.434$, i.e. stability amplitude margin is $K_G = 1.3$, see [15]. Formula (18) with respect to (19), $m_0 = 0.095$, leads to

$$G_R(s) = \frac{1.775s + 0.124 - 0.095e^{-0.8s}}{3\left[s + 0.095\left(1 - e^{-4s}\right)\right]} \tag{20}$$

Again, the suitability of the "equalization method" and the influence of m_0 are illustrated in Fig. 3. Responses in these simulations are relatively slow in contrast to "unstable" time constant T of the plant. The approach suffers from high overshoots after step reference signal changes. This overshoots can be attenuated by increasing of m_0, which is sacrificed by faster changes of controlled signal. Much better responses are obtained by an improvement of the method using inner feedback loop or internal model control (IMC) structure [15] or 2DOF structure [16].

The example shows possibility of using the presented algebraic approach for unstable (anisochronic) delayed systems; nevertheless, it also demonstrates infelicity of the "equalization method" which is herein utilized after some frequency adjustment of a derived controller.

5.3 Higher order approximation

The transcendental character of anisochronic functions is able to estimate and approximate models of high order dynamics. This following example demonstrates the ability.

Assume a plant exactly governed by the transfer function (21).

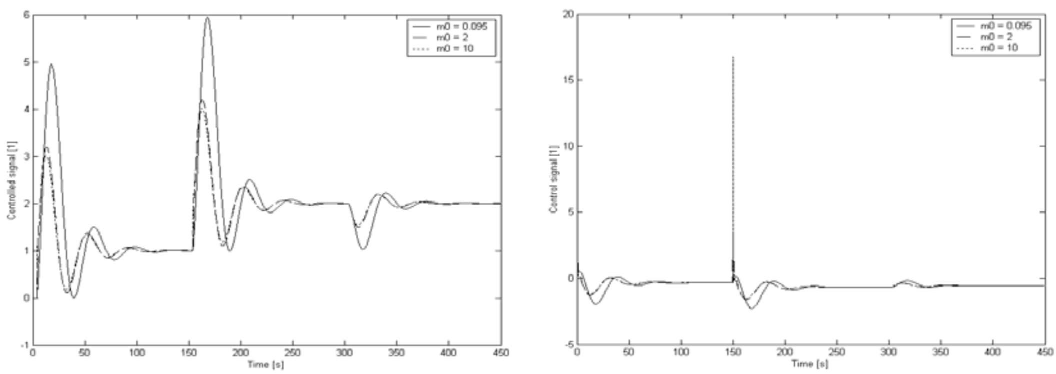


Fig. 3 First order unstable plant - output and input for various m_0 values

$$G_S(s) = \frac{e^{-3s}}{(3s+1)^5} \tag{21}$$

After some autotuning experiments with asymmetric relay technique [3], this process can be approximated by the transfer function of form (10) as in (22)

$$\hat{G}_S(s) = \frac{e^{-8.96s}}{15.59s + e^{-7.03s}} \tag{22}$$

The mentioned design approach according to (11) – (13) results in the controller

$$G_R(s) = \frac{Q(s)}{P(s)} = 0.045 \frac{15.59s + e^{-7.03s}}{s + 0.045\left(1 - e^{-8.96s}\right)} \tag{23}$$

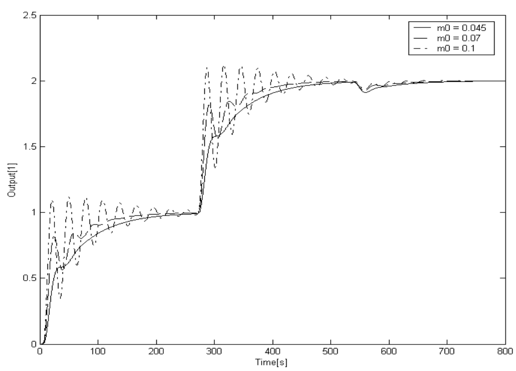
where the "tuning knob" m_0 is calculated according to (14) as $m_0 = 0.045$.

Fig. 4 demonstrates efficiency of the controller to control plant (21). Simulation control responses of output and input system signals, respectively, are displayed in the pictures. Various values of m_0 are chosen to verify the "equalization principle". The responses in Figure 4 clearly indicate that higher values of m_0 contribute to the faster but more oscillating control responses. Overall, the simulation responses are too slow in contrast to the step response of (21), where the settling time is approximately equal to 40s. The value m_0 = 0.045 calculated from (14) based on considerably modified "equalization method" gives quite satisfactory control response in comparison with other options of m_0. However, it is reasonable suspicious that frequency simplification of the controller can notably contribute to uselessness of the mentioned tuning method.

Model (10) is capable to describe in particular non-oscillatory plants. This feature is due to absence of more descriptive parameters in the model, especially in the nominator. Thus, for systems of more complex dynamics it is better to choose a different anisochronic model which has more parameters. However, this choice insists an investigation of identification techniques.

6 CONSLUSIONS

The contribution is focused on algebraic control approach in the special ring of proper and stable RQ-meromorphic functions (R_{MS}). A delayed plant is described as a ratio of two terms in R_{MS}. Both the input-output delayed systems and anisochronic (dynamic-delayed) systems are assumed. For the plant model, the control synthesis is then performed through a solution of a Diophantine equation in this ring. The methodology generates a class of Smith-like delay compensating controllers. The design method brings a scalar tuning parameter $m_0 > 0$ that can be adjusted by various strategies; the "equalization method" can be one of them.



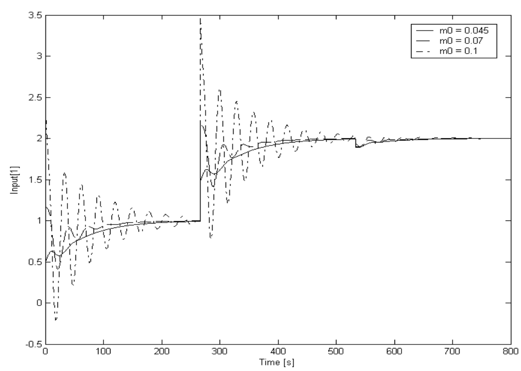


Fig. 4 Fifth order plant modelled by first order anisochronic model and controlled by (23) - output and input for various m_0 values

The methodology is illustrated by the simulation examples for stable and unstable first order systems. Higher order dynamics approximation using anisochronic model is also presented in the example. The simulations show simplicity and usability of the proposed methodology for delayed systems. For further study, the problems of anisochronic model estimation, validation and searching of suitable controller tuning method are opened.

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