Sborník vědeckých prací Vysoké školy báňské - Technické univerzity Ostrava
číslo 2, rok 2007, ročník LIII, řada strojní
článek č. 1560

Darja NOSKIEVIČOVÁ*

OPTIMAL EWMA CONTROL CHART DESIGN USING SELECTED STATISTICAL SOFTWARE

NÁVRH OPTIMÁLNÍHO EWMA REGULAČNÍHO DIAGRAMU S VYUŽITÍM VYBRANÉHO STATISTICKÉHO SOFTWARE

Abstract

Today there are many statistical software packages wholly or partially specialized in statistical process control methods (SPC). They offer not only classical Shewhart control charts but also advanced methods suitable for non-standard situations when classical Shewhart methods fail. The ability to select the suitable control chart and insert data into the PC program is not sufficient for the effective and correct application of the selected control chart. The PC programs ask for some parameters whose optimal setting determinates correct application of the selected control chart. And information in Helps often is not sufficient. In this paper these problems are analyzed when designing optimal EWMA chart parameters using Statgraphics Plus, version 5.0.

Abstrakt

Dnes existuje řada statistických softwarových balíků, které jsou zcela nebo zčásti zaměřeny na metody statistické regulace procesu (SPC). Nabízejí nejen klasické Shewhartovy regulační diagramy, ale také pokročilejší metody vhodné pro nestandardní situace, ve kterých klasické Shewhartovy regulační diagramy selhávají. Pro efektivní a správnou aplikaci vybraného regulačního diagramu nestačí umět diagram správně vybrat a zadat data do počítače. PC programy dále vyžadují zadání určitých parametrů, jejichž správné stanovení rozhoduje o efektivnosti aplikace vybraného regulačního diagramu. Informace obsažené v Helpech programu často nejsou dostatečné. V tomto článku je uvedený problém analyzován v rámci návrhu optimálních hodnot parametrů EWMA regulačního diagramu za použití programu Statgraphics Plus, verze 5.0.

1 EWMA CONTROL CHART

The EWMA (exponentially weighted moving average) control chart is a solution for the situation when we are interested in detecting small shifts (about 1,5σ or less) supposing uncorrelated data. For correlated data there exists special type of the EWMA control chart – dynamic EWMA control chart. In this paper we do not deal with this special EWMA control chart. The test criterion in the EWMA chart Y_k is defined as follows:

$$y_k = (1-\lambda)^k . Y_0 + \lambda \sum_{j=1}^{k} (1-\lambda)^{k-j} . f(x_j) \tag{1}$$

where:

$f(x_j)$ - the value of the sample measure,

* doc., Ing., CSc., Department of QualityManagement, Faculty of Metallurgy and Material Engineering, VŠB – Technical University of Ostrava, 17. listopadu 15, Ostrava-Poruba, (+420) 59 732 4503, darja.noskievicova@vsb.cz

k - the subgroup order,

Y_0 - the target level of the parameter of the controlled quantity distribution.

If $Y_0 = \mu_0$ (μ_0 is the process target) and function $f(x_j)$ is a sample mean $\bar{x}_j$, then we have the EWMA control chart for sample means. When subgroup size $n = 1$, then we have the EWMA control chart for individuals.

EWMA control charts are charts with memory which is unlimited and unequal. Properties of this memory are determined by the parameter λ $(0 < \lambda < 1)$.

Central line in the EWMA control chart for sample means $CL = \mu_0$.

Control limits for this control chart are as follows:

$$UCL = CL + K\sigma_{EWMA} = \mu_0 + K\sigma_{EWMA}, \tag{2}$$

$$LCL = CL - K\sigma_{EWMA} = \mu_0 - K\sigma_{EWMA}.$$

where:

K - the constant for setting control limits.

Standard deviation σ_{EWMA} is:

$$\sigma_{EWMA} = \frac{\sigma_0}{\sqrt{n}} \cdot \sqrt{\frac{\lambda}{2-\lambda} \cdot \left[1-(1-\lambda)^{2k}\right]} \tag{3}$$

where:

σ_0 - a target level of standard deviation of controlled quantity,

n - a subgroup (sample) size,

k - a subgroup order.

As opposed to the Shewhart control charts control limits in the EWMA chart depend on the sample time moment k but quite quickly they approach steady state values

$$UCL_a = CL + K.\sigma_a = \mu_0 + K.\sigma_a,$$

$$LCL_a = CL - K.\sigma_a = \mu_0 - K.\sigma_a, \tag{4}$$

where σ_a we compute as

$$\sigma_a = \frac{\sigma_0}{\sqrt{n}} \cdot \sqrt{\frac{\lambda}{2-\lambda}}. \tag{5}$$

To be effective against a small process shift the EWMA control chart must be optimally designed. That means we must design suitable combination of parameters λ a K so that this combination gives suitable ARL performance for detecting small shift of the predetermined size. What does it mean? Does every user know what ARL performance is and how the values of parameters λ a K are connected with it? When he does not know it can he find sufficient information in the SW helps? We try to find answers to these questions in the next chapter.

2 THE BASIC DESCRIPTION OF „THE EWMA CHART ANALYSIS" IN THE ANALYZED STATISTICAL SOFTWARE

From the main Menu bar [4] we must choose Special – Quality Control – Time Weighted Charts – EWMA Chart (for the EWMA control chart for sample means) or EWMA Individuals Chart (for the EWMA control chart for individual observations).

After this selection the Analysis dialog box is displayed. In this window we select variable or variables with analyzed data (measured observations or computed sample means and sample ranges). In the both cases next we must enter a subgroup size.

After the data insertion we can see Analysis summary on the left and EWMA control chart on the right (Fig.1) in the next windows.

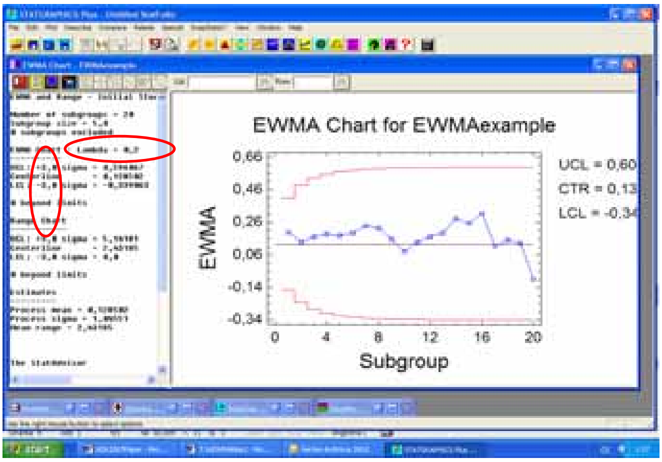

Fig. 1 Analysis summary

The report on the left also provides information about values of parameters λ and K. Some users will be satisfied with the values set by the system in spite of the fact that they do not know how the values λ and K were set and if these values are optimal for their situation, what is the ARL performance of this chart and so on. Answers to this questions and possibility to change design of the EWMA chart we can find in the EWMA Chart Options. This dialog box is available from all the tabular and graphical panes using the right mouse (Fig.2).

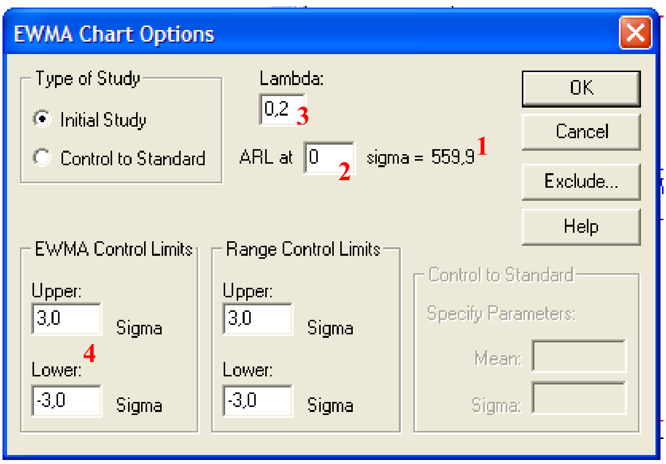

Fig. 2 Chart Options dialog box

This Dialog Box is decisive one for the optimal design of the EWMA chart. Here we can read all information about effectiveness of the chart offered automatically by the system. When we find out that this chart design does not meet our requirements we can change parameters λ a K so that their combination ensures ARL performance that satisfies our demands in a better way. But is a common user able to read this information in the dialog box or to do effective changes in the chart design? Does he know

1. what is ARL and can he set the adequate value of ARL?
2. when to change the number in the field next to the ARL?
3. what is the best value of λ ?
4. what is the value for the computing control limits in the EWMA chart and what is the best value of this parameter?

Can such common user find answers in the helps?

3 A BRIEF ANALYSIS OF HELPS IN THE APPLIED STATISTICAL SOFTWARE

For the comparison there were realized analyses of the Helps in Statgraphics Plus version 5.0 and Statgraphics Centurion. The answers to questions 1-4 are in these versions of Statgraphics insufficient. In the Statgraphics Centurion manual answers to questions 1-4 are more detailed but it is not sufficient for a correct design of the EWMA chart, too.

Detailed answers could be found in the next chapter.

4 DESIGN OF THE OPTIMAL EWMA CONTROL CHART

In this chapter answers to questions 1–4 formulated in the previous chapter will be given.

4.1 ARL

ARL is one of the measurements for description of the performance of a control chart. ARL (average run length) is the average number of points that must be plotted in control chart before a point indicates an out-of –control state.

When data are uncorrelated then for any Shewhart control chart the ARL can be computed as:

$$ARL = \frac{1}{p} \tag{6}$$

where:

p - the probability that any point falls outside the control limit in a control chart.

For instance in the control chart for variables where we suppose normal distribution of the controlled quantity and three-sigma control limits, probability that the point will lie between these control limits is 0,9973. Thus probability that any point will lie outside the limit is 1-0,9973=0,0027 (= p). From this ARL(0) = 1/0,0027 ≅ 370. That means that an out-of-control signal will be given on average every 370 samples (points in a chart) even if the process remains in control.

For the EWMA chart ARL can be calculated as follows assuming that $L(u)$ is the ARL and the EWMA starts with $\text{EWMA}_0 = u$ *(1)* [1]:

$$L(u) = 1 + (1/\lambda)\int_{LCL}^{UCL} L(y) f\{[y - (1-\lambda)u]/\lambda\} dy \tag{7}$$

where:

UCL, LCL - control limits,

$f(x)$ - the $N(\mu(\sigma^2/n)$ density function,

μ - the true process mean,

σ - the nominal process standard deviation,

n - the sample size.

The ARL corresponding to a risk of the false alarm α (i.e. probability that a point in a control chart falls outside the limits even if the process is in control) is called ARL(0). It is the average number of points before an out-of-control signal is given when the process is actually in control. The risk α needs to be as low as possible so ARL(0) should be as high as possible. In practice decision about ARL(0) depends on economical factors such as costs for the process interruption and the looking for the out-of-control state causes costs. When we want to have the EWMA control chart corresponding to three-sigma limits Shewhart control chart with $\alpha = 0{,}0027$ then we use ARL(0) = 370.

The ARL corresponding to a risk of missing signal β (i.e. probability of not detecting the shift in the process mean of a given size δ on the first subsequent sample) is called $\mathrm{ARL}(\delta)$. It is the average number of points in control chart that must be taken to detect a true process shift once one has occurred. For instance when $\mathrm{ARL}(\delta) = 8$, it takes on average 8 points after the true process shift has occurred before detection (point out of limit) of such out-of-control state. We want $\mathrm{ARL}(\delta)$ to be minimal.

When we want to compute ARL(0) in the EWMA Chart Options dialog box in Statgraphics Plus version 5.0 we must have number 0 in the field signed with the green arrowhead in Fig. 3. When we want to compute $\mathrm{ARL}(\delta)$, we must enter into this field the value of δ. δ is a size of the shift expressed in the process sigma (σ) which we want to detect as soon as possible (critical size of the shift). Its magnitude will likely depend on factors such as a process capability relative to specifications and costs of adjusting the process [1]. As we can see on Fig.3 values of ARL change with changes of lambda (λ) and upper and lower sigma constant K for setting EWMA control limits.

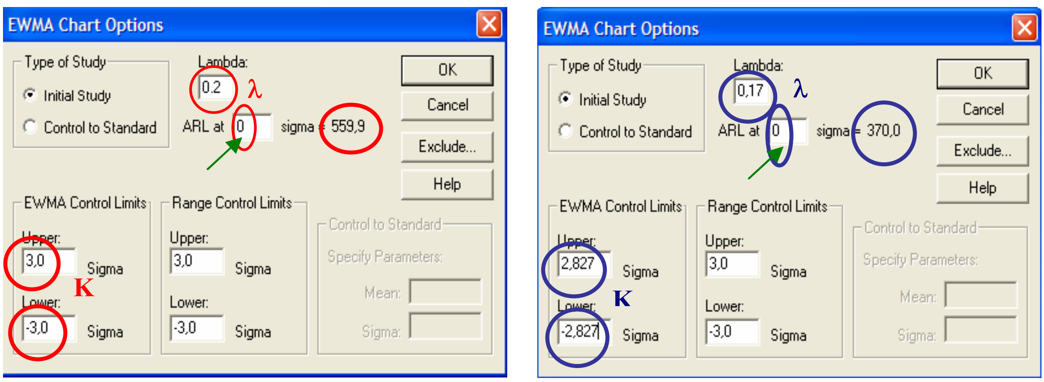

Fig. 3 Influence of changes of λ and K

Combination of $\lambda = 0{,}2$ and $K = 3$ gives ARL(0) nearly 560 (see the dialog box on the left on Fig. 3), combination of $\lambda = 0{,}17$ and $K = 2{,}827$ gives ARL(0) = 370 (see the dialog box on the right on Fig.3). How to set parameters λ and K will be explained in the next paragraph.

4.2 Setting optimal combination of parameters λ and K

Parameter λ is a smoothing parameter which determinates properties of the EWMA control chart memory and it pays that $0 < \lambda < 1$. Parameter K is the constant for setting control limits.

The combination of parameters λ and K is considered to be optimal when for a fixed false alarm α this combination produces the smallest possible risk β for a specified shift in the process mean. For setting of optimal values of λ and K we can use nomograms [1].

We recommend the following steps for setting optimal values λ and K:

1. Choose the smallest acceptable ARL(0).
2. Decide what magnitude of the shift in the process must be detected as quickly as possible (critical shift).

3. Choose the parameter λ which produces a minimal ARL for the critical shift using Crowder´s nomogram (Fig. 4). This nomogram is for ARL(0) = 370 or 250 or 100 or 50 and for the shift from 0,25 to 4 where the shift is expressed as a multiple of the standard deviation of the sample average Δ ($\Delta = \delta.\sqrt{n}$, where *n* is a sample size).

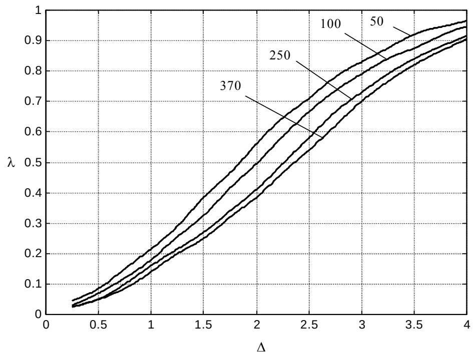

Fig. 4 Nomogram for optimal λ vs. shift Δ

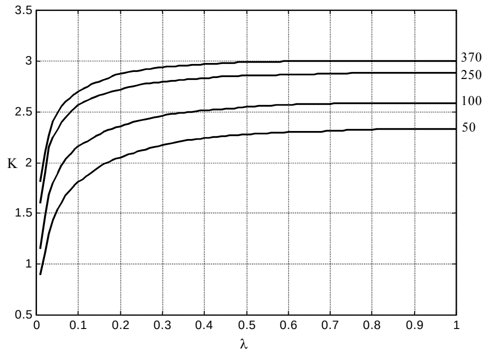

Fig. 5 Nomogram of *K* vs. λ

Optimal λ is read from the nomogram so that we find on the x-axis the critical shift δ expressed as Δ and through the curve for ARL(0) selected in the step 1 we read optimal value of λ on the y-axis.

4. Find optimal parameter *K* for setting control limits in the EWMA chart which satisfies the in-control ARL(0) set in the step 1. Use for it the next Crowder´s nomogram (Fig. 5). Each curve in this nomogram represents all possible combinations of λ and *K* ($\lambda \geq 0{,}01$) producing the in-control ARL(0) associated with that curve. For the documentation of it there is a table 1 in which you can see combinations of λ and *K* (with the most preferable values of λ) that approximately produce ARL(0) = 370.

Tab. 1 Combinations of λ and K for the same ARL(0)

λ	0,05	0,10	0,15	0,20	0,25	0,4
K	2,490	2,701	2,8005	2,859	2,898	2,959
ARL(0)	370,3	370,0	370,3	370,0	370,4	370,5

Optimal K can be found so that we find on the x-axis value of λ set in the previous step and through the curve for ARL(0) selected in the step 1 we read optimal value of *K* on the y-axis.

5. Enter this optimal λ and *K* to the EWMA Chart Options dialog box (see Fig. 2, numbers 3,4) and watch if this combination gives ARL(0) set in the step 1 (see Fig. 2, number 1).
6. As the reading from the nomograms need not to be precise enough, it is possible that there is some difference between target ARL(0) (step 1) and ARL(0) in the used SW. In that case we must try to change precision of the parameters λ and *K* to obtain the target value of ARL(0).
7. Compute ARL(δ) for the critical shift so that you will change number 0 in the field next to ARL (number 2 in Fig. 2) for δ - the critical shift expressed as a multiple of the process standard deviation σ.

These steps are shown on the example in the next chapter.

5. AN APPLICATION

5.1 Description of the situation

We have 20 samples with 5 independent measurements of the quality characteristics (in mm) in each (sample size is 5). Sample interval is 10 minutes. We want to detect as soon as possible shift from $\mu_0 = 100\text{mm}$ (target value of the process mean) to $\mu_1 = 110\text{mm}$ (critical value of the process mean with which the process produces unacceptable portion of nonconforming units). The target process standard deviation σ_0 is 20 mm. We want to apply the two-sided EWMA control chart for sample means corresponding to the $\text{ARL}(0)$ performance of classical two-sided Shewhart chart for sample means based on the target values of μ_0 and σ_0.

5.2 Solution

A. For the first time we must select in the EWMA Chart Options dialog box the Control to Standard type of study and specify parameters mean ($\mu_0 = 100\text{mm}$) and sigma (σ_0 is 20 mm) – see Fig. 6, character A.

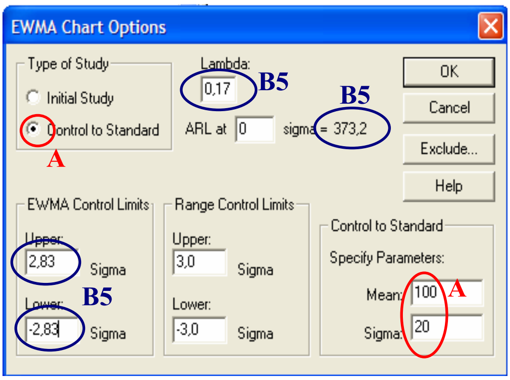

Fig. 6 Steps A and B5 of the example solution

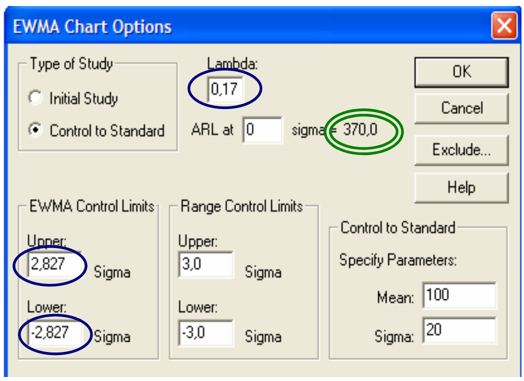

Fig. 7 Step B6 of the example solution

B. Then we find optimal λ and K applying the steps described in the previous chapter.

1. We decided to select ARL(0) = 370. It corresponds to ARL(0) of the three sigma limits Shewhart control chart with false alarm $\alpha = 0{,}0027$.
2. The critical magnitude which we want to detect in control chart as soon as possible is 10 mm ($\mu_1 - \mu_0$), i.e. $\delta = (\mu_1 - \mu_0)/\sigma_0 = 10/20 = 0{,}5$, $\Delta = \delta.\sqrt{n} = 0{,}5.\sqrt{5} = 1{,}118$.
3. For $\Delta = 1{,}118$ through the curve for ARL(0) = 370 in nomogram on the Fig. 4 we have found that the optimal parameter λ is approximately 0,17.
4. For optimal $\lambda = 0{,}17$ through the curve for ARL(0) = 370 in nomogram on the Fig. 5 we have read that the optimal parameter K is approximately 2,83.
5. We entered read values of λ and K into the EWMA Chart Options dialog box and the program computed ARL(0) = 373,2 (see Fig.6, character B5).
6. Because obtained ARL(0) is different from value 370, we try to change precision of the parameter K to produce ARL(0) closer to 370. We left $\lambda = 0{,}17$ and we change K from 2,83 to 2,827 and after that ARL(0) is exactly 370 (see Fig.7). We consider $\lambda = 0{,}17$ and K = 2,827 to be optimal values that give minimal $\text{ARL}(\delta)$.
7. We change value next to ARL from 0 to 0,5 (see δ in the step 2) and the program immediately computes $\text{ARL}(\delta) = 8{,}1$ (see Fig. 8). That means that the EWMA chart for sample means with $\lambda = 0{,}17$ and K =2,827 will give signal about the shift $0{,}5\sigma_0$ on average 8

samples after the shift has occurred. Considering the sample interval 10 minutes the chart will give the signal about this shift via point out of the control limit on average 80 minutes after the critical shift occurred.

For now it is possible to run the computation of the control limits and construction of the EWMA control chart using optimal parameters $\lambda = 0{,}17$ and $K = 2{,}827$ (see Fig. 9).

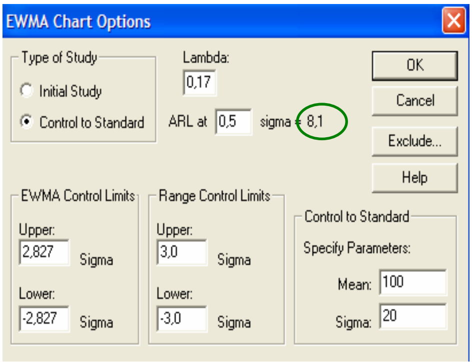

Fig. 8 Step B7

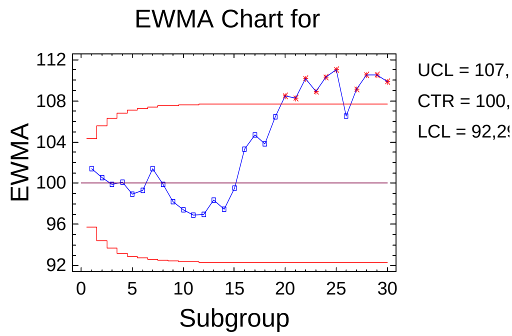

Fig. 9 EWMA control chart for example

From the Fig. 9 we can conclude that the analyzed process is not statistically stable (it is out of control).

6 CONCLUSIONS

It this paper there was analyzed the rate of information given by SW Statgraphics Plus version 5.0 and Statgraphics Centurion XV as to the correct design of the EWMA control chart. It was concluded that given information is not sufficient. The paper offers the procedure for setting optimal parameters λ and K and shows it on the example using SW Statgaphics Plus version 5.0.

REFERENCES

[1] CROWDER, S.V. Design of Exponentially Weighted Moving Average Schemes. *Journal of Quality Technology*. 1989, Vol. 21, Nr. 3, pp. 155-162. ISSN 0022-4065.

[2] *Statgraphics Plus Version 5.0 Online Manual.*

[3] *Statgraphics Centurion XV.* On-line User Manual. Available from www.statgraphics.com/documents.htm.

[4] *Statgraphics Plus Version 5.0.*

Reviewer: doc. Ing. Lenka Landrová, CSc., VŠB - Technical University of Ostrava