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ROBUST CONTROL VERIFICATION

OVĚŘOVÁNÍ ROBUSTNÍCH AGORITMŮ ŘÍZENÍ

Abstract

The paper shows robust control design by chosen methods. The properties of robust controls will be described on their application. Particularly, state variables aggregation method, sliding mode control, will be compared. Results with corresponding accuracy for each method applied on example will be published.

Abstrakt

Příspěvek popisuje návrh robustních algoritmů řízení pomocí vybraných metod. Budou popsány vlastnosti jednotlivých algoritmů řízení a jejich aplikace pro řízení polohy levitujícího předmětu v magnetickém poli. Navržené algoritmy řízení budou porovnány dle obdržené kvality řízení.

1 INTRODUCTION

At present time there is no general method for nonlinear control systems synthesis. In addition each method is applicable for certain type of nonlinear control subsystems. The contribution describes nonlinear control systems synthesis with help of state variables aggregation method.

The mathematical model of controlled dynamic subsystem is suppose in form

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x},\boldsymbol{v},t) + \boldsymbol{G}(\boldsymbol{x},\boldsymbol{v},t)\,\boldsymbol{u} + z(\boldsymbol{x},\boldsymbol{u},t) \quad (1)$$

$$z(\boldsymbol{x},\boldsymbol{u},t) = \Delta\boldsymbol{f}(\boldsymbol{x},t) + \Delta\boldsymbol{G}(\boldsymbol{x},t)\,\boldsymbol{u} \quad (2)$$

where:

$\boldsymbol{x}$ – vector of state variables with dimension n,

$\boldsymbol{u}$ – vector of control variables with dimension m,

$\boldsymbol{v}$ – vector of measurable disturbances with dimension p,

$\boldsymbol{f}$ and $\boldsymbol{\Delta f}$ – continuous nonlinear function with dimension n,

$\boldsymbol{G}$ and $\boldsymbol{\Delta G}$ – matrices of continuous functions dimension (n,m).

The expressions $\boldsymbol{\Delta f}$ and $\boldsymbol{\Delta G}$ can describe for example uncertainty in behaviour of nonlinear subsystem, inaccuracy of identification, influence of immeasurable disturbances, etc.

The goal of program control is to ensure accurate and fast tracking of required state variable trajectory $\boldsymbol{x}^w(t)$ by real state variable trajectory $\boldsymbol{x}(t)$. That is why the quality of program control is evaluated by quadratic functional in form

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$$J = \int_0^t \left(\boldsymbol{e}^T \boldsymbol{Q} \boldsymbol{e} + \dot{\boldsymbol{e}}^T \boldsymbol{Q}_0 \dot{\boldsymbol{e}} \right) d\tau, \boldsymbol{e} = \boldsymbol{x}^w - \boldsymbol{x}, \tag{3}$$

where:

$\boldsymbol{e}$ – error vector dimension n,

$\boldsymbol{Q} = \boldsymbol{D}^T \boldsymbol{D}$, $\boldsymbol{Q}_0 = \boldsymbol{D}^T \boldsymbol{T}^2 \boldsymbol{D}$ – symmetric positive definite matrices dimension (n,n), which elements are constant parameters.

This problem can be easily solved with help of state variables method which is detailed described in [ZÍTEK, P & VÍTEČEK. 1999]. In case the accurate mathematical model of controlled subsystems is known non-robust control can be designed.

In real conditions properties of controlled subsystem may change and the control quality must not be established. This problem can be removed by using robust control algorithm. It can be obtained by numerical solution of differential equation, which results from the minimization of the quadratic functional (3),

$$\boldsymbol{m}^w\left(\boldsymbol{u}^*\right) = \boldsymbol{0}\ ,\ \boldsymbol{m}^w\left(\boldsymbol{u}^*\right) = \boldsymbol{D}\dot{\boldsymbol{e}}\left(\boldsymbol{u}^*\right) + \boldsymbol{T}^{-1}\boldsymbol{D}\boldsymbol{e} \tag{4}$$

where:

$\boldsymbol{D}$ – aggregation matrix dimension (m,n) with constant elements,

$\boldsymbol{T}$ – time constant matrix dimension (m,m).

The robust control algorithm is written by equation

$$\boldsymbol{u}^x = \boldsymbol{\Theta}\left[\boldsymbol{D}\left(\boldsymbol{e} - \boldsymbol{e}_0\right) + \boldsymbol{T}^{-1}\boldsymbol{D}\int_0^t \boldsymbol{e}\, d\tau \right] + \boldsymbol{u}_0, \tag{5}$$

where:

$\boldsymbol{u}_0$ – initial value of control (performs in initial time).

A suitable choice of matrix $\boldsymbol{\Theta}$ ensures faster computation of optimal close control; it means computation has to stop in time interval in which each elements are constant.

The control algorithm is robust because it does not required exact knowledge of mathematical model of controlled subsystems. Disadvantage is necessary usage of high value of elements in matrix $\boldsymbol{\Theta}$ and it may cause irregular high value of control.

2 SLIDING MODE CONTROL

The quality of close control systems without knowledge of mathematical model or measurable disturbances can be also ensured by using sliding mode control. Sliding mode control means discontinuous control, where according to value switching function control has marginal value [UTKIN, V. I. 1992]. The control is written by equation

$$\boldsymbol{u}^{sl} = \left[u_1^{sl}, u_2^{sl}, \ldots, u_m^{sl}\right]^T, \tag{6}$$

$$u_j^{sl} = \begin{cases} u_j^+ & \text{for} \quad m_j > 0, \\ u_j^- & \text{for} \quad m_j < 0, \end{cases} \tag{7}$$

where:

u_j^+, u_j^- - marginal value of control,

m_j – element of switching function.

The form of switching function m_j can come out from state variables aggregation method. Then it is describes

$$\boldsymbol{u}^{sl} = \boldsymbol{U}^{m} \boldsymbol{sgn}(\boldsymbol{m}), \tag{8}$$

$$\boldsymbol{m} = \boldsymbol{D}(\boldsymbol{e} - \boldsymbol{e}_0) + \boldsymbol{T}^{-1}\boldsymbol{D}\int_0^t \boldsymbol{e} d\tau, \tag{9}$$

$$\boldsymbol{U}^{m} = \mathrm{diag}\left[u_1^m, u_2^m, \ldots, u_m^m\right], \tag{10}$$

$$\mathbf{sgn}(\boldsymbol{m}) = \left[\mathrm{sgn}(m_1), \mathrm{sgn}(m_2), \ldots, \mathrm{sgn}(m_m)\right]^T, \tag{11}$$

where:

$\boldsymbol{U}^m$ - diagonal matrix, whose elements u_j^m are marginal values of control variables,

sgn – sign fuction.

Sliding mode control is discontinuous, robust and simple, but its disadvantage is control high activities; it means quick switching between marginal values. It can be removed by using saturation function (continuous approximation of sign function) instead of sign function.

$$\boldsymbol{u}^{sa} = \boldsymbol{U}^{m} \mathbf{sat}(\Theta^{m} \boldsymbol{m}), \tag{12}$$

$$\boldsymbol{\Theta}^{m} = \mathrm{diag}\left[\theta_1^m, \theta_2^m, \ldots, \theta_m^m\right], \tag{13}$$

$$\mathbf{sat}(\boldsymbol{\Theta}^{m} \boldsymbol{m}) = \left[\mathrm{sat}(\theta_1^m m_1), \ldots, \mathrm{sat}(\theta_m^m m_m)\right]^T, \tag{14}$$

where:

$\boldsymbol{\Theta}^m$ - positive diagonal matrix,

sat – saturation function.

By using saturation function continuous control with restriction is obtained.

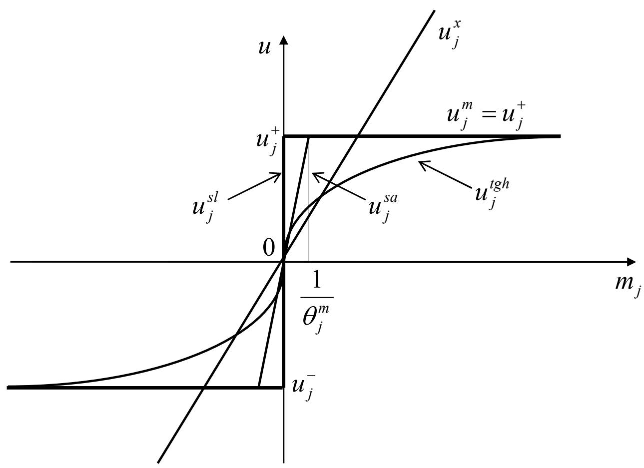

Fig. 1 The relation between sliding mode controls

Or it can be used hyperbolic tangent function as a smooth continuous approximation of sign function. The control will be described by equations:

$$\boldsymbol{u}^{tgh} = \boldsymbol{U}^{m} \mathbf{tgh}(\Theta^{m} \boldsymbol{m}), \tag{15}$$

$$\mathbf{tgh}(\boldsymbol{\Theta}^{m}\boldsymbol{m}) = \left[\mathrm{tgh}(\theta_1^m m_1), \ldots, \mathrm{tgh}(\theta_3^m m_m)\right]^T. \tag{16}$$

The relation between the three described sliding mode controls is shown on Fig. 1. We can see courses of robust control with high gain u_j^x and sliding mode controls (u^{sl}, u^{sa}, u^{tgh}). The sliding mode control with sign function u_j^{sl} has nonlinear two-state dependence. The sliding mode control with saturation function u_j^{sa} is linear between marginal values with restrictions. Inverse value of θ_j^m specifies slope of curve; the higher value θ_j^m means close course to sign function; the lower value θ_j^m means close course to control with high gain.

3 ROBUST CONTROL APPLICATION

The described robust control algorithms were applied to position control of ball levitating in magnetic field, which structure is shown on Fig. 2. [HUMUSOFT. 2002]

Fig. 2 Model of levitation in magnetic field

The behavior of controlled subsystem is described by second-order nonlinear differential equation that is why the aggregation matrix and time constant matrix have presentation

$$\boldsymbol{D} = \left[\frac{1}{T_1}\ 1\right], \ \boldsymbol{T} = [T_1], \tag{17}$$

where:

T_1 – time constant.

Therefore robust control with high gain is written by equation

$$\boldsymbol{u}^x = \boldsymbol{\Theta}\left\{\frac{2}{T_1}e_1 + \dot{e}_1 + \frac{1}{T_1^2}\int_0^t e_1\, d\tau\right\}, \tag{18}$$

where:

e_1 – difference between required and real position.

Sliding model controls are described:

$$\boldsymbol{u}^{sl} = \boldsymbol{U}^m \mathrm{sgn}\left[\frac{2}{T_1}e_1 + \dot{e}_1 + \frac{1}{T_1^2}\int_0^t e_1\, d\tau\right], \tag{19}$$

$$\boldsymbol{u}^{sa} = \boldsymbol{U}^{m}\operatorname{sat}\left[\boldsymbol{\Theta}^{m}\left(\frac{2}{T_1}e_1 + \dot{e}_1 + \frac{1}{T_1^2}\int_0^t e_1\, d\tau\right)\right], \tag{20}$$

$$\boldsymbol{u}^{tgh} = \boldsymbol{U}^{m}\operatorname{tgh}\left[\boldsymbol{\Theta}^{m}\left(\frac{2}{T_1}e_1 + \dot{e}_1 + \frac{1}{T_1^2}\int_0^t e_1\, d\tau\right)\right]. \tag{21}$$

The designed control algorithms were verified by computer simulation and directly on laboratory model. The model is connected to PC with data acquisition card MF614 and control algorithms were implemented in Matlab environment.

The Fig. 3 shows result of position control when the robust control with high gain was used. The control algorithm parameters were T_1=0.1 and Θ=0.05.

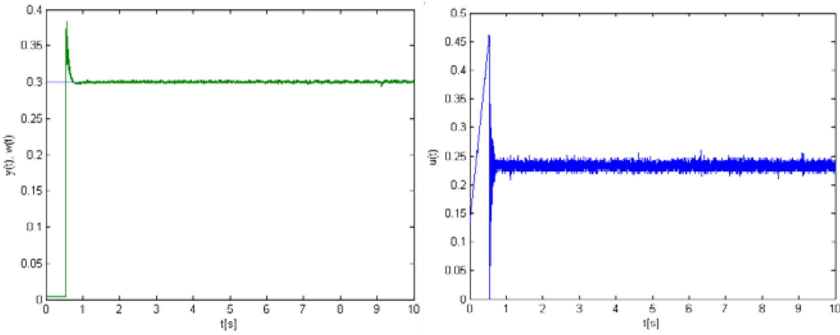

Fig. 3 Course of position and robust control with high gain

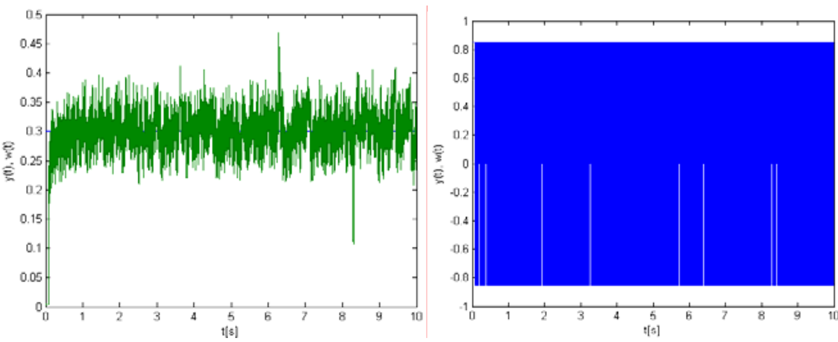

Fig. 4 Course of position and sliding mode control with sign function

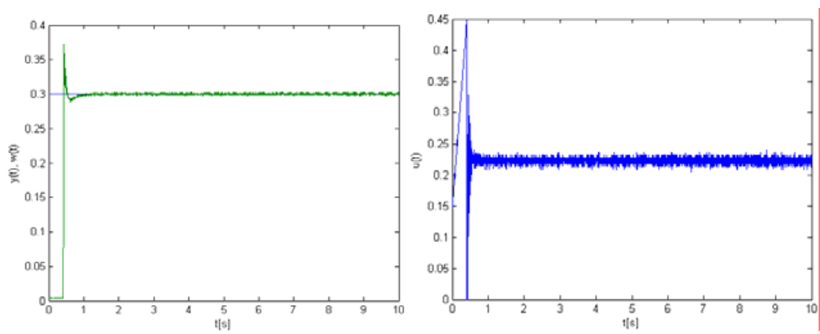

Fig. 5 Course of position and sliding mode control with saturation function

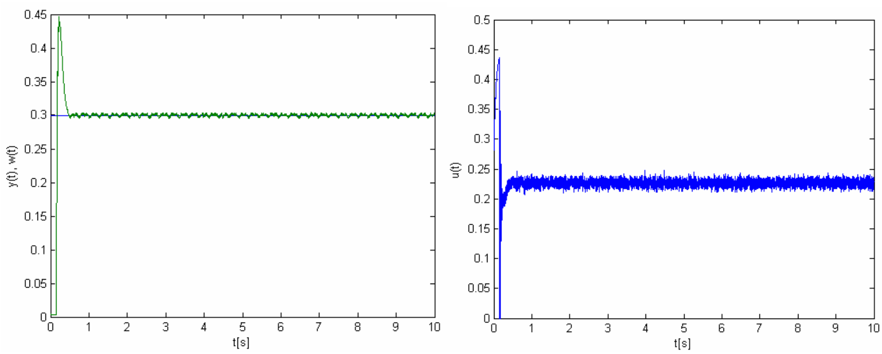

Fig. 6 Course of position and sliding mode control with hyperbolic tangent function

Fig. 4 – 6 show course of position when the sliding mode controls were used. The control parameters were:

Sign function: T_1= 0.1, U^m = 0.85.

Saturation function: T_1=0.1, U^m =1, Θ^m =0.05.

Hyperbolic tangent function: T_1=0.04, U^m =0.9, Θ^m =0.05.

4 Conclusion

The contribution presents nonlinear control systems synthesis. There are described properties of robust control and three types of sliding mode control: discontinuous, continuous and smooth continuous. The controls were used to position control of levitating object in magnetic field. All designed algorithms ensure reaching the required position. The best quality is obtained by using sliding mode control with saturation function.

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