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COMPUTATION OF THE STEADY-STATE RESPONSE OF A ROTOR SYSTEM TO THE PRESENCE OF A TRANSVERSE CRACK

VÝPOČET USTÁLENÉ SLOŽKY ODEZVY ROTOROVÉ SOUSTAVY S PŘÍČNOU TRHLINOU

Abstract

A transverse crack is an often type of damage of rotor systems, which can lead, if undetected, to serious failure. Computational procedures for investigation of the lateral vibration of a rotor system with an open crack are developed and applied on Rotor Kit 4 (RK 4) with a shaft containing a notch. A comparison of experimentally measured stiffness of the shaft, the vibration responses and the trajectories of the centre of the shaft is used for the judgement of the computational model of rotor system RK 4.

Abstrakt

Častým způsobem poškození rotorových soustav je příčná trhlina, která může způsobit vážnou poruchu, pokud není včas detekována. Vytvořené výpočetní postupy ke zkoumání příčného kmitání rotorové soustavy s otevřenou trhlinou jsou použity pro Rotor Kit 4 (RK 4) s vrubem v hřídeli. Výpočtový model rotorové soustavy RK 4 je zhodnocen pomocí experimentálně změřených tuhostí hřídele, rezonančních křivek a trajektorií středu hřídele.

1 INTRODUCTION

An increased cyclic-bending load of the shaft may initiate a transverse fatigue crack in the places of stress concentration (construction or structural notch, etc.). The transverse crack can be permanently opened [1], closed or also periodically open and close, during a revolution of the shaft [2]. Based on the theory of the fracture mechanics, the matrix of the flexibility of the shaft element with the fatigue transverse crack located in the centre of the element is derived in [1] and [3].

The following text shows a comparison between numerical calculations and experimental results for the stability of rotor system RK 4 and its response to unbalance excitation. The numerical calculations of the response to the unbalance force are calculated from a discretized model and the stability is judged using Floquet theorem. The experiments have been carried out for different depths of the notch. The vibration responses, the trajectories of the centre of the shaft and the stiffness of the shaft were measured at two planes perpendicular to each other.

2 DETERMINATION OF THE LATERAL VIBRATION OF THE ROTOR SYSTEM WITH AN OPEN TRANSVERSE CRACK

The model rotor systems are assumed to have the following properties: (i) the shaft is represented by a beam-like body that is discretized with finite elements, (ii) the stationary part is rigid, (iii) the discs are axis-symmetric rigid bodies, (iv) the inertia and the gyroscopic effects of the rotating parts are taken into account, (v) the material damping of the shaft and other kinds of damping are linear, (vi) the rotor is loaded either with constant or periodically time-varying forces, (vii) the shaft contains an open transverse crack, that influences only the stiffness matrix of the rotor system, (viii) the angular velocity of the rotor remains constant.

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The equations of motion of the lateral vibration of a rotor system with a transverse fatigue crack become of a following form in the stationary co-ordinate system:

$$\mathbf{M}\ddot{\mathbf{q}}(t)+(\mathbf{B}+\eta_v\mathbf{K}_{SH}+\omega\mathbf{G})\dot{\mathbf{q}}(t)+[\mathbf{K}(t)+\omega\mathbf{K}_C]\mathbf{q}(t)=\mathbf{f}_A(t)+\mathbf{f}_V\,, \tag{1}$$

$$\mathbf{q}_{BC}=\mathbf{q}_{BC}(t),\qquad \mathbf{q}(0)=\mathbf{q}_0\,,\qquad \dot{\mathbf{q}}(0)=\dot{\mathbf{q}}_0\,. \tag{2}$$

The nomenclature in equations (1) and (2) $\mathbf{M}, \mathbf{B}, \mathbf{G}, \mathbf{K}(t), \mathbf{K}_{SH}, \mathbf{K}_C$ denote the mass matrix, the damping matrix (external, damping of material), the matrix of gyroscopic effects, the stiffness matrix of the rotor system with the transverse crack, the stiffness matrix of the shaft and the circulatory matrix of the rotor system, $\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}$ are vectors of generalized displacements, and its time derivates, $\mathbf{q}_{BC}, \mathbf{q}_0, \dot{\mathbf{q}}_0$ are vectors of boundary and initial conditions, $\mathbf{f}_A, \mathbf{f}_V$ are vectors representing generalized forces on the rotor system (external and constraint forces), ω is the angular velocity of the shaft, η_V is the coefficient of viscous damping of the material and t denotes the time.

2.1 Determination of the stiffness matrix with the open transverse crack

The presence of the transverse crack on a shaft element introduces local flexibility $\mathbf{C}^e_{cr}$ (see [1]). Calculation of the flexibility matrix of shaft element $\mathbf{C}^e$ is determined as a sum of local flexibility $\mathbf{C}^e_{cr}$ (the shaft element with the crack) and local flexibility $\mathbf{C}^e_{ncr}$ (the shaft element without the crack).

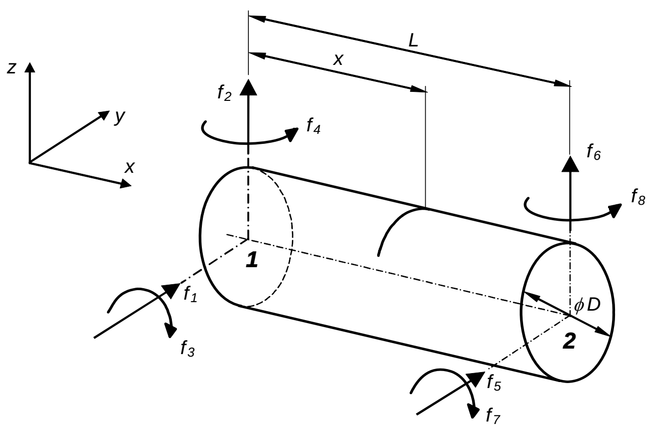

Fig. 1 An example of the shaft element with the transverse crack

Let us consider a shaft element with diameter D, length L and a transverse crack situated at distance x (measured as shown in Fig. 1). The shaft element (from Fig. 1) is loaded in the first node with shear forces f_1, f_2 in the direction of axes y, z and bending moments f_3, f_4 around co-ordinates y, z. The second node is loaded with shear forces f_5, f_6 and bending moments f_7, f_8. The local flexibility due to the crack $\left(c^e_{cr}\right)_{ij}$ is determined according to [1]

$$\left(c^e_{cr}\right)_{ij}=\frac{\partial q_i}{\partial f_j}=\frac{\partial^2}{\partial f_i\,\partial f_j}\left[\int_{-b}^{b}\int_{0}^{a}J(\alpha)\,d\alpha\,d\beta\right], \tag{3}$$

where q_i is additional displacement due to the crack, f_i, f_j are corresponding loads (forces, bending moments) in the i,j directions $(i,j=1,2,3,4)$ from Fig. 1, α is the immediate depth of the crack, a is the total depth of the crack, b is the half of width of crack, β is the distance between element and z axis (Fig. 2) and $J(\alpha)$ is the Strain Energy Density Function, that for plane stress equates:

$$J=\frac{1-\mu^2}{E}\left[\left(\sum_{i=1}^{4}K_{Ii}\right)^2+\left(\sum_{i=1}^{4}K_{IIi}\right)^2+[1+\mu]\left(\sum_{i=1}^{4}K_{IIIi}\right)^2\right], \tag{4}$$

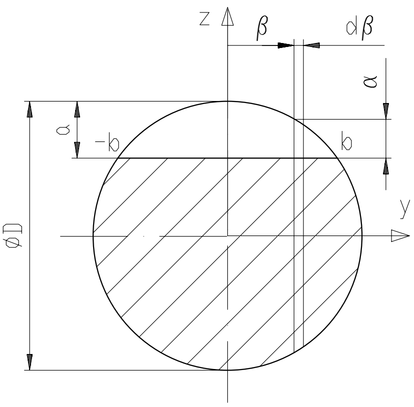

Fig. 2 A Cross-sectional drawing of the shaft with the crack

where E is the modulus of elasticity, μ is the Poisson ratio and K_{Ii}, K_{IIi}, K_{IIIi} are the crack Stress Intensity Factors for the 1^{st} , 2^{nd} *and* 3^{rd} mode and for the load index $i=1,\ 2,\ 3,\ 4$. The order of matrix of local flexibility $\mathbf{C}^{e}_{cr}$ is *4x4* – and it corresponds the degree of freedom of the shaft element.

The stiffness matrix of crack element $\mathbf{K}^{e}_{cr}$ in rotor-fixed co-ordinate system have the following form

$$\mathbf{K}^{e}_{cr} = \mathbf{T}\left(\mathbf{C}^{e}\right)^{-1}\mathbf{T}^{T} \,, \tag{5}$$

where $\mathbf{T}$ denotes the transformation matrix that is formed based on the condition of static equilibrium of the shaft element, see [1]. Stiffness matrix of cracked element $\mathbf{K}^{e}_{cr}$ has to be transferwed into stationary co-ordinate system, according to [1].

2.2 Computation of the response excited by unbalance of the rotating parts and the stability investigation

After the initial transient component of the response dies out, the steady-state response of the rotor system (1) subjected to periodic excitation may be analysed. For this purpose, the trigonometric collocation method can be applied, see [4]. This approach assumes that: (i) the steady-state vibration is a periodic function of time, (ii) its period is real multiple of the period of the excitation and (iii) the response can be approximated by a finite number of terms of the Fourier series. Because the stiffness matrix of the investigated rotor system is a periodic function of time, there is danger of formation of parametric resonance. The stability of the steady-state periodic vibration of the rotor system is investigated via the Floquet theory.

3 RESULTS OF COMPUTER SIMULATION WITH ROTOR SYSTEM RK 4

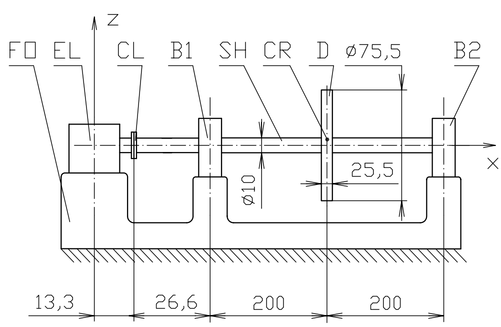

Fig. 3 A sketch of rotor system RK 4

The investigated rotor system RK 4 (Fig. 3) consists of shaft (SH) that is driven by electromotor (EL) through clutch (CL) and disc (D) attached to the midspan. The connection of the rotor to foundation plate (FD) is realised via two bearings (B1 and B2). Shaft (SH) of the rotor system contains transverse crack (CR) in the midspan.

Rotor system RK 4 has following parameters: $m = 0{,}8\,kg$ weight of the disc, $\rho = 7800\,kgm^{-3}$ density of the material, $E = 2{,}1 \cdot 10^{11}\,Pa$ modulus of elasticity, stiffness of bearings B1 and B2 in the horizontal $k_y = 1{,}3 \cdot 10^{5}\,Nm^{-1}$ and in the vertical $k_Z = 1{,}3 \cdot 10^{5}\,Nm^{-1}$ direction, $\eta_V = 0{,}8 \cdot 10^{-6}\,s^{-1}$ coefficient of viscous damping of material, $\alpha = 4\,s^{-1}$ coefficient of Rayleigh damping (external damping), $\beta = 0\,s$ coefficient of Rayleigh damping (structural damping), $D = 0{,}01\,m$ diameter of shaft, $a = 0\,mm$, $a = 1\,mm$, $a = 3\,mm$ and $a = 5\,mm$ depth of the crack (notch), initial position of the notch is situated in the direction of z from Fig. 2 and unbalance of the disc lies on positive axis z (Fig. 2).

Using finite element program ANSYS, the stiffness of the centre of the shaft under the disk was determined for various loading directions (Fig. 4, right). Subsequently, the stiffness was compared to the results obtained via software package MATLAB, which predicted damage of the shaft of rotor system RK 4 due to open transverse crack (right Fig. 4). Using the finite element program ANSYS, it was found that under given operation conditions, the surfaces of the notch never come into contact; this predominantly happens due to the dimensions of the notch (etc. depth notch *3mm* is width *0,4mm*). Because the difference in the stiffness is minor, it is possible to analyse the dynamic characteristics of rotor system RK 4 analogously to a shaft containing an open transverse crack.

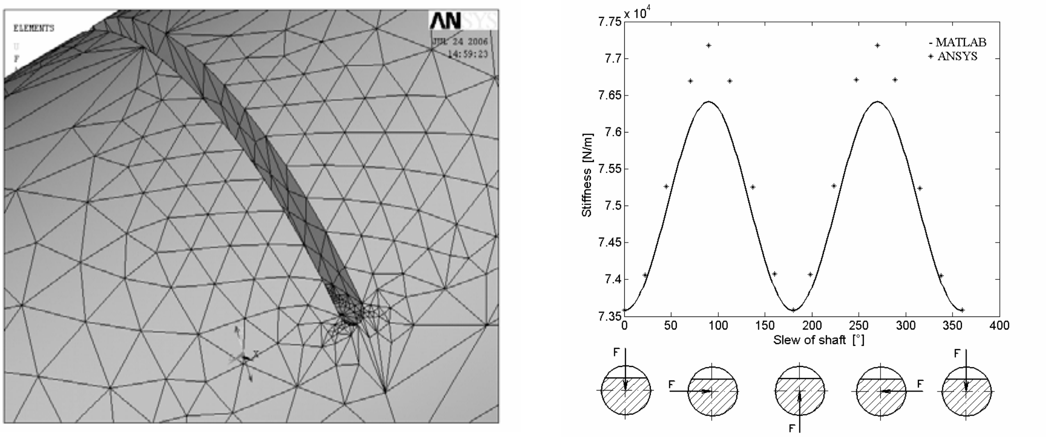

Fig. 4 A finite element model (left) of the shaft containing of the notch (rotor system RK 4) and the stiffness of the shaft near the disc in relation to the load direction *F* (right)

After the initial transient component dies out, the steady-state response of rotor system RK 4 is determined with the trigonometric collocation method. An approximation of the response is computed on the assumption that the period of the response equates the period of the excitation if expanded into the Fourier series with one absolute and two harmonic components. Fig. 5 – left presents the steady-state response of the centre of the shaft calculated for *1375rpm* and right for *1298rpm*.

Tab. 1 Range of unstable rotational speed

Depth of the notch [mm]	1	3	5
Upper bound of unstable rotational speed [±0,5rpm]	-	2564	2455
Lower bound of unstable rotational speed [±0,5rpm]	-	2556	2443

The stability of the vibration of rotor system RK 4 is judged based on the equation of motion (1) using the Floquet theory. The range of unstable rotational speed of rotor system RK 4 with depth of the crack *1mm* is suppressed (Tab. 1). Increasing the depth of the notch lowers the stiffness of the shaft; therefore, the unstable rotational speed shifts to lower ranges.

4 RESULTS OF EXPERIMENTAL MEASUREMENTS WITH ROTOR SYSTEM RK 4

Obtained results were compared to the experimental measurements performed on rotor system RK 4, pictured in Fig. 6. With rotor system RK 4 it is possible to simulate various kinds of lateral vibrations. In order to perform the experiment measurements, several shafts with various depth of the notch (*0mm*, *1mm*, *3mm* and *5mm*) and a limiter of the vibration (component on the left side of the disc in photograph Fig. 6) were fabricated.

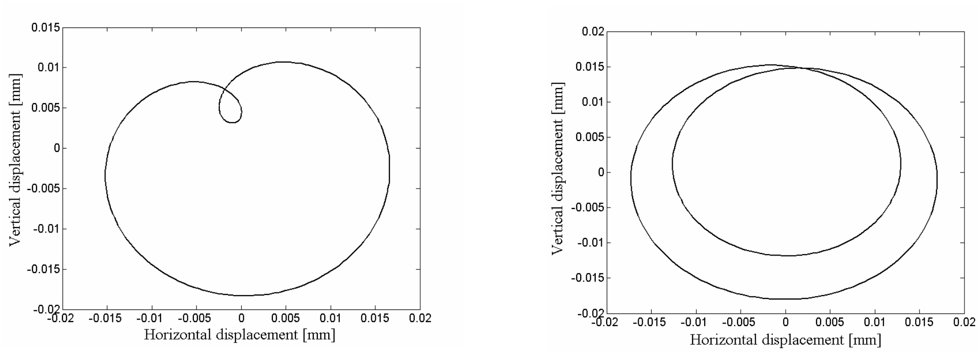

Fig. 5 Trajectory of centre of the shaft in the proximity of the probes and shaft with the crack of depth *3mm* (left) and *5mm* (right) – shift of static displacement of the shaft without the crack

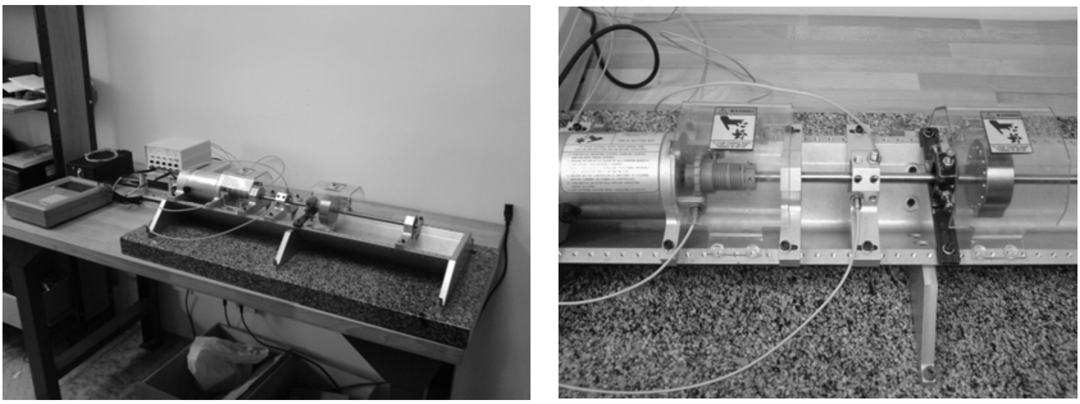

Fig. 6 Rotor system RK 4 with measurement instruments (left photograph) and a detail of the electromotor, the clutch, the bearing house, the stand of the probes, the limiter of the vibration and the disc of RK 4 rotor system (right photograph)

Following measurements were carried out: (i) measurements of the vibration responses, (ii) measurements of the trajectory of the centre of the shaft, (iii) measurement of the stiffness of the shafts. All measurements relate to rotor system RK 4.

Waterfall diagrams (Fig. 7, Fig. 8) are formed for the measured values of the Fourier coefficients computed from the response of the shaft for horizontal and vertical plane vibration. Opposite to the shaft without the notch (see on Fig. 7), there is (see on Fig. 8) an obvious increase in the second harmonic component near the second resonance in the case of the shaft with the notch of the depth of *3mm*. One of possible causes of the increase of the vibration near the second resonance is the fluctuation of the stiffness of the shaft during a revolution.

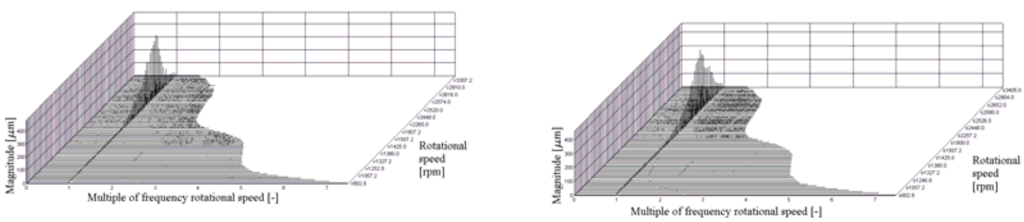

Fig. 7 Waterfall diagram of multiple first harmonic frequency component of displacements response in horizontal plane (left) and vertical plane (right) for the vibration of the rotor system without the crack

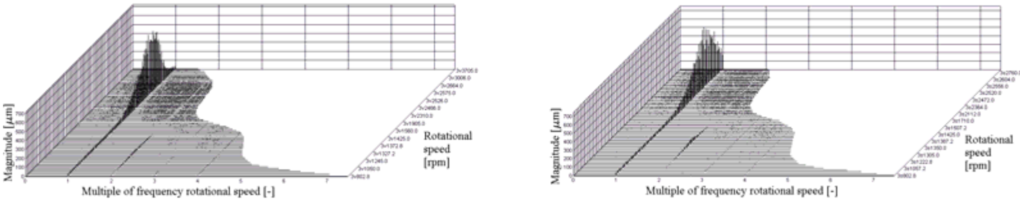

Fig. 8 Waterfall diagram of multiple first harmonic frequency component of displacements response in horizontal plane (left) and vertical plane (right) for the vibration of the rotor system with crack depth *3mm*

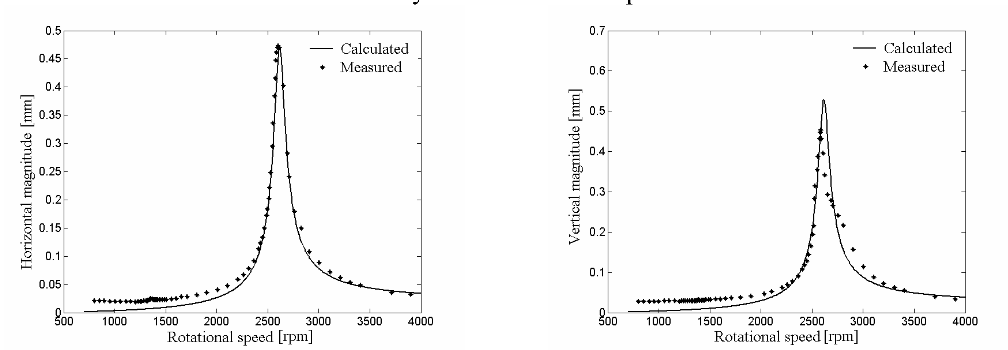

Fig. 9 Vibration response in horizontal (left) and vertical (right) plane of vibration for the shaft without the notch

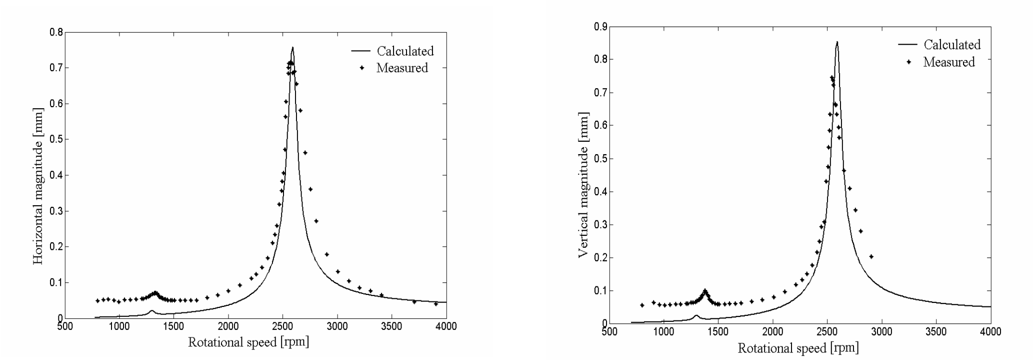

Fig. 10 Vibration response in horizontal (left) and vertical (right) plane of vibration for the shaft with notch depth *3mm*

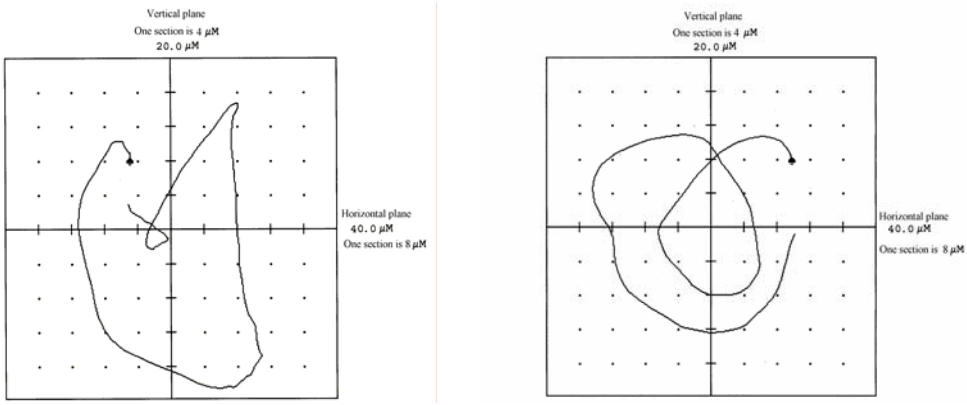

Fig. 11 Measured trajectories of the centre of the shaft near the gauging probes for notch depth *3mm* (left) and *5mm* (right)

Tab. 2 Stiffness of the shaft in two perpendicular directions

Depth of the notch [mm]	1	3
Computed/experimentally measured stiffness of the shaft, k_z [N/m]	77014/76894	73581/73022
Computed/experimentally measured stiffness of the shaft, k_y [N/m]	77293/77186	76302/76176

Computed and experimentally measured vibration responses of the shafts without the notch and with the notch depth *3mm* are presented in Fig. 9 and Fig. 10. The vibration responses were computed using the trigonometric collocation method and the assumption that the period of the response is equal to the period of the excitation. This condition is not valid in the range of unstable rotational speed Tab. 1. Experimentally measured and numerically computed vibration responses for the shaft with the notch depth (*0mm*, *3mm* and *5mm)* were obtained practically identical.

It was found, that the numerically computed and experimentally measured trajectories of the centre of the shaft are practically identical for following combinations: *1375rpm* and *3mm* notch, and *1298rpm* and *5mm* notch (see on Fig. 5 and Fig. 11).

In order to determine the stiffness of shaft of rotor system RK 4 with notch, the shaft was supported with two rigid stands. The loading force the shaft was subjected to was stepwise increased and the displacement of the centre of the shaft under the notch was measured. Based on the known loading force, the measured displacement and the least square method, the shaft stiffness in two perpendicular directions k_y and k_z was determined. The shaft stiffness (k_y, k_z) in Tab. 2 was calculated using static load and yielding displacement of the centre of the shaft under the notch (equation (1), with conditions that $\ddot{\mathbf{q}} = \mathbf{0}$, $\dot{\mathbf{q}} = \mathbf{0}$ and $\omega = 0rad \cdot s^{-1}$).

5 CONCLUSIONS

Using computer simulation with finite element model of rotor system RK 4, the paper has presented the calculation of the stiffness of the shaft and the steady-state response when the system is excited by unbalance of the rotating parts; the stability of the system has been judged via the Floquet theory. The results from of numerical computations with suggested computational procedures were verified by experimental measurements carried out on rotor system RK 4 with the shaft containing the notch.

The obtained data showed that the shaft stiffness computed for the shaft with the open transverse fatigue crack are practically identical to experimental results of the shaft with the notch (Tab. 2). Good qualitative and quantitative correspondence was reached for experimentally measured and numerical computed vibration responses of the shaft containing the notch of depth *(0mm, 3mm and 5mm)*.

Numerical computations and experimental measurements revealed that the notch in the shaft (open crack) influences the lateral vibration of the rotor system close to rotational speed of the secondary resonance (Fig. 8 and Fig. 10). Therefore, increase of vibration near the secondary resonance can be used as one of the indicators of the presence of a crack in the shaft.

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