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THE HANDLING OF UNIT LOADS USING A POWERED ROLLER CONVEYOR  
MANIPULACE KUSOVÝCH BŘEMEN PROSTŘEDNICTVÍM VÁLEČKOVÉ TRATĚ

**Abstract**

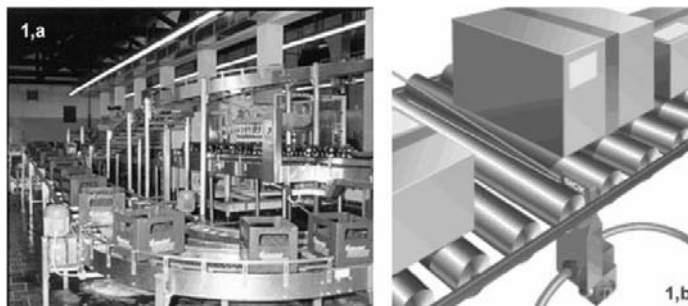
This article elaborates the handling of unit loads using a powered roller conveyor. Due to the requirement of using a gravitational conveyor for further handling, unit loads, comprising rectangular-footprint steel-sheet storage crates, need to be turned around their vertical axis by the required angle during transport on the powered roller conveyor. Turning of the storage crate is provided for by a stopper mounted transversely on the roller conveyor.

**Abstrakt**

Příspěvek pojednává o manipulaci kusových břemen prostřednictvím poháněné válečkové trati. Kusová břemena, tvořená plechovými ukládacími bednami obdélníkové podstavy, je nutno v důsledku požadavku následné manipulace gravitační válečkovou tratí, v průběhu jejich dopravy poháněnou válečkovou tratí, otočit o požadovaný úhel kolem své těžištní osy. Otáčení ukládací bedny je zajištěno příčně uchycenou zarážkou v daném místě dopravní trasy válečkové tratě.

## 1 INTRODUCTION

In many industrial areas unit objects (storage boxes, crates, cartons, etc.) are handled using powered and/or gravitational roller conveyors (Fig. 1).



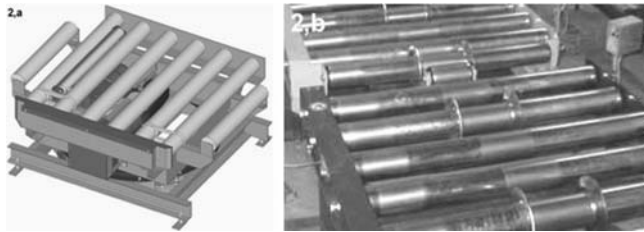
**Fig. 1** Powered and gravitational roller conveyor

Practical application often requires for loads to be handled during their actual transportation, such requirement can be substantiated by saved handling time, absence of additional handling devices, increase of transport performance, and requirement for transport of load in a specific physical configuration (specified in advance).

The layout of roller conveyor routes often requires a change in direction of transported unit loads. In practice this requirement is usually implemented using turntables (see Fig. 2,a), which are short sections of roller conveyors mounted on rotating chassis. Small curve angles in transport routes can be provided for by bending the roller conveyor, however, storage crates (cartons, pallets, etc.) must be guided (Fig. 2,b) in such a way as to prevent their jamming along the conveyor. Turntables require certain handling time which limits the performance capacity of roller conveyors during high outputs. Transported material must be moved to the turntable with a certain time delay which is

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gained by breaking down the roller conveyor route into sections that are controlled automatically. The drawback of this solution is the number of drives for numerous sections of the roller conveyor route.



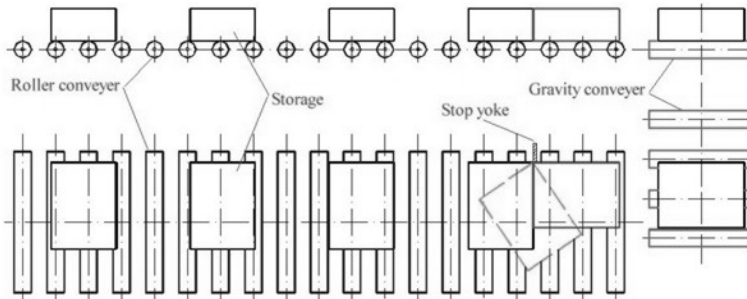
**Fig. 2** a) Handling turntable, b) Structural modification of roller for good pallet guidance

Turning transported loads (up to 90 deg) around a stopper mounted transversely on the roller conveyor is a possible method of handling transported unit materials on roller conveyors. However, this handling method requires an increase in the roller length in the handling space, but significantly cuts the number of essential drives per section of conveyor.

The following chapters of this article detail mathematically the method of handling a transported box, its turning on the roller conveyor by the required angle.

## 2 ASSUMPTIONS

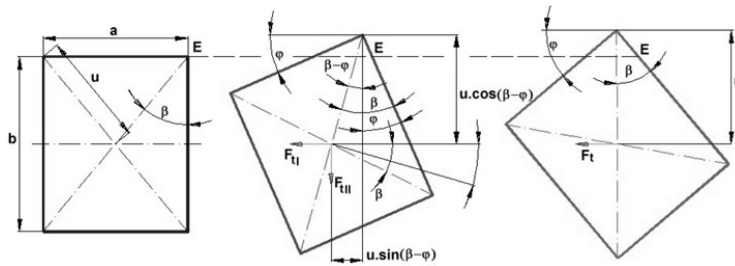
Storage crates are transported with regular spacing on a powered roller conveyor, at the end of the transport route the crate must be turned by (Fig. 3) due to the transition of the crate onto a gravitational roller conveyor (positioned square to the axis of the powered roller conveyor) where crates move by gravitational force and are stored on the gravitational conveyor for a certain time.



**Fig. 3** Schematic drawing of transport of storage crates

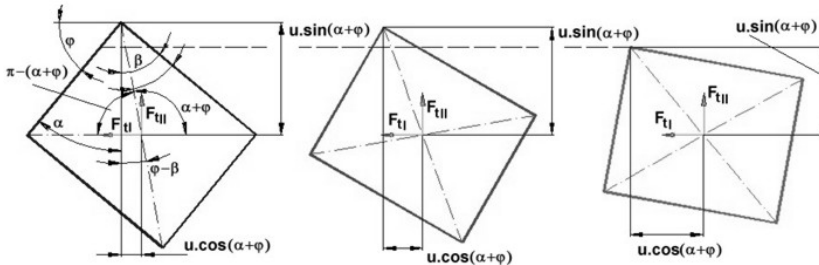
To determine the spacing of crates (spacing is defined by the periodicity of crates supply onto the conveyor) transported on a roller conveyor, it is necessary to first determine the time needed for turning a crate. Not knowing the exact time causes collisions of the crate being turned with the subsequent crate entering onto the turntable which leads to system failure.

Calculation of the necessary time for turning a load assumes the following input data: the storage crate has a flat bottom of rectangular footprint with sides  $a$ ,  $b$  [m]; the transported storage crate is supported by at least two rollers at any one time during transportation; the centre of gravity of the crate lies in the intersection of the two diagonals of the crate; friction between the contact surface of the storage crate (crate bottom) and contact surface of a roller is regarded as Coulomb's friction; during turning of the crate it is subject simultaneously to drag friction in the direction of crate movement and friction that moves the crate (by rotation of the powered rollers) square to the direction of movement; dynamic and kinematical effects were related to the centre of gravity, which is positioned in the centre of the rectangular footprint for homogeneity reasons (bottom of storage crate).



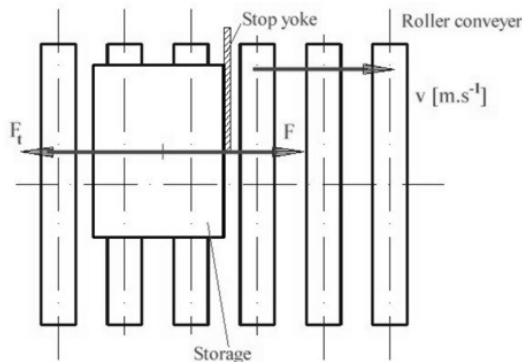
**Fig. 4** First stage of crate rotation around the transverse stopper on the roller conveyor

Experimental observation of movement of storage crates during their rotation around the stopper on the powered roller conveyor it follows that the movement of the storage crate can be broken down into two stages. During the first stage (Fig. 4) the storage crate moves along the axis of the powered roller conveyor and simultaneously square to the roller conveyor axis (in the direction of the transverse stopper). At the moment when the crate's centre of gravity reaches position  $(\pi/2 - \alpha)$  the crate begins to move against the former direction square to the transverse direction (see Fig. 5) and simultaneously it is moved by friction along the roller conveyor axis.



**Fig. 5** Second stage of crate rotation around the transverse stopper on the roller conveyor

The time necessary for rotation of the crate around the transverse stopper depends on the crate dimensions, crate weight including transported material, angular speed of the supporting roller, friction coefficient between the contact surfaces of the storage crate and supporting roller, and the length of the transverse stopper.



**Fig. 6** Movement of storage crate along the roller conveyor with the given length of transverse stopper

From this assumption the first conclusion can be derived – if the stopper length, square to the powered roller conveyor longitudinal axis, should reach a length equal to at least half the crate length  $b/2$  [m], the crate would not be turned around the transverse stopper. A transverse stopper of this length prevents rotation of the crate; the crate is not moved by the supporting rollers and overcomes friction  $F_t = m \cdot g \cdot \mu$  (see Fig. 6).

### 3 THEORETICAL EXPRESSION

According to Fig. 7 we express angle  $\alpha$  [deg]:

$$\sin \alpha = \frac{b}{2 \cdot u}, \quad (1)$$

During rotation (Fig. 7) of a rectangular crate, it applies:

$$\frac{1}{2} \cdot m \cdot v_i^2 = \frac{1}{2} \cdot J_I \cdot \omega_i^2 \text{ [kg.m}^2/\text{s}^2 \text{]}, \quad (2)$$

where:

$m$  – load weight [kg],

$v$  – load motion speed [m/s],

$J_I$  – load moment of inertia [kg/m<sup>2</sup>],

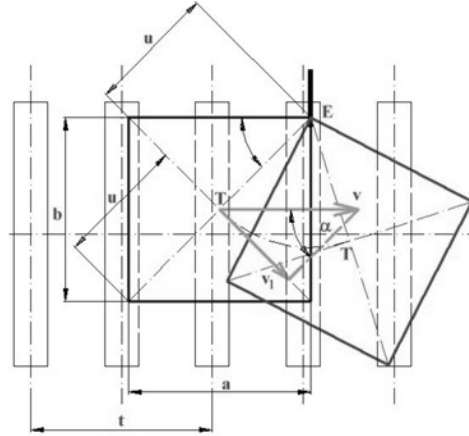
$\omega_i$  – initial angular speed of load [rad/s].

According to Fig. 7 we can derive speed  $v_i$  [m/s], see equation (3):

$$v_i = u \cdot \omega_i \text{ [m/s]} \text{ and simultaneously } v_i = v \cdot \sin(\alpha + \varphi) \text{ [m.s}^{-1} \text{]}, \quad (3)$$

If we substitute expression  $u$  [m] into equation (3) we get:

$$v_i = u \cdot \omega_i = v \cdot \sin(\alpha + \varphi) \Rightarrow v_i^2 = v^2 \cdot \sin^2(\alpha + \varphi) = u^2 \cdot \omega_i^2 \text{ [m/s]}, \quad (4)$$



**Fig. 7** Movement of storage crate along the roller conveyor with the given length of transverse stopper

According to Steiner's theorem we can express the moment of inertia of the crate to the axis passing through point E (Fig. 7), see equation (5).

$$J_I = J + m \cdot u^2 = m \cdot \left( \frac{a^2 + b^2}{12} + u^2 \right) \text{ [kg.m}^2 \text{]}, \quad (5)$$

According to Fig. 7 it applies:

$$u^2 = \left( \frac{a}{2} \right)^2 + \left( \frac{b}{2} \right)^2 = \frac{a^2}{4} + \frac{b^2}{4} = \frac{a^2 + b^2}{4} \text{ [m}^2 \text{]}, \quad (6)$$

By substituting equation (6) into equation (5) it follows that:

$$J_1 = m \cdot \left( \frac{a^2 + b^2}{12} + u^2 \right) = m \cdot \left( \frac{a^2 + b^2}{12} + \frac{a^2 + b^2}{4} \right) = m \cdot \frac{4 \cdot (a^2 + b^2)}{12} = m \cdot \frac{(a^2 + b^2)}{3} \text{ [kg.m}^2\text{]}, \quad (7)$$

Into equation (2) we substitute for the speed value  $v_1^2$  [m/s] expression (4) and for the moment of inertia  $J_1$  [kg.m<sup>2</sup>] equation (7), whereby we get:

$$\begin{aligned} J_1 \cdot \omega_1^2 &= m \cdot v^2 \cdot \sin^2(\alpha + \varphi) \Rightarrow m \cdot \frac{(a^2 + b^2)}{3} \cdot \omega_1^2 = m \cdot v^2 \cdot \sin^2(\alpha + \varphi) \Rightarrow \\ &\Rightarrow \omega_1 = \sqrt{\frac{3 \cdot v^2 \cdot \sin^2(\alpha + \varphi)}{a^2 + b^2}} \text{ [s}^{-1}\text{]}, \end{aligned} \quad (8)$$

We break down the rotation of the storage crate on the powered roller conveyor into two stages that follow immediately one after the other:

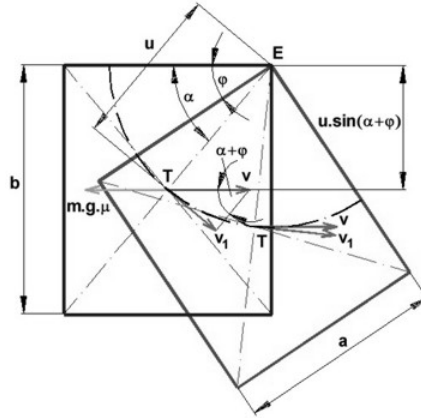
$$1^{\text{st}} \text{ stage: } \varphi = 0 \div \frac{\pi}{2} - \alpha \text{ [deg]} \dots t_1 \text{ [s]}, \quad (9)$$

$$2^{\text{nd}} \text{ stage: } \varphi = \frac{\pi}{2} - \alpha \div \frac{\pi}{2} \text{ [deg]} \dots t_{II} \text{ [s]}, \quad (10)$$

#### 4 INITIAL STAGE OF STORAGE CRATE ROTATION ON THE ROLLER CONVEYOR

According to Fig. 8 it is possible to determine the respective moment of inertia in equation (11).

$$M_1 = G \cdot \mu \cdot u \cdot \sin(\beta - \varphi) = G \cdot \mu \cdot u \cdot \cos(\alpha + \varphi) \text{ [Nm]}, \quad (11)$$



**Fig. 8** Turning of storage crate on roller conveyor in 1<sup>st</sup> stage

By mathematical analysis it is possible to express the sum of angles in equation (11):

$$\sin(\beta - \varphi) = \sin\beta \cdot \cos\varphi - \sin\varphi \cdot \cos\beta; \quad \cos(\alpha + \varphi) = \cos\alpha \cdot \cos\varphi - \sin\varphi \cdot \sin\alpha, \quad (12)$$

By substituting equation (12) into equation (11) we get:

$$M_1 = G \cdot \mu \cdot u \cdot \cos(\alpha + \varphi) = G \cdot \mu \cdot u \cdot (\cos\alpha \cdot \cos\varphi - \sin\varphi \cdot \sin\alpha) \text{ [Nm]},$$

$$M_1 = G \cdot \mu \cdot u \cdot \sin(\beta - \varphi) = G \cdot \mu \cdot u \cdot (\sin\beta \cdot \cos\varphi - \sin\varphi \cdot \cos\beta) \text{ [Nm]}, \quad (13)$$

According to Fig. 8 we can express  $\cos\alpha = \frac{a}{2 \cdot u}$ ;  $\cos\beta = \frac{b}{2 \cdot u}$ , and simultaneously

$$\sin\alpha = \frac{b}{2 \cdot u}; \quad \sin\beta = \frac{a}{2 \cdot u}.$$

Substitute angles into equation (13):

$$M_I = G \cdot \mu \cdot u \cdot \cos(\alpha + \varphi) = G \cdot \mu \cdot u \cdot \left( \frac{a}{2 \cdot u} \cdot \cos\varphi - \sin\varphi \cdot \frac{b}{2 \cdot u} \right) [Nm], \quad (14)$$

Respective moments  $M_I$  [Nm] in equation (14) are equal, further we shall use only moment:

$$M_I = G \cdot \mu \cdot u \cdot \cos(\alpha + \varphi) [Nm], \quad (15)$$

where:

$G$  – weight of transported storage crate [N],

$$M_I = J_I \cdot \varphi'' \Rightarrow \varphi'' = \frac{M_I}{J_I} = \frac{G \cdot \mu}{2 \cdot J_I} \cdot (a \cdot \cos\varphi - b \cdot \sin\varphi) \Rightarrow \varphi'' - \frac{G \cdot \mu}{2 \cdot J_I} \cdot (a \cdot \cos\varphi - b \cdot \sin\varphi) = 0, \quad (16)$$

In equation (16) we mark:

$$A = - \frac{G \cdot \mu \cdot a}{2 \cdot J_I} [s^{-2}]; B = \frac{G \cdot \mu \cdot b}{2 \cdot J_I} [s^{-2}], \quad (17)$$

By substituting equation (17) and modification of equation (16) we get:

$$\varphi'' + A \cdot \cos\varphi + B \cdot \sin\varphi = 0, \quad (18)$$

To solve differential equation (18) it is suitable to multiply the whole equation by  $2 \cdot \varphi'$ :

$$\begin{aligned} \varphi'' + A \cdot \cos\varphi + B \cdot \sin\varphi = 0 \cdot 2 \cdot \varphi' &\Rightarrow 2 \cdot \varphi' \cdot \varphi'' + 2 \cdot \varphi' \cdot A \cdot \cos\varphi + 2 \cdot \varphi' \cdot B \cdot \sin\varphi = 0 \Rightarrow \\ \Rightarrow [( \varphi' )^2]' + (2 \cdot A \cdot \sin\varphi)' - (2 \cdot B \cdot \cos\varphi)' = (C_1)' &\Rightarrow (\varphi')^2 = -2 \cdot A \cdot \sin\varphi + 2 \cdot B \cdot \cos\varphi + C_1, \end{aligned} \quad (19)$$

Integration constant  $C_1$  is expressed from boundary conditions, for  $t = 0$  is  $\varphi = 0$  and  $\varphi' = \omega_1$ . If we substitute boundary conditions into equation (19), we get the integration constant  $C_1$ , provided that  $\varphi' = d\varphi/dt = \omega_1$ :

$$(\varphi')^2 = \omega_1^2 = -2 \cdot A \cdot \sin 0 + 2 \cdot B \cdot \cos 0 + C_1 \Rightarrow \omega_1^2 = C_1 + 2 \cdot B \Rightarrow C_1 = \omega_1^2 - 2 \cdot B, \quad (20)$$

Substituting the integration constant from equation (20) to equation (19):

$$\begin{aligned} (\varphi')^2 = -2 \cdot A \cdot \sin\varphi + 2 \cdot B \cdot \cos\varphi + \omega_1^2 - 2 \cdot B &\Rightarrow \frac{d\varphi}{dt} = \sqrt{2 \cdot B \cdot \cos\varphi - 2 \cdot A \cdot \sin\varphi + \omega_1^2 - 2 \cdot B} \Rightarrow \\ dt = \frac{d\varphi}{\sqrt{2 \cdot B \cdot \cos\varphi - 2 \cdot A \cdot \sin\varphi + \omega_1^2 - 2 \cdot B}} &\Rightarrow t_1 = \int_{\varphi=0}^{\varphi=\frac{\pi}{2}-\alpha} \frac{d\varphi}{\sqrt{2 \cdot B \cdot \cos\varphi - 2 \cdot A \cdot \sin\varphi + \omega_1^2 - 2 \cdot B}} [s], \end{aligned} \quad (21)$$

The time  $t_1$  [s] value in equation (21) cannot be calculated analytically, the time  $t_1$  [s] must be calculated using a numerical method. Equation (21) is an elliptical integral; the solution is found by applying a numerical method using MathCad Professional software.

## 5 FINAL STAGE OF STORAGE CRATE ROTATION ON THE POWERED ROLLER CONVEYOR

According to Fig. 9 we can determine the respective moment described in equation (22).

$$M_{II} = G \cdot \mu \cdot u \cdot \cos[\pi - (\alpha + \varphi)] = m \cdot g \cdot \mu \cdot u \cdot \cos[\pi - (\alpha + \varphi)] [Nm], \quad (22)$$

According to mathematical analysis it applies that:

$$\cos[\pi - (\alpha + \varphi)] = \sin(\alpha + \varphi); \sin(\alpha + \varphi) = \sin\alpha \cdot \cos\varphi + \sin\varphi \cdot \cos\alpha, \quad (23)$$

By substituting equation (23) into equation (22) we get:

$$\begin{aligned}
M_{II} &= m \cdot g \cdot \mu \cdot u \cdot \cos[\pi - (\alpha + \varphi)] = m \cdot g \cdot \mu \cdot u \cdot \sin(\alpha + \varphi) = \\
&= m \cdot g \cdot \mu \cdot u \cdot (\sin\alpha \cdot \cos\varphi + \sin\varphi \cdot \cos\alpha) [Nm],
\end{aligned} \tag{24}$$

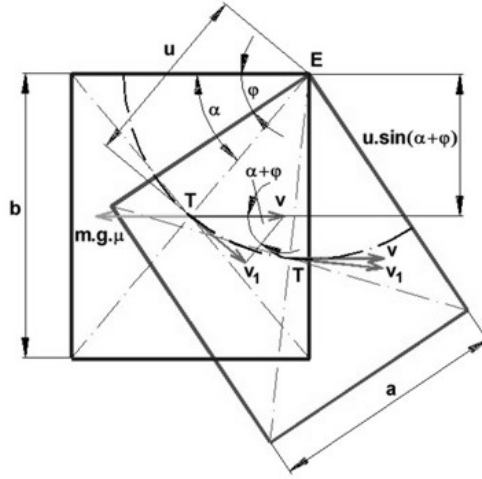
By substituting for respective angles into equation (24) it applies that:

$$M_{II} = m \cdot g \cdot \mu \cdot u \cdot \left( \frac{b}{2 \cdot u} \cdot \cos\varphi + \sin\varphi \cdot \frac{a}{2 \cdot u} \right) [Nm], \tag{25}$$

$$M_{II} = J_1 \cdot \varphi'' \Rightarrow \varphi'' = \frac{M_{II}}{J_1} = \frac{m \cdot g \cdot \mu}{2 \cdot J_1} \cdot (b \cdot \cos\varphi + a \cdot \sin\varphi), \tag{26}$$

Modify equation (26) into form (27):

$$\varphi'' - \frac{m \cdot g \cdot \mu}{2 \cdot J_1} \cdot (b \cdot \cos\varphi + a \cdot \sin\varphi) = 0, \tag{27}$$



**Fig. 9** Turning of storage crate on roller conveyor in 1st stage

In equation (27) we mark expressions according to equation (17) and modify equation as follows:

$$\varphi'' - B \cdot \cos\varphi + A \cdot \sin\varphi = 0, \tag{28}$$

To solve differential equation (28) it is suitable to multiply the whole equation by  $2 \cdot \varphi'$ :

$$\begin{aligned}
\varphi'' - B \cdot \cos\varphi + A \cdot \sin\varphi = 0 \cdot 2 \cdot \varphi' &\Rightarrow \varphi'' \cdot 2 \cdot \varphi' - 2 \cdot \varphi' \cdot B \cdot \cos\varphi + 2 \cdot \varphi' \cdot A \cdot \sin\varphi = 0 \Rightarrow \\
\Rightarrow [(\varphi')^2]' - (2 \cdot B \cdot \sin\varphi)' + (-2 \cdot A \cdot \cos\varphi)' &= (C_2)' \Rightarrow (\varphi')^2 = 2 \cdot B \cdot \sin\varphi + 2 \cdot A \cdot \cos\varphi + C_2,
\end{aligned} \tag{29}$$

Integration constant  $C_2$  is expressed from boundary conditions, for  $t = 0$  is  $\varphi = \pi/2 - \alpha$  and  $\varphi' = \omega_2$ . If we substitute boundary conditions into equation (29) we get the integration constant  $C_2$ , provided that  $\varphi' = d\varphi/dt = \omega_2$  [s<sup>-1</sup>].

$$\begin{aligned}
(\varphi')^2 = \omega_2^2 = 2 \cdot B \cdot \sin\varphi + 2 \cdot A \cdot \cos\varphi + C_2 &= 2 \cdot B \cdot \sin\left(\frac{\pi}{2} - \alpha\right) + 2 \cdot A \cdot \cos\left(\frac{\pi}{2} - \alpha\right) + C_2 = \\
= 2 \cdot B \cdot \cos\alpha + 2 \cdot A \cdot \sin\alpha + C_2 &= 2 \cdot B \cdot \frac{a}{2 \cdot u} + 2 \cdot A \cdot \frac{b}{2 \cdot u} + C_2 \Rightarrow C_2 = \omega_2^2 - \frac{B \cdot a}{u} - \frac{A \cdot b}{u} = \\
= \omega_2^2 - \frac{1}{u} \cdot (A \cdot b + B \cdot a),
\end{aligned} \tag{30}$$

In equation (30) it applies:

$$\sin\left(\frac{\pi}{2} - \alpha\right) = \cos\alpha = \frac{a}{2 \cdot u}; \quad \cos\left(\frac{\pi}{2} - \alpha\right) = \sin\alpha = \frac{b}{2 \cdot u}, \quad (31)$$

In equation (31) we mark  $C_3 = 1/u \cdot (A \cdot b + B \cdot a)$ , equation (31) changes to:

$$C_2 = \omega_2^2 - C_3, \quad (32)$$

By substituting the integration constant  $C_2$  from equation (32) into equation (29) we get:

$$\begin{aligned} (\varphi')^2 &= \left(\frac{d\varphi}{dt}\right)^2 = 2 \cdot B \cdot \sin\varphi + 2 \cdot A \cdot \cos\varphi + \omega_2^2 - C_3 \Rightarrow \frac{d\varphi}{dt} = \sqrt{2 \cdot A \cdot \cos\varphi + 2 \cdot B \cdot \sin\varphi + \omega_2^2 - C_3} \Rightarrow \\ \Rightarrow dt &= \frac{d\varphi}{\sqrt{2 \cdot A \cdot \cos\varphi + 2 \cdot B \cdot \sin\varphi + \omega_2^2 - C_3}} \Rightarrow t_{II} = \int_{\varphi = \frac{\pi}{2} - \alpha}^{\varphi = \frac{\pi}{2}} \frac{d\varphi}{\sqrt{2 \cdot A \cdot \cos\varphi + 2 \cdot B \cdot \sin\varphi + \omega_2^2 - C_3}}, \quad (33) \end{aligned}$$

The time value  $t_{II}$  [s] in equation (33) cannot be calculated analytically, the time  $t_{II}$  [s] must be calculated using a numerical method. Equation (33) is an elliptical integral; the solution is found by applying a numerical method using MathCad Professional software.



**Fig. 10** Transport and handling (turning) of storage crate on roller conveyor

## 6 TRANSITION STAGE FROM THE INITIAL TO THE FINAL STAGE OF CRATE TURNING ON THE ROLLER CONVEYOR

At the moment of transition of the grain from the initial stage  $\varphi_I = 0 \div \pi/2 - \alpha$  [deg] to the final stage  $\varphi_{II} = \pi/2 - \alpha \div \pi/2$  [deg] the respective angle of rotation  $\varphi$  [deg] of the crate equals:

$$\varphi = \varphi_I = \varphi_{II} = \frac{\pi}{2} - \alpha \text{ [deg]}, \quad (34)$$

If we substitute the angle  $\varphi$  [deg] into equation (21):

$$(\varphi_I')^2 = -2 \cdot A \cdot \sin\left(\frac{\pi}{2} - \alpha\right) + 2 \cdot B \cdot \cos\left(\frac{\pi}{2} - \alpha\right) + \omega_1^2 - 2 \cdot B, \quad (35)$$

By substituting into equation (35) expression according to equation (31), we get:

$$\begin{aligned} (\varphi_I')^2 &= -2 \cdot A \cdot \cos\alpha + 2 \cdot B \cdot \sin\alpha + \omega_1^2 - 2 \cdot B \Rightarrow -2 \cdot A \cdot \frac{a}{2 \cdot u} + 2 \cdot B \cdot \frac{b}{2 \cdot u} + \omega_1^2 - 2 \cdot B = \\ &= \frac{1}{u} \cdot (B \cdot b - A \cdot a) + \omega_1^2 - 2 \cdot B, \quad (36) \end{aligned}$$

In equation (36) we mark expression  $1/u \cdot (B \cdot b - A \cdot a)$  as  $C_4$ , whereby equation (36) can be rewritten as (37):



$$(\varphi'_1)^2 = \frac{1}{u} \cdot (B \cdot b - A \cdot a) + \omega_1^2 - 2 \cdot B = C_4 + \omega_1^2 - 2 \cdot B, \quad (37)$$

If we substitute the angle  $\varphi$  [deg] from equation (34) into equation (33) we get:

$$\begin{aligned} (\varphi'_{II})^2 &= \left( \frac{d\varphi}{dt} \right)^2 = 2 \cdot B \cdot \sin\varphi + 2 \cdot A \cdot \cos\varphi + \omega_2^2 - C_3 \Rightarrow 2 \cdot B \cdot \sin\left(\frac{\pi}{2} - \alpha\right) + 2 \cdot A \cdot \cos\left(\frac{\pi}{2} - \alpha\right) + \\ &+ \omega_2^2 - C_3 = 2 \cdot B \cdot \cos\alpha + 2 \cdot A \cdot \sin\alpha + \omega_2^2 - C_3, \end{aligned} \quad (38)$$

By substituting into equation (38) expression from equation (14) we get:

$$\begin{aligned} (\varphi'_{II})^2 &= 2 \cdot B \cdot \cos\alpha + 2 \cdot A \cdot \sin\alpha + \omega_2^2 - C_3 = 2 \cdot B \cdot \frac{a}{2 \cdot u} + 2 \cdot A \cdot \frac{b}{2 \cdot u} + \omega_2^2 - C_3 = \\ &= \frac{1}{u} \cdot (B \cdot a + A \cdot b) + \omega_2^2 - C_3 = C_3 + \omega_2^2 - C_3 = \omega_2^2, \end{aligned} \quad (39)$$

From equation (34) it applies that:

$$\varphi = \varphi_I = \varphi_{II} \Rightarrow \varphi'^2 = \varphi_I'^2 = \varphi_{II}'^2 \Rightarrow \omega_2^2 = C_4 + \omega_1^2 - 2 \cdot B, \quad (40)$$

If we substitute into equation (32) for  $\omega_2^2$  expression according to equation (40), we get:

$$t_{II} = \int_{\varphi = \frac{\pi}{2} - \alpha}^{\varphi = \frac{\pi}{2}} \frac{d\varphi}{\sqrt{-2 \cdot A \cdot \cos\varphi - 2 \cdot B \cdot \sin\varphi + C_4 + \omega_1^2 - 2 \cdot B - C_3}} [s], \quad (41)$$

We express the value of constant  $C_3$  and  $C_4$ . If we substitute for constants A and B from equation (17), we get:

$$C_3 = \frac{1}{u} \cdot (A \cdot b + B \cdot a) = - \frac{G \cdot \mu}{2 \cdot J_1 \cdot u} \cdot (a \cdot b - a \cdot b) = 0, \quad (42)$$

$$C_4 = \frac{1}{u} \cdot (B \cdot b - A \cdot a) = \frac{G \cdot \mu}{2 \cdot J_1 \cdot u} \cdot (b \cdot b - a \cdot a) = \frac{G \cdot \mu}{2 \cdot J_1 \cdot u} \cdot (b^2 - a^2), \quad (43)$$

By substituting constants  $C_3$  and  $C_4$  into equation (42) we modify the expression into its final form (44):

$$t_{II} = \int_{\varphi = \frac{\pi}{2} - \alpha}^{\varphi = \frac{\pi}{2}} \frac{d\varphi}{\sqrt{-2 \cdot A \cdot \cos\varphi - 2 \cdot B \cdot \sin\varphi + \omega_1^2 - 2 \cdot B + C_4}} [s], \quad (44)$$

The storage crate being turned on the powered roller conveyor must not collide with the next crate, therefore, it is necessary to determine a regular spacing  $l$  [m] (according to equation 45) between crates that are transported on the roller conveyor.

$$l = v \cdot (t_I + t_{II}) + \Delta l [m], \quad (45)$$

where:

$l$  – spacing (distance) of storage crates [m],

$v_2$  – speed of crate travel on the roller conveyor [m/s],

$t_I, t_{II}$  – turning time, 1<sup>st</sup> stage and 2<sup>nd</sup> stage of crate turning around transverse stopper [s],

$\Delta l$  – additional distance of crates [m], safety allowance, adjustable  $\Delta l \approx (0,15 \div 0,20)$ .

Based on the assumption of achieving maximum transport output  $Q$  [ks/h] it is necessary to optimally select the safety allowance for the inter-crate distance  $\Delta l$  [m]. The safety allowance for inter-crate distance is chosen based on experimental measurements and is verified according to equation (45).

## 7 EXPERIMENTAL DETERMINATION OF CRATE TURNING TIME ON THE ROLLER CONVEYOR

The laboratory of the Research and testing Institute, Department of Transport, Technical University of Ostrava, is equipped with a powered roller conveyor, 3,5 m long, conveyor slope angle  $\alpha = 3$  deg. The roller conveyor comprises 35 rollers, diameter 60 mm, roller spacing  $t = 100$  mm. The conveyor is powered by a three-phase asynchronous squirrel cage motor, sealed, model 4A71-4 - four-pole, 1500 synchronous revolutions, manufacturer MEZ Mohelnice k.p., parameters: motor output 370 W, revs  $1370 \text{ min}^{-1}$ , stator current 1,1 A/380V,  $\cos\phi$  0,78,  $h = 68$  %,  $M_z/M_n = 1,9$ ,  $I_z/I_n = 3,1$ , motor weight 6,5 kg.

Gearbox – worm gear type 63, 0,60 [kW], manufacturer ZŤS š.p. Moldava n/B. Drawing force from the electromotor is transferred via the worm gearbox which has a chain gear (teeth  $n = 20$ ) spring-mounted on the output shaft, to a single-row chain, type 08B to ČSN 02 3311, which drives one roller. Closed loop roller chains subsequently drive the neighbouring rollers. The real turning time  $t$  [s] of the storage crate around the stopper mounted on the roller conveyor is experimentally determined by measuring time; this value is compared to the theoretical time calculated from equations (21) and (44). The measuring equipment comprises the roller conveyor and storage crates, dimensions  $a = 280$  mm and  $b = 320$  mm.

## CONCLUSION

The theoretical relations of turning times (first  $t_I$  [s] and second  $t_{II}$  [s] stage) of the storage crate on the conveyor cannot be solved analytically; numerical method employing MathCad 2000 Professional software was used to solve integrals and determine times  $t_I$  [s] and  $t_{II}$  [s]. The storage crate being turned on the belt conveyor must not collide with the next crate. The regular spacing  $l$  [m] (according to equation 13) between subsequent crates on the belt conveyor must be determined. Under the assumption of achieving the maximum transport output value  $Q$  [ks/h] it is necessary to optimally select the safety allowance of the inter-crate distance  $\Delta l$  [m]. The safety allowance of the inter-crate distance is selected on the basis of experimental measurements and is verified by a series of practical tests. Due to the scope of the article it is not possible to detail the whole methodology of the theoretical calculation of the crate rotating around the transverse stopper on the belt conveyor, or the practical measurements that were performed in the laboratory of the Research and testing Institute, Department of Transport, Technical University of Ostrava. The author of this article are prepared to provide more detailed information about theoretical calculations and practical measurements of the turning time and simulation of crate motion around the transverse stopper.

## REFERENCES

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