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STUDIES ON THE ACCURACY OF VARIOUS STRUCTURES OF CONTROL SYSTEMS

STUDIE PŘESNOSTI VYBRANÝCH STRUKTUR ŘÍDICÍCH SYSTÉMŮ

Abstract

This paper presents a comparison of the accuracy of closed-loop systems with static and astatic regulator and static even astatic plant with time delay, and an astatic regulator and astatic plant with delay. An effect the elements of feedback on the static accuracy of the closed-loop system was also determined. The steady state regulation error was accepted as the criterion of the static accuracy under step variations of both control and disturbance signal.

Abstrakt

Příspěvek představuje srovnání přesnosti uzavřených zpětnovazebních systémů se statickým a astatickým regulátorem s proporcionální a integrační regulovanou soustavou s dopravním zpožděním. Je zde vymezen vliv prvků zpětnovazebního obvodu na statickou přesnost. Jako kritérium pro posouzení statické přesnosti, tj. kvality byla využita trvalá regulační odchylka po krokových změnách žádané hodnoty a poruchové veličiny.

1 INTRODUCTION

In automation there are many methods of studying the accuracy of control systems, working in deterministic and stochastic conditions [2,3,4,5]. This research is conducted in states of both stable nd unstable conditions of work of the control systems. A popular criterion of judging their accuracy for stable work conditions is a steady regulation error value when the both steering and disturbing signals have step characteristics.

2 PRELIMINARIES

The base for the analysis is a one-dimensional regulation circuit consisting of a regulator, an object with transfer function respectively $G_R(s)$ and $G_O(s)$ and measurement sensor with S(s) transmittance.

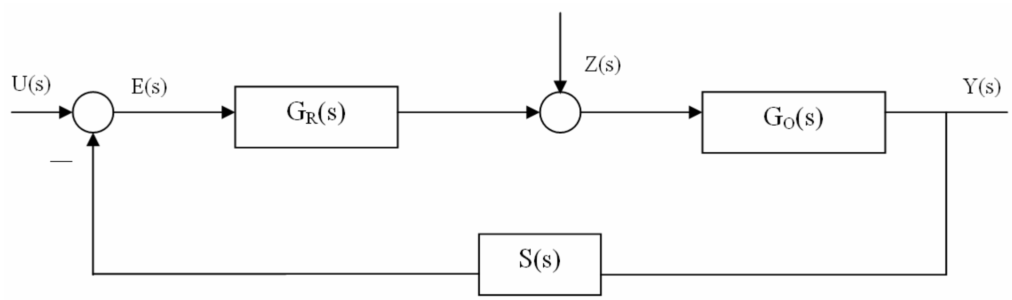

Fig.1 Block-diagram of one-dimensional regulation circuit

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Stable regulation errors will be calculated basing on avoidance and disturbance transfer function of the circuit. They are presented in forms of expressions:

$$G_\varepsilon(s) = \frac{E_n(s)}{U(s)} = \frac{1}{1 + G_R(s)G_O(s)S(s)} \tag{1}$$

assuming that Z(s) = 0, and

$$G_z(s) = \frac{E_z(s)}{Z(s)} = \frac{G_O(s)S(s)}{1 + G_R(s)G_O(s)S(s)} \tag{2}$$

assuming that U(s)=0,

where:

$G_\varepsilon(s)$ – error transfer function,

$G_z(s)$ – disturbance transfer function,

$E_n(s)$ – following component of error control,

$U(s)$ – reference signal

$E_z(s)$ – disturbance component of error control,

$Z(s)$ – disturbance signal.

Laplace's transformation of following factor of error control is

$$E_n(s) = U(s)G_\varepsilon(s) \tag{3}$$

while disturbance factor of this error is

$$E_z(s) = U(s)G_z(s) \tag{4}$$

The value of regulation's error steady following factor of $\varepsilon_{nu} = \lim_{t\to\infty} \varepsilon_n(t)$ may be set using the theorem about border value of Laplace's transformation [1]

$$\varepsilon_{nu} = \lim_{s\to 0} sE_n(s) = \lim_{s\to 0} sU(s)G_\varepsilon(s) \tag{5}$$

The value of regulation's error disturbance factor $\varepsilon_{zu} = \lim_{t\to\infty} \varepsilon_z(t)$ is equal to

$$\varepsilon_{zu} = \lim_{s\to 0} sE_z(s) = \lim_{s\to 0} sZ(s)G_z(s) \tag{6}$$

The value of regulation's error in regulation circuit is a sum of both factors, that is

$$\varepsilon_u = \varepsilon_{nu} + \varepsilon_{zu} \tag{7}$$

Value of this error is the accuracy criterion of regulation circuit in stable state.

3 ACCURACY ANALYSIS OF REGULATION CIRCUIT WITH STATIC AND ASTATIC REGULATORS AND PLANTS

The basis of this research will be regulation circuits containing:

a) static regulator type P and static object with delay,
b) static regulator type P and astatic object with delay,

c) astatic regulator type PI and static object with delay,
d) astatic regulator type PI and astatic object with delay,
e) static regulator type P, static object and sensor being an 1st rank inertial element,
f) static regulator, astatic object and measuring transducer.

The schematics of circuit presented in point (a) are shown on Fig. 2

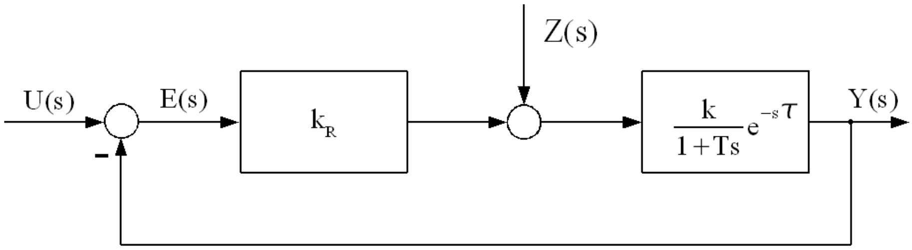

Fig. 2 Schematics of control system with P-type regulator and inertia-delaying object

Avoidance transfer function of this circuit, defined by expression (1) is following:

$$G_{\varepsilon}(s) = \frac{1+Ts}{k_R k e^{-s\tau} + Ts + 1}$$

Regulation's avoidance following factor in stable state with step changes of reference signal

$$U(s) = \frac{U}{s}$$

Calculated basing on relation (5) expresses itself as

$$\varepsilon_{nu} = \frac{U}{k_R k + 1}$$

Disturbance transmittance of the regulation circuit is based on relation (2) and is equal to

$$G_z(s) = \frac{k e^{-s\tau}}{k_R k e^{-s\tau} + Ts + 1}$$

The regulation's error disturbance factor in stable state with step changes of disturbance signal

$$Z(s) = \frac{Z}{s}$$

calculated basing on relation (6) is equal to

$$\varepsilon_{zu} = \frac{Zk}{k_R k + 1}$$

The steady value of regulation's error is

$$\varepsilon_{zu} = \frac{U + Zk}{k_R k + 1}$$

Circuit described in point (b) is shown in Fig. 3

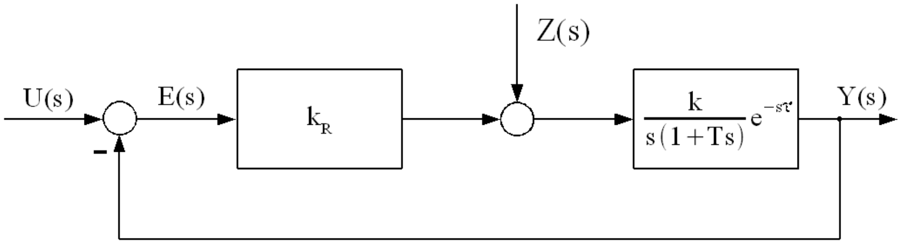

Fig.3 Schematics of regulation circuit with P-type regulator and astatic object with delay

Error transfer function of this control system, steady avoidance following factor with step changes in reference signal, disturbance transfer function, regulation's avoidance disturbance factor in stable state with step changes in disturbance, were calculated basing on corresponding relations (1), (5), (2) and (6) respectively, have the form of

$$G_{\varepsilon}(s) = \frac{s(1+Ts)}{k_R k e^{-s\tau} + Ts^2 + s}$$

$$\varepsilon_{nu} = 0$$

$$G_z(s) = \frac{k e^{-s\tau}}{k_R k e^{-s\tau} + Ts^2 + s}$$

$$\varepsilon_{zu} = \frac{Z}{k_R}$$

Therefore:

$$\varepsilon_u = \frac{Z}{k_R}$$

For the circuit presented in point (c), which schematics are shown in Fig. 4

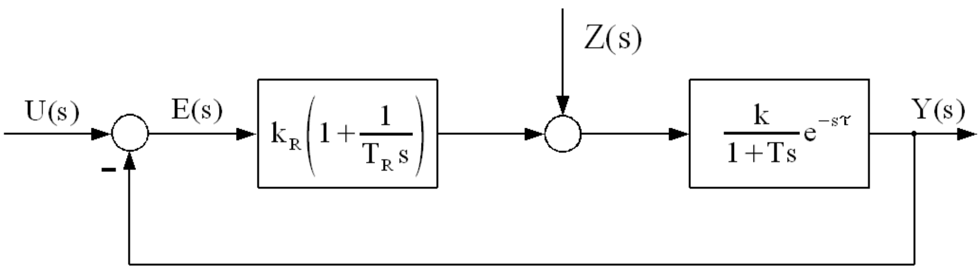

Fig. 4 Block-diagram of regulation circuit with astatic regulator type PI and intertial-delaying object

values of $G_\varepsilon(s), \varepsilon_{nu}, G_z(s), \varepsilon_{zu}$ are calculated basing on the same expressions (1), (5), (2) and (6) are the following

$$G_\varepsilon(s) = \frac{T_R s(1+Ts)}{k_R k(1+T_R s)e^{-s\tau} + T_R s(1+Ts)}$$

$$\varepsilon_{nu} = 0$$

$$G_z(s) = \frac{kT_R s e^{-s\tau}}{k_R k(1+T_R s)e^{-s\tau} + T_R s(1+Ts)}$$

$$\varepsilon_{zu} = 0$$

and

$$\varepsilon_u = 0$$

Circuit in point (d) is presented on Fig. 5

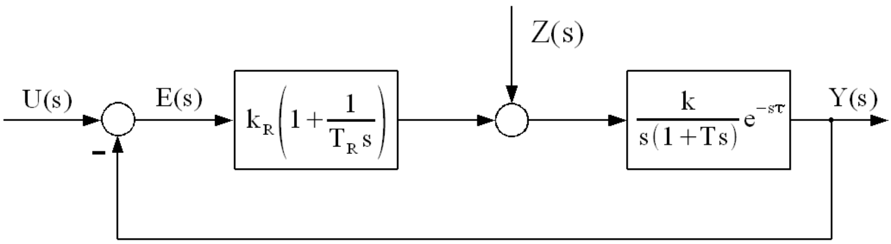

Fig. 5 Circuit with astatic both regulator and object

Variables $G_\varepsilon(s), \varepsilon_{nu}, G_z(s), \varepsilon_{zu}$ are connected in following relations

$$G_\varepsilon(s) = \frac{T_R s^2(1+Ts)}{k_R k(1+T_R s)e^{-s\tau} + T_R s^2(1+Ts)}$$

$$\varepsilon_{nu} = 0$$

$$G_z(s) = \frac{kT_R s e^{-s\tau}}{k_R k(1+T_R s)e^{-s\tau} + T_R s^2(1+Ts)}$$

$$\varepsilon_{zu} = 0$$

and

$$\varepsilon_u = 0$$

Let's discuss the circuit presented in point (e) (Fig. 6)

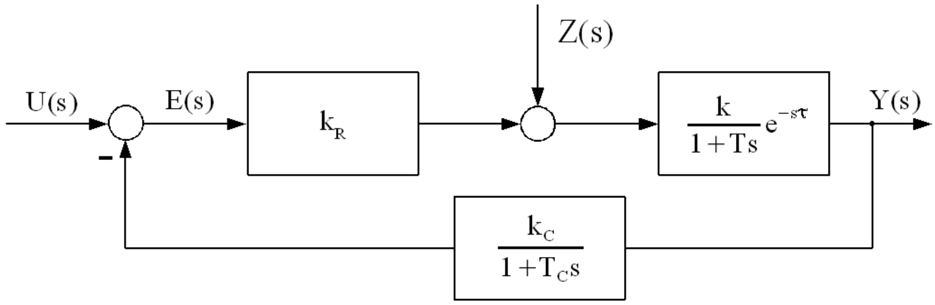

Fig. 6 Block-diagram with static regulator and intertia-delaying object, and a interia sensor

For this circuit respective variables of $G_\varepsilon(s), \varepsilon_{nu}, G_z(s), \varepsilon_{zu}$ are shown in relations

$$G_\varepsilon(s) = \frac{(1+Ts)(1+T_C s)}{k_R k k_C e^{-s\tau} + (1+Ts)(1+T_C s)}$$

$$\varepsilon_{nu} = \frac{U}{k_R k k_C + 1}$$

$$G_z(s) = \frac{k k_C e^{-s\tau}}{k_R k k_C e^{-s\tau} + (1+Ts)(1+T_C s)}$$

$$\varepsilon_{zu} = \frac{Z k k_C}{k_R k k_C + 1}$$

$$\varepsilon = \frac{U + Z k k_C}{k_R k k_C + 1}$$

CONCLUSIONS

In the control systems described above, steady following components of error signal ε_u when using static regulators (type P, for example). In case of astatic regulators (i.e. type PI), regardless of objects character, regulation's error stable factor is equal zero.

Measurement sensor's inertia does not make significant impact on regulation's error value. It is mostly influenced by amplification of the regulator and regulation's object.

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