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MODELING OF WORKING PRESSURE CHANGING PROCESSES ORIENTED ONTO THERMODYNAMIC ANALYZES

MODELOVÁNÍ PRACOVNÍCH PROCESŮ SE ZMĚNOU TLAKU S OHLEDEM NA TERMODYNAMICKOU ANALÝZU

Abstract

The modeling approach using dimensionless parameters oriented onto thermodynamic analyzes has been presented for the so-called working or pressure change processes. With help of only one dimensionless parameter, which almost exactly matches the polytropic efficiency, all important processes in turbines and compressors of the power engineering systems can be univocally determined. Furthermore the important properties of the exergy concept, which differ from such properties of the common thermodynamic caloric state parameters like inner energy or enthalpy have been entirely discussed and presented a new general method of applying them to process rating. The rating quotient obtained, the thermodynamic effectivity, is an exergy characteristics of the energy conversions by means of only one numerical value between zero and one. It has been pointed out the very good applicability of the dimensionless modeling parameter, which matches the known polytropic efficiency, for thermodynamic analyzes, especially the exergy ones.

Abstrakt

Metodika modelování používající bezrozměrných parametrů je orientována na termodynamickou analýzu tepelných procesů se změnou tlaku. Pomocí pouze jednoho dimenzionálního parametru, kterým je většinou polytropická účinnost, je analyzována většina důležitých procesů v turbínách, kompresorech a energetických systémech. Důležité vlastnosti exergetického konceptu, které se liší od vlastností známých termodynamických veličin jako je vnitřní energie nebo entalpie jsou diskutovány a prezentovány v nové metodě zhodnocení energetických systémů. Termodynamická efektivita jako exergetická charakteristika energetických konverzních procesů je parametr v mezích od nuly do jedné. Je to rovněž velmi dobrý dimenzionální parametr pro modelování pro stanovení polytropické účinnosti termodynamických a exergetických analýz.

1. INTRODUCTION

For the basic thermodynamic analysis of any gas change in a power device the perfect gas equation of state and the polytropic process model are used. This polytropic model, however, doesn't take into account process irreversibilities. It assumes process reversibility in the same way, as it was assumed by the author of this model, Gustav Zeuner. The simplicity of the polytropic model is the most valuable advantage.

In many cases it is important to estimate the process irreversibilities. It is because of the Second Law Analysis or the Exergy Method in particular.

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In the following the application possibility of the irreversible polytropic process model for the gas compression modeling and analysis will be discussed. While analyzing the adiabatic compression process, the irreversibilities are taken into account by the polytropic exponent numerical value. That's why such a process is often called the pseudo-polytropic (the entropy dS_{irr} will be generated or produced due to irreversibilities).

2. THE IRREVERSIBLE PROCESS MODEL FOR VARIABLE AMOUNT OF WORKING AGENT

To explain the application of the polytropic efficiency of a process to the non-adiabatic compression (the appropriate expansion is a very rarely used) the following derivation will be presented, [03]. From the First Law there is

$$dI = dQ + Vdp + \bar{i}dn \quad (01)$$

where dQ is the heat supplied or (the usually applied case) carried away (i.e. abstracted). For processes, in which irreversibilities are observed, the last equation can be written down as

$$dI = dQ_{\text{rev}} + Vdp + dQ_{\text{irr}} + \bar{i}dn = dQ_{\text{rev}} + dL_t + \bar{i}dn$$

i.e. the total heat dQ in the balance equation of the 1. Law consists of two parts

$$dQ = dQ_{\text{rev}} + dQ_{\text{irr}}$$

where dQ_{rev} is the heat exchanged with an outer system and dS_{irr} the dissipative heat TdQ_{irr} due to process irreversibilities. The useful (technical or shift) work is [01]–[02]

$$dL_t = Vdp + dQ_{\text{irr}}$$

The in the expansion process the really obtained technical (useful or shift) work is

$$-dL_t = -Vdp - dQ_{\text{irr}} \quad (\text{for } dp < 0)$$

and in the compression one really supplied, or more exact the observed technical work obtained by the working agent (gaseous substance, e.g. the ideal gas)

$$Vdp = dL_t - dQ_{\text{irr}}$$

or the supplied technical (or useful, which is the same) work

$$dL_t = Vdp + dQ_{\text{irr}} \quad (\text{for } dp > 0)$$

The first dimensionless partial coefficient σ will take into account the process irreversibilities (dissipative energy), namely

$$\sigma^{\text{exp}} = \frac{-dL_t}{-Vdp} = \frac{-Vdp - dQ_{\text{irr}}}{Vdp + dQ_{\text{irr}}} = 1 - \left(\frac{dQ_{\text{irr}}}{-Vdp} \right) \quad (\text{for } dp > 0) \quad (02)$$

$$\sigma^{\text{com}} = \frac{Vdp}{dL_t} = \frac{Vdp}{Vdp + dQ_{\text{irr}}} = 1 - \left(\frac{dQ_{\text{irr}}}{dL_t} \right) \quad (\text{for } dp < 0) \quad (03)$$

This coefficient gives an information about the relation of process irreversibilities and technical (useful) work in a reversible case by the same state change of working agent (gas) in both processes. To achieve a generality it can be assumed

$$\text{expansion} \quad (dp < 0): \quad \sigma = \sigma^{\text{exp}} \quad (04)$$

$$\text{compression} \quad (dp > 0): \quad \sigma = \frac{1}{\sigma^{\text{com}}} \quad (05)$$

Herewith is the 1. Law, Equation (01) to

$$dI = dQ + \sigma Vdp + \bar{i}dn$$

For processes with heat exchange with surroundings (carrying away or supplying it from or to outer systems), the second partial dimensionless coefficient can be formulated:

$$\text{expansion} \quad (dp < 0): \quad \omega = \omega^{\text{exp}} = \frac{dQ_{\text{rev}}}{-Vdp}$$

$$\text{compression} \quad (dp > 0): \quad \omega = \omega^{\text{com}} = \frac{-dQ_{\text{rev}}}{Vdp}$$

whereby the heat amount dQ_{rev} corresponds to the limit case of the reversible process and means the heat exchanged (or transferred). The third partial dimensionless coefficient, the mole number change coefficient, can be eventually formulated, namely in just the same way as the previous one

$$\begin{array}{ll} \text{expansion} & (dp < 0): \quad \nu = \nu^{\text{exp}} = \frac{\bar{i} dn}{-V dp} \\ \text{compression} & (dp > 0): \quad \nu = \nu^{\text{com}} = \frac{-\bar{i} dn}{V dp} \end{array}$$

This coefficient is important, when the leakages in the compression or expansion process are to be analyzed. In a real process these partial coefficients σ , ω and ν obviously change, but it can be assumed that their value is constant in the whole analyzed process.

Hence, the First Law Equation (01) will be to

$$dI = dQ_{\text{rev}} + V dp + dQ_{\text{irr}} + \bar{i} dn = dQ + dL_t + \bar{i} dn = (\sigma - \omega - \nu) \cdot V dp$$

e.g.

$$\frac{dI}{V dp} = \sigma - \omega - \nu \quad (06)$$

Taking into account very good known dependencies for ideal gases, i.e.

$$dI = n \bar{c}_p dT; \quad \bar{c}_p = \frac{k \bar{R}}{k-1}$$

and the Clapeyron equation of state $pV = n \bar{R} T$, it follows

$$d \ln(Tn) = \frac{k-1}{k} (\sigma - \omega - \nu) d \ln p \quad (07)$$

whereby the perfect gas was assumed as the working agent (the system). The integration between the state 1 and state 2 yields

$$\frac{T_2 n_2}{T_1 n_1} = \left(\frac{p_2}{p_1} \right)^{\frac{k-1}{k} (\sigma - \omega - \nu)} \quad \text{and} \quad \frac{V_2}{V_1} = \left(\frac{p_2}{p_1} \right)^{\frac{k-1}{k} (\sigma - \omega - \nu) - 1}$$

The appropriate substitute polytropic exponent can be written down, e.g.

$$m = \frac{k}{k - (k-1)(\sigma - \omega - \nu)} \quad (08)$$

Thus, taking into account the three characteristic dimensionless coefficients the substitute polytropic equation has been obtained [03], i.e.

$$pV^m = \text{const.}$$

whereby the polytropic exponent describes process irreversibilities, heat transferred in a process and leakages of the working agent (perfect gas), [04]–[05].

3. THE CASE OF CONSTANT AMOUNT OF WORKING AGENT

The Equation (06) for $\nu=0$ will be to

$$\frac{dI}{V dp} = \sigma - \omega \quad \text{or} \quad \frac{d\bar{i}}{\bar{v} dp} = \sigma - \omega \quad (\text{i.e. } \frac{di}{v dp} = \sigma - \omega) \quad (09)$$

and Equation (07) to

$$\frac{k}{k-1} \cdot \frac{dT}{T} = (\sigma - \omega) \frac{dp}{p} \quad (10)$$

After integration one becomes

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1} \right)^{\frac{k-1}{k} (\sigma - \omega)}$$

i.e. the polytropic process equation. Comparing exponent of it with the appropriate polytropic process exponent it follows

$$\frac{m-1}{m} = \frac{k-1}{k}(\sigma - \omega) \quad (11)$$

Is there no heat exchange with outer systems (heating up or cooling), then $\omega=0$ (in process & chemical engineering the so-called one phase process) and are there no irreversibilities ($\sigma=1$), the analyzed process is an isentropic one, for which obviously yields $m=k$. Is there $\sigma=1$ and $\omega=1$, the process is a reversible isotherm change and the appropriate polytropic exponent $m=1$. In other cases the exponent equals to

$$m = \frac{1}{1 - \frac{k-1}{k}(\sigma - \omega)} \quad \text{or} \quad m = \frac{k}{k - (k-1)(\sigma - \omega)} \quad (12)$$

cf Equation (08).

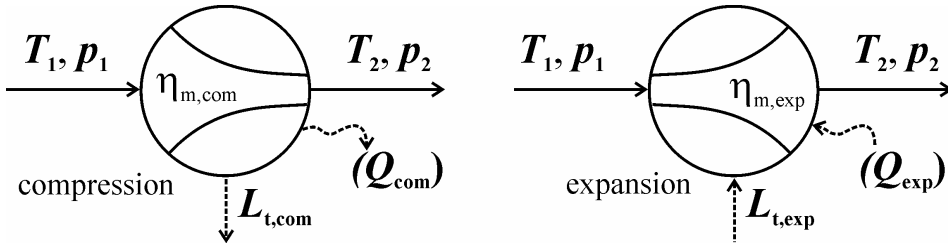


Fig. 1 Schemes of the compression and expansion processes as typical pressure changing ones

Analyzing the two typical processes (Fig. 1), one becomes with the above Equation (09) following relations: by the adiabatic expansion is the enthalpy change in a technical process less, than the reversible useful work, i.e. $di/vdp \leq 1$ or $di \leq vdp$, and in the case of adiabatic compression on the contrary, i.e. $di \geq vdp$ or $di/vdp \geq 1$. Defining an appropriate dimensionless parameter suitable for further process modeling, another influence factors should be taken into account, namely in such a manner that numerical values of this dimensionless parameter does not exceed the only logical range between zero and one. So, one should distinguish four possible typical cases, in particular

$$\eta = \frac{di}{vdp} = \sigma - \omega = \sigma^{\text{exp}} - \omega^{\text{exp}} \quad (\text{expansion with } di \leq vdp) \quad (13a)$$

$$\eta = \frac{di}{vdp} = \sigma - \omega = \frac{1 - \sigma^{\text{com}} \omega^{\text{com}}}{\sigma^{\text{com}}} \quad (\text{compression with } di \leq vdp) \quad (13b)$$

$$\eta = \frac{vdp}{di} = \frac{1}{\sigma - \omega} = \frac{1}{\sigma^{\text{exp}} - \omega^{\text{exp}}} \quad (\text{expansion with } di \geq vdp) \quad (13c)$$

$$\eta = \frac{vdp}{di} = \frac{1}{\sigma - \omega} = \frac{\sigma^{\text{com}}}{1 - \sigma^{\text{com}} \omega^{\text{com}}} \quad (\text{compression with } di \geq vdp) \quad (13d)$$

In the special case of adiabatic pressure changing processes the Equation (13a) refers to the expansion and the Equation (13d) to the compression. Taking into account Equation (11) it follows

$$\eta = \frac{k}{m} \frac{m-1}{k-1} \quad \text{for} \quad (di \leq vdp) \quad (14a)$$

and

$$\eta = \frac{m}{k} \frac{k-1}{m-1} \quad \text{for} \quad (di \geq vdp) \quad (14b)$$

respectively. There is not differentiated between expansion and compression because there is known in advance what a process should be determined with the dimensionless parameter η . These relations are the same as the polytropic efficiencies $\eta_{m,\text{com}}$ or $\eta_{m,\text{exp}}$ for the appropriate processes. Taking into account Equation (12), which is valid for a general case (i.e. not only adiabatic), one can become for the polytropic exponent:

$$m = \frac{1}{1 - \frac{k-1}{k} \cdot \eta_{m,\text{com}}} \quad \text{and} \quad m = \frac{1}{1 - \frac{k-1}{k} \cdot \frac{1}{\eta_{m,\text{exp}}}}$$

for a compression and expansion, respectively. In the special case of adiabatic ones, which are also called the pseudo-polytropic processes, the appropriate useful (technical) work can be determined as

$$l_{t,\text{com}} = \frac{\Delta i}{\eta_{m,\text{com}}} \quad \text{and} \quad l_{t,\text{exp}} = \eta_{m,\text{exp}} \Delta i$$

respectively. The polytropic process specific heat,

$$c_m = \frac{m-k}{k(m-1)} c_p$$

can be expressed as an η_m (or in general η) dependence, thus

$$c_m = \left(1 - \frac{1}{\eta}\right) c_p \quad \text{and} \quad c_m = (1 - \eta) c_p$$

for the compression and expansion, respectively. Using Equations (02)–(05) and (09) the appropriate total entropy change (reversible and irreversible) in a process can be determined. From the obvious relation

$$Tds = dq + dq_{\text{irr}}$$

yields

$$ds = (\sigma - \omega - 1) \frac{v}{T} dp \quad \text{and} \quad ds = \frac{\sigma - \omega - 1}{\sigma} dl_t$$

for the compression and expansion, respectively. Applying Equations (10) and (14) it becomes further

$$ds = (\eta - 1) R \frac{dp}{p} \quad \text{and} \quad ds = \left(\frac{1}{\eta} - 1\right) R \frac{dp}{p}$$

respectively. From the two last equations the appropriate exergy change in these processes can be calculated. From $de = di - T_0 ds$ it follows

$$de = [(T - T_0)\eta + T_0] R \frac{dp}{p} \quad \text{and} \quad de = \left[(T - T_0)\frac{1}{\eta} + T_0\right] R \frac{dp}{p}$$

respectively. There is also no need to distinguish between η_{com} and η_{exp} , because at the very beginning of modeling procedure it is known, which one of the two processes is to analyze.

It has been shown above, how easy it can be to proceed by formulating an exergy balance and appropriate rating quotient of the pressure changing processes, e.g. [06]. The working agent change in a process can be also taken into account. The discussion of such a case has been already presented in [05], and in a more detailed version in [04].

The process heats can be determined very simple, if only the compression or expansion analyzed are not assumed to be adiabatic.

One more pressure changing process, which is important in power systems modeling and analysis is the liquid pressure changing in pumps, where $p_2 > p_1$. The appropriate dimensionless parameter for incompressible working agent can be formulated as

$$\eta_{\text{wpp}} = \frac{v(p_2 - p_1)}{l_t}$$

For water turbines is can be defined as

$$\eta_{\text{wtr}} = \frac{l_t}{v(p_2 - p_1)}$$

The practical experience shows, the work spent for pressure increase of liquids is usually negligible.

4. EXERGY ANALYSIS OF FLOW SYSTEM PROCESSES

The exergy method of thermodynamic analysis was developed in the sixties and seventies and was a very important step in improving the most of mechanical engineering processes and devices. The main goal of these improvements was to reduce the energy consumption, i.e. to make the energy conversions and energy converters more effective. The greatest disadvantage of the exergy analysis, however, was the lack of the unified rating quotient. It seemed and seems to be obvious that on the base of the thermodynamic quantity, which allows the objective comparison of different forms of energies and energy interactions, the exergy, can be doubtless achieved the possibility of the objective comparison of energy conversions with a help of the appropriate generalized rating quotients. Alas, it wasn't so. The number of trials has been done, e.g. [07]–[10], but the presented algorithms couldn't be applied in an enough wide range of particular process cases.

In this paper there is discussed the unified exergy rating quotient, which bases on the exergy balance of the flow system process and which takes into account the specific property of the exergy, namely the existence of its zero value point (or inversion point of its changes in a process).

The exergy analysis and rating of processes discussed below is to find in the main exergy monographs [05], [11]–[18].

A flow thermodynamic system, the energy state of which is characterized by the enthalpy I , converts heat dQ into technical (or useful) work dL_t . The energy balance condition (1. Law) yields $dI = dQ + dL_t = 0$ and that of the entropy balance $dS = dQ/T + dQ_0/T_0 + dS_{\text{irr}} = 0$ (thermal condition) should be taken into account. The analyzed energy converter is not an entropy accumulator — dQ_0 is the additive heat carried away from the system by the temperature of the natural environment T_0 (i.e. $dQ_0 < 0$) to fulfill the energy balance and the thermal condition (entropy balance) at the same time, and dS_{irr} the irreversible entropy production (2. Law: $dS_{\text{irr}} > 0$). There is no volume compensation condition, because of the different character of the technical (useful or shaft) work $dL_t = Vdp$ in comparison with the volume absolute work $dL = -pdV$ of a closed thermodynamic system. With the thermal condition the additive heat equals to dQ_0 , and must be carried away from the flow system converting the heat dQ into the useful work dL_t

$$dQ_0 = -\frac{T_0}{T} dQ - T_0 dS_{\text{irr}} \quad (15)$$

and hence

$$\left(1 - \frac{T_0}{T}\right) dQ + dL_t = T_0 dS_{\text{irr}}$$

The exergy formula for the flow thermodynamic system can be obtained from

$$dQ = \left(1 - \frac{T_0}{T}\right) dQ + \frac{T_0}{T} dQ = dE_Q + dA_Q$$

as the sum of the heat exergy and heat anergy, and

$$dI = dQ + dL_t = \left(1 - \frac{T_0}{T}\right) dQ + \frac{T_0}{T} dQ + dL_t$$

to

$$dE_t = dE_Q + dL_t = dI - T_0 dS$$

There is further

$$E_t = (I - I_0) - T_0(S - S_0) \quad (16)$$

because for the intensity parameters of the natural (given) environment always valid

$$E_{t,0} = I_0 - T_0 S_0 = 0$$

The exergy of the flow thermodynamic system is called sometimes the exergy of the enthalpy.

The very good known behavior of this so-called thermo mechanical exergy wasn't used yet by the formulating the exergy rating quotients, [07]–[10]. Taking into account the special property of the system exergy as a thermodynamic parameter of state it was possible to formulate the objective

exergy rating quotients, independently on the temperature or pressure range in a process: below or above the appropriate values in the natural environment (T_0 or p_0), [05], [18]–[19]. For the perfect gas the thermo mechanical exergy can be calculated from Equation (16),

$$\bar{e}_{i,TP} = \bar{c}_p \left(T - T_0 - T_0 \ln \frac{T}{T_0} \right) + \bar{R} T_0 \ln \frac{p}{p_0} \quad (17)$$

It is to state, that there can be $\bar{e}_{i,TP} \geq 0$ or $\bar{e}_{i,TP} \leq 0$. The appropriate diagrams for the Equation (17) are presented in [14], page 41 (Figure 2.6), and [17], page 61 (Figure 2.10) in the coordinates $T/T_0 - p/p_0$ and the most interesting of them in [13], page 69 (Figure 22), and [16], page 41 (Figure 2.9), in the coordinates $e_{U,TP} - T$ with the pressure value p as the parameter (the thermo mechanical exergy is there called the physical exergy). The particular parts of the thermo mechanical exergy for the perfect gas, i.e. the thermal and the mechanical (pressure) exergy can be found noting, that

$$de_{TP} = \left(\frac{\partial e_{TP}}{\partial T} \right)_p dT + \left(\frac{\partial e_{TP}}{\partial p} \right)_T dp$$

and the further integration can be realized in an arbitrary way, e.g. assuming $p=p_0$ ($dp=0$) and then $T=T_0$ ($dT=0$), if only T_0 and p_0 are considered to be constant in the time of analysis. Using the appropriate differentials [17], the following formulas can be obtained

$$\bar{e}_{i,T} = \bar{c}_p \left(T - T_0 - T_0 \ln \frac{T}{T_0} \right) = \bar{c}_p T_0 \left(\frac{T}{T_0} - 1 - \ln \frac{T}{T_0} \right) \quad (18)$$

$$\bar{e}_{i,p} = \bar{R} T_0 \ln \frac{p}{p_0} \quad (19)$$

Thus, each of the parts described by Equations (18)–(19) can be taken separately in the exergy balance of any particular thermo mechanical process analyzed, i.e.

$$\Delta \bar{e}_{i,TP} = \Delta \bar{e}_{i,T} + \Delta \bar{e}_{i,p} = \bar{e}_Q + \bar{l}_1 - T_0 \Delta \bar{s}_{irr} \quad (20)$$

where

$$\bar{e}_Q = \int_1^2 \left(1 - \frac{T_0}{T} \right) d\bar{q} \quad (21)$$

is the exergy of heat exchanged with the surroundings and \bar{l}_1 the shaft work of the flow system analyzed. It is obvious, that the exergy Equation (21), as the process quantity, does change its algebraic sign, i.e. the direction in relation to the system balanced, while crossing in a process the value of $T=T_0$. It can be then divided into arising e_j^+ and vanishing e_i^- parts, i.e.

$$\bar{e}_Q = |\bar{e}_{Q+}^>| - |\bar{e}_{Q+}^<| - |\bar{e}_{Q-}^>| + |\bar{e}_{Q-}^<| \quad (22)$$

whereby + is for delivering and – for carrying away the appropriate quantity, whereby

$$\text{sgn}(\bar{e}_j^+) = \text{sgn}(T_0 d\bar{s}_{irr}) = \text{sgn}(d\bar{s}_{irr}) \quad \text{and} \quad \text{sgn}(\bar{e}_i^-) = -\text{sgn}(T_0 d\bar{s}_{irr}) = -\text{sgn}(d\bar{s}_{irr}) \quad (23)$$

because there is always $T_0 > 0$. The absolute value brackets have been used for calculation convenience only. The superscripts > or < indicate the range of changes of the appropriate intensity (here temperature T_0): above or below its value in the natural environment, respectively. The terms of Equation (22), which do not concern the process analyzed, will be simply set to zero in the exergy balance Equation (20).

The Equation (17) shows, that the thermal part of the total exergy change $\Delta e_{i,T}$ has an inversion point while crossing the value $T=T_0$ in a process. Such a behavior is peculiar for the exergy; other caloric parameters of state, such as the enthalpy I , have no inversion point of their algebraic sign in any process. The zero point of these parameters is selected in a fully optional way. The mechanical exergy change $\Delta e_{i,p}$ on the contrary, has no algebraic sign inversion point, even while crossing the pressure value of the natural environment p_0 . These facts should be taken into account by balancing processes in order to rate them in an objective, independent way. Dividing the exergy change given by Equation (17) in a similar way as the Equation (22), yields

$$\Delta\bar{e}_{1,T} = |\Delta\bar{e}_T^>| + |\Delta\bar{e}_T^<| - |\nabla\bar{e}_T^>| - |\nabla\bar{e}_T^<|$$

but from Equation (19) there is

$$\Delta\bar{e}_{1,p} = |\Delta\bar{e}_p| - |\nabla\bar{e}_p|$$

whereby conditions Equation (23) are valid. The differential symbol Δ indicates an increase and ∇ a decrease of the appropriate exergy change, respectively (according to [14], where adopted from J.H. Keenan). The balance Equation (20) can be then written in a form of the „thermodynamic transformation” (or „thermodynamic transition”), namely

$$|\nabla\bar{e}_T^>| + |\nabla\bar{e}_T^<| + |\nabla\bar{e}_p| + |\bar{e}_{Q^+}^>| + |\bar{e}_{Q^-}^<| + |\bar{l}_{1+}| \rightarrow |\Delta\bar{e}_T^>| + |\Delta\bar{e}_T^<| + |\Delta\bar{e}_p| + |\bar{e}_{Q^-}^>| + |\bar{e}_{Q^+}^<| + |\bar{l}_{1-}| + T_0\Delta\bar{s}_{irr} \quad (24)$$

or in general

$$\sum |\bar{e}_i^-| \rightarrow \sum |\bar{e}_j^+| + T_0\Delta\bar{s}_{irr} \quad (25)$$

All terms of the above balances are set to zero, when they do not concern the appropriate process analyzed. The universal, independent and objective rating quotient can be formulated as the process thermodynamic effectivity

$$\varepsilon = \frac{\sum |\bar{e}_j^+|}{\sum |\bar{e}_i^-|} \leq 1 \quad (26)$$

The application of this quotient, the process (or system) thermodynamic effectivity was detailed discussed in the monograph [05], [18] and [20]. It has been found, that it is the generalization of the exergy rating quotients used usually and, what’s more, it can be formulated without any subjective factor.

5. THERMODYNAMIC EFFECTIVITY OF A PROCESS IN A TURBINE

Power turbines do work usually adiabatic, so the „thermodynamic transformation”, Equation (24) will be to

$$|\nabla\bar{e}_T^>| + |\nabla\bar{e}_p| \rightarrow |\Delta\bar{e}_T^<| + |\bar{l}_{1-}| + T_0\Delta\bar{s}_{irr}$$

The terms $\Delta\bar{e}_p$ and \bar{l}_{1+} were also set equal to zero because they do not concern the gas expansion process. During expansion the gas temperature gets lower which means, that by temperatures above T_0 there is always $\Delta\bar{e}_T^>=0$ and under T_0 $\nabla\bar{e}_T^<=0$. Thus, the thermodynamic effectivity quotient is

$$\varepsilon_{dp<0,dQ=0} = \frac{|\Delta\bar{e}_T^<| + |\bar{l}_{1-}|}{|\nabla\bar{e}_T^>| + |\nabla\bar{e}_p|} \quad (27)$$

The particular forms of this quotient for the ranges above or under the natural environment temperature T_0 are

$$\varepsilon_{dp<0,dQ=0}^{\geq} = \frac{|\bar{l}_{1-}|}{|\nabla\bar{e}_T^>| + |\nabla\bar{e}_p|} = \frac{|\bar{l}_{1-}|}{|\nabla\bar{e}|} \quad \text{and} \quad \varepsilon_{dp<0,dQ=0}^{\leq} = \frac{|\Delta\bar{e}_T^<| + |\bar{l}_{1-}|}{|\nabla\bar{e}_p|}$$

The first quotient were very often presented in the literature as the exergy effectiveness of the gas expansion process in a turbine, but there wasn’t defined the appropriate temperature range under or above the T_0 . Noting $\tau_1=T_1/T_0$ and $\tau_2=T_2/T_0$ as the perfect gas dimensionless temperatures at the start and at the end of a process, respectively, which can be calculated i.e. with a help of the isentropic (or adiabatic) effectiveness, all the three presented quotients are to

$$\varepsilon_{dp<0,dQ=0}^{\geq} = \frac{\tau_1 - \tau_2}{\tau_1 - \tau_2 + \ln \frac{\tau_2}{\tau_1} - \frac{k-1}{k} \ln \frac{p_2}{p_1}} \quad \varepsilon_{dp<0,dQ=0}^{\leq} = \frac{k}{k-1} \frac{\ln \frac{\tau_2}{\tau_1}}{\ln \frac{p_2}{p_1}} = \frac{k}{k-1} \frac{m-1}{m} = \eta_{m,exp}$$

where $\eta_{m,exp}$ is the polytropic process effectiveness, and in the case, the temperatures $T_1>T_0$ and $T_2<T_0$.

$$\varepsilon_{dp<0,dQ=0} = \frac{\tau_1 - 1 - \ln \tau_2}{\tau_1 - 1 - \ln \tau_1 - \frac{k-1}{k} \ln \frac{p_2}{p_1}}$$

If $\tau_2=1$ (or $T_2=T_0$), the last of these three equations is identical with the first of them. The second equation, however, shows the independence of the thermodynamic effectivity value on process intensities — it depends only on the gas properties (k) and irreversibilities, which are expressed by the polytropic (or pseudo-polytropic) exponent m .

For non-adiabatic gas expansion processes the expression of the „thermodynamic transformation” can be written as

$$|\bar{e}_{Q+}^>| + |\bar{e}_{Q-}^<| + |\nabla \bar{e}_p| + |\nabla \bar{e}_T^>| \rightarrow |\bar{e}_{Q+}^<| + |\bar{e}_{Q-}^>| + |\bar{l}_{t-}| + |\Delta \bar{e}_T^<| + T_0 \Delta \bar{s}_{irr}$$

and the appropriate thermodynamic effectivity quotient

$$\varepsilon_{dp<0} = \frac{|\bar{e}_{Q+}^<| + |\bar{e}_{Q-}^>| + |\bar{l}_{t-}| + |\Delta \bar{e}_T^<|}{|\bar{e}_{Q+}^>| + |\bar{e}_{Q-}^<| + |\nabla \bar{e}_p| + |\nabla \bar{e}_T^>|}$$

When the heat delivered (subscript +) and carried away (subscript –) will be equal to zero, the above quotient will be the same as the Equation (27). In the practical technology such a non-adiabatic process is to analyze in the refrigerating engineering as a process in so-called expansion engines. It takes places in the temperature range under the natural environment value T_0 , where the heat is delivered. For such processes the thermodynamic effectivity quotient will be to

$$\varepsilon_{dp<0,dQ>0}^< = \frac{|\bar{e}_{Q+}^<| + |\bar{l}_{t-}| + |\Delta \bar{e}_T^<|}{|\nabla \bar{e}_p|}$$

This form of the exergy rating quotient for the low temperature expansion engines are to be find among others in [14], [21]–[24].

The expansion engines, however, very often do work in the temperature range which crosses the natural environment temperature T_0 . The non-adiabatic behavior is a result of the temperature difference between the working agent and the surroundings. By temperatures above T_0 there is to observe the heat flow from the working agent to the surroundings and under the T_0 from the surroundings to the working agent. The expanding gas will change its temperature from the T_1 to T_0 and further to T_2 . All the terms with superscripts $>$ will be integrated in the limits from T_1 to T_0 , and all the terms with superscript $<$ from T_0 to T_2 . Additionally $\bar{e}_{Q+}^> = \bar{e}_{Q-}^< = 0$ is taken into account. The thermodynamic effectivity of the process discussed will be therefore to

$$\varepsilon_{dp<0,dQ\neq 0} = \frac{|\bar{e}_{Q+}^<| + |\bar{e}_{Q-}^>| + |\bar{l}_{t-}| + |\Delta \bar{e}_T^<|}{|\nabla \bar{e}_T^>| + |\nabla \bar{e}_p|}$$

6 THERMODYNAMIC EFFECTIVITY OF PROCESSES IN A COMPRESSOR

For the adiabatic compression process the „thermodynamic transformation” can be formulated as

$$|\bar{l}_{t+}| + |\nabla \bar{e}_T^<| \rightarrow |\Delta \bar{e}_T^>| + |\Delta \bar{e}_p| + T_0 \Delta \bar{s}_{irr}$$

and the thermodynamic process effectivity in the whole temperature range

$$\varepsilon_{dp>0,dQ=0} = \frac{|\Delta \bar{e}_T^>| + |\Delta \bar{e}_p|}{|\bar{l}_{t+}| + |\nabla \bar{e}_T^<|}$$

whereby appropriate integrations are to be made between T_1 and T_0 or T_0 and T_2 , if only the T_0 value lies between the two process temperatures. It follows from the last equation for the range above T_0 :

$$\varepsilon_{dp>0,dQ=0}^{\geq} = \frac{|\Delta\bar{e}_T^{\geq}| + |\Delta\bar{e}_p|}{|\bar{l}_{t+}|} = \frac{|\Delta\bar{e}|}{|\bar{l}_{t+}|}$$

and for the range below T_0 :

$$\varepsilon_{dp>0,dQ=0}^{\leq} = \frac{|\Delta\bar{e}_p|}{|\bar{l}_{t+}| + |\nabla\bar{e}_T^{\leq}|}$$

The first of the two above rating quotients is the usually formulated in the literature exergy effectiveness of the adiabatic compression process, e.g. [14], [21]. For the perfect gas as working fluid yields (at $T_1=T_0$):

$$\varepsilon_{dp>0,dQ=0}^{\geq} = 1 - \frac{T_0 \ln \frac{T_2}{T_0} - \frac{k-1}{k} T_0 \ln \frac{p_2}{p_1}}{T_2 - T_0} = 1 - \frac{\ln \tau_2 - \frac{k-1}{k} \ln \pi}{\tau_2 - 1} \quad (28)$$

or assuming a polytropic process (for $T_1=T_0$, as well),

$$\varepsilon_{dp>0,dQ=0}^{\geq} = 1 - \left(\frac{m-1}{m} - \frac{k-1}{k} \right) \frac{\ln \pi}{\pi^{\frac{m-1}{m}} - 1}$$

where π equals to p_2/p_1 . For the process below natural environment temperature T_0 one gets

$$\varepsilon_{dp>0,dQ=0}^{\leq} = \frac{k-1}{k} \frac{m}{m-1} = \eta_{m,com}$$

This result was obtained by assuming the polytropic compression process. In fact, the assumed polytropic process is the so-called pseudo-polytropic one, because the exponent m gives the information about process irreversibilities (irreversible adiabatic process). In the same way the thermodynamic effectivity of the adiabatic expansion process of the perfect gas equaled to the polytropic effectiveness. For the adiabatic compression, however, the $\eta \equiv \eta_m$ value changes in the range between $(k-1)/k$ and 1, [05], [18], [20].

For the general case $T_1 \neq T_0$ and perfect gas as working fluid the thermodynamic effectivity by $T_1 < T_0$ but $T_2 > T_0$ it becomes

$$\varepsilon_{dp>0,dQ=0} = \frac{\frac{k-1}{k} T_0 \ln \frac{p_2}{p_1} + (T_2 - T_0) - T_0 \ln \frac{T_2}{T_0}}{(T_2 - T_0) - T_0 \ln \frac{T_1}{T_0}} = \frac{\frac{k-1}{k} \ln \pi + \tau_2 - 1 - \ln \tau_2}{\tau_2 - 1 - \ln \tau_2}$$

and by $T_1 > T_0$

$$\varepsilon_{dp>0,dQ=0}^{\geq} = \frac{T_0 \ln \frac{T_2}{T_0} - \frac{k-1}{k} T_0 \ln \frac{p_2}{p_1}}{T_2 - T_1} = \frac{\ln \tau_2 - \frac{k-1}{k} \ln \pi}{\tau_2 - \tau_1}$$

If only $T_1=T_0$, (or $\tau_1=1$) the last formula will be the same as the quotient obtained for such a case above, Equation (28), because the natural logarithm of one equals to zero.

„Thermodynamic transformation” for compression processes above T_0 with concurrent cooling by $T_2 \geq T_1$ is

$$|\bar{l}_{t+}| \rightarrow |\bar{e}_{Q-}^{\geq}| + |\Delta\bar{e}_T^{\geq}| + |\Delta\bar{e}_p| + T_0 \Delta\bar{s}_{irr}$$

and the appropriate thermodynamic effectivity quotient

$$\varepsilon_{dp>0,dQ<0}^{\geq} = \frac{|\bar{e}_{Q-}^{\geq}| + |\Delta\bar{e}_T^{\geq}| + |\Delta\bar{e}_p|}{|\bar{l}_{t+}|} = \frac{|\bar{e}_{Q-}^{\geq}| + |\Delta\bar{e}|}{|\bar{l}_{t+}|} \quad (29)$$

For the isothermal compression in the same way the quotient

$$\varepsilon_{dp>0,dQ<0}^{dT=0} = \frac{|\Delta \bar{e}_p|}{|\bar{l}_{t+}|} \quad (30)$$

can be obtained, that corresponds to the exergy effectiveness formulated e.g. in [14], [21] or [25]. It was proposed (e.g. in [14], [25]) as the general exergy rating quotient for gas compression processes of all kinds. But, if only the analyzed isothermal process runs above the natural environment temperature T_0 , than from the Equation (29) yields

$$\varepsilon_{dp>0,dQ<0}^{dT=0,>} = \left(1 - \frac{T_0}{T}\right) + \frac{|\Delta \bar{e}_p|}{|\bar{l}_{t+}|}$$

So, the Equation (30) is the special case of the last expression, when only $T \neq T_0$ (especially $T \geq T_0$).

7. NUMERICAL VALUES OF RATING QUOTIENTS

The numerical values of dimensionless parameter η for common compression and expansion processes are needed to determine a proper description of real processes in real power devices. They can be obtained directly an indirectly using another rating quotients, e.g. the isentropic efficiency. Results of an extensive study have been presented in [03] and [05]. The study based on data found in scientific and engineering papers, specializing handbooks, textbooks and manufacturers information.

Practical numerical values of the expansion isentropic efficiency in steam turbines are about 0.8, and for gas turbines are in the range between 0.85 and 0.92. The appropriate value of the dimensionless parameter η that determines the expansion process can be chosen using the left-hand Equation (31) or the diagram in Fig. 2, taking into account the given pressure change. The polytropic efficiency for adiabatic expansion processes in real power devices equal to 0.7–0.9. For adiabatic compressions the isentropic efficiency lies in the range from 0.8 to 0.9, but there are units that are characterized by the isentropic efficiency of even 0.7. The dimensionless parameter η can be then found using right-hand Equation (31) or the diagram in Fig. 3.

$$\eta_{s,\text{exp}} = \frac{\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} - 1}{\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k\eta}} - 1} \quad \text{and} \quad \eta_{s,\text{com}} = \frac{\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} - 1}{\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k\eta}} - 1} \quad (31)$$

These equations can be rearranged to get

$$\eta = \frac{k \ln \left\{ 1 + \eta_{s,\text{exp}} \left[\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} - 1 \right] \right\}}{(k-1) \ln \left(\frac{p_2}{p_1}\right)} \quad \text{and} \quad \eta = \frac{(k-1) \ln \left(\frac{p_2}{p_1}\right)}{k \ln \left\{ 1 + \frac{1}{\eta_{s,\text{com}}} \left[\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} - 1 \right] \right\}}$$

respectively. The polytropic efficiency, which in fact matches the dimensionless parameter η , is for big compressors equal to 0.80–0.85, for the middle ones 0.75–0.80 and for little units 0.70–0.75. The appropriate polytropic exponent is in the range between 1.45 and 1.50.

The expansion devices, called detanders, used in the low temperature technologies, e.g. in air or helium liquefaction systems are characterized by the „isentropic efficiency”, although they do not operate adiabatic (usually a little heat supply below the natural environment temperature T_0). The appropriate rating quotient refers to that of adiabatic expansion, Equation (31). For air expansion machines the appropriate numerical values are

- for the start temperature 30 °C $\eta_{s,\text{det}} = 0.66\text{--}0.80$
- for the start temperature –40 °C $\eta_{s,\text{det}} = 0.62\text{--}0.76$

- for the start temperature $-50\text{ }^{\circ}\text{C}$ $\eta_{s,\text{det}} = 0.60\text{--}0.68$
- for the start temperature $-120\text{ }^{\circ}\text{C}$ $\eta_{s,\text{det}} = 0.52\text{--}0.60$

whereby the less values refer to the machines of less capacity. In the practice there is assumed the value of isentropic efficiency in the range between 0.65 and 0.85.

Piston compressors with cooling are characterized by the very good known isothermal efficiency quotient (usually from 0.4 to 0.7), but very often the appropriate polytropic exponent is given. Its numerical values lie in the range between 1.18 and 1.35. For little piston compressors the isothermal efficiency equals 0.25–0.40, for one cylinder high-speed ones 0.35–0.55, for little multistage or medium units with the mean piston speed of about 5 m/s 0.40–0.60 and for medium multistage ones with the mean piston speed equal or below 3.5 m/s 0.55–0.70. The isothermal efficiency of a single stage in multi-stage compressors with inter-stage cooling equals about 0.75–0.80. For single stage piston compressors the isentropic efficiency can be also given. It equals about 0.85. Rotary compressors with cooling can be characterized by the isothermal efficiency of 0.62–0.67. Processes in single stage piston compressors with cylinder cooling with compression number of 3–6 can be described by the substitute polytropic exponent $m = (0.92 \dots 0.98)k$. For the multistage units processes in every next $z > 1$ stage the appropriate polytropic exponent can be calculated from the formula

$$m_z = m_{z-1} + 0.015k \quad \text{or} \quad m_z = m_1 + 0.015(z-1)k$$

where m_1 is the exponent of a compression process in the first stage that could be determined with help of the previous formula.

At last the processes in water pumps and water turbines should be determined. It can be assumed that the appropriate efficiency (or dimensionless modeling parameter) η equals from 0.8 to 0.99.

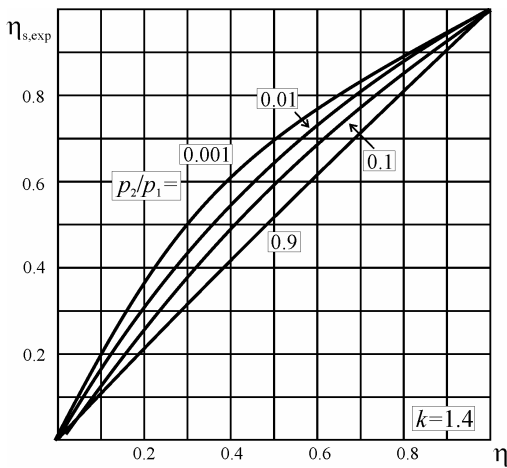


Fig. 2 Isentropic efficiency $\eta_{s,\text{exp}}$ vs. dimensionless parameter η for expansion, [03]

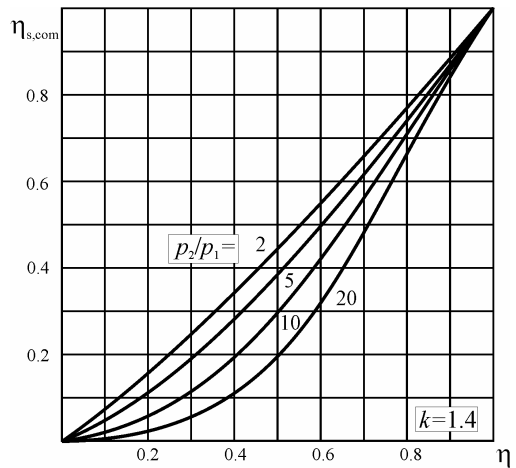


Fig. 3 Isentropic efficiency $\eta_{s,\text{com}}$ vs. dimensionless parameter η for compression, [03]

8. CONCLUSIONS

The presented method of modeling the so-called working or pressure change processes in power (also chemical) engineering systems with use of only one dimensionless parameter is very good oriented onto thermodynamic analyzes, which has been shown above.

With the already worked out universal exergy rating procedure for processes of the different kind using the thermodynamic effectivity quotient, the overall complex system thermodynamic effectivity can be expressed by means of such effectivities determined for constituent processes, [18]–[19]. The simplicity of the presented approach is a very important feature.

The presented special behavior of the system exergy as the caloric parameter of state differs very clearly from the behavior of other caloric parameters of state, such as the inner energy or enthalpy.

Changing in a process it shows an inversion point of the algebraic sign while crossing the value of the natural environment intensities (or in general their functional dependencies). Another words, the flow system exergy has the very exact defined zero-value point, whereas for the inner energy or enthalpy this point is usually assumed by the investigator in order to attain a certain calculation convenience.

The stated special behavior of the exergy as the caloric parameter of state, which is very good known and has been already many times described in exergy monographs, makes its properties very similar to them of the caloric process quantities like heat or work (exergy of heat or exergy of work, as well). The zero point by the last mentioned quantities corresponds to the „no heat” or „no work” transferred. The exergy of heat (or absolute work for closed systems) changes its algebraic sign, i.e. the direction of delivery, by $T=T_0$ (or $p=p_0$, respectively). This is because of the Carnot’s thermal (or mechanical) coefficient value, which equals to

$$\eta_t^c = 1 - \frac{T_0}{T} \quad (\text{or } \eta_m^c = 1 - \frac{p_0}{p})$$

It can be seen, that the form of the thermal exergy part, Equation (18), do agree with the form of the heat or absolute work exergy, respectively. That is why it was a trial undertaken to use this statement in the objective rating of thermo mechanical processes. It has been proved that by dividing the system exergy changes into thermal and mechanical (pressure) parts with further assigning them to the disappearing or created exergy terms alike the parts of the heat exergy and work exergy above or below the appropriate intensities T_0 and p_0 , the generalization could be achieved in a formulation algorithm of the exergy rating quotients, Equations (25)–(26) with Equation (23) as the generalized condition. The process thermodynamic effectivity, Equation (26), can be treated as the thermodynamic (or energy conversion) rating quotient, which has been formulated in a pure objective way. The so-called „thermodynamic transformation” or „thermodynamic transition”, Equation (24) or Equation (25), is not an equation because of the different value of each exergy part — they can be balanced only in a traditional exergy balance formula, which is represented by Equation (20).

The presented thermodynamic effectivities of processes in turbines and compressors do mostly agree with the exergy rating quotients formulated in the literature, what’s more, they are defined very strictly for the certain temperature ranges above or under T_0 . One of the greatest advantages of the proposed algorithm is the possibility of the system rating in a unified or and objective way (e.g. [05], [18]–[19], the numerical example in [26]). In Fig. 3 and Fig. 4 the dependencies of the thermodynamic effectivity in adiabatic expansion and compression on the dimensionless modeling parameter have been presented.

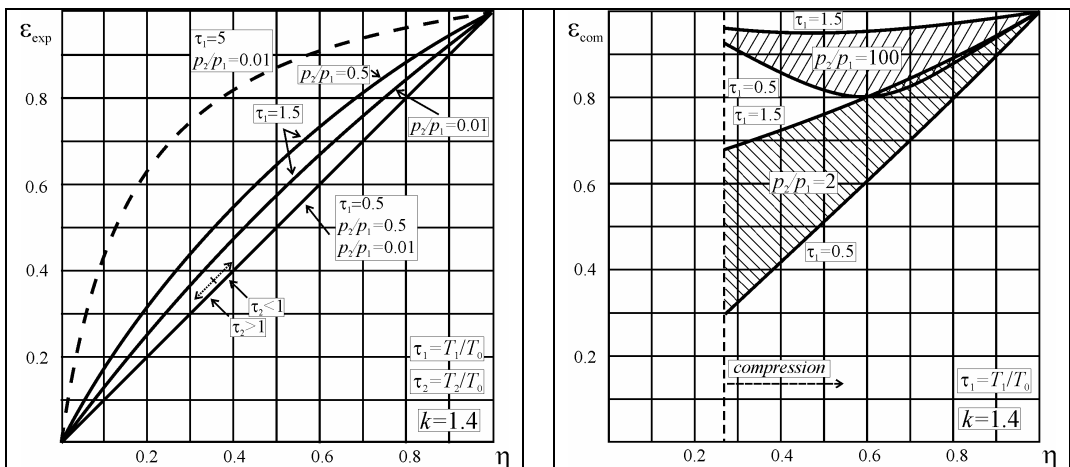


Fig. 4 Thermodynamic effectivity ϵ_{exp} vs. dimensionless parameter η for expansion, [05]

Fig. 5 Thermodynamic effectivity ϵ_{com} vs. dimensionless parameter η for compression, [05]

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