Sborník vědeckých prací Vysoké školy báňské - Technické univerzity Ostrava
číslo 2, rok 2006, ročník LII, řada strojní
článek č. 1562

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SIMPLE PI AND PID CONTROLLER TUNING

JEDNODUCHÉ SEŘIZOVÁNÍ REGULÁTORŮ PI A PID

Abstract

The article is devoted to the simple PI and PID analog and digital controller tuning for nonoscillatory proportional plants of the first and second order with time delay. The approach is a modification of the desired model method and it is effected by varying of the one parameter – controller gain to tune the control system for the desired course of steps response. Resultant formulas are brought out in easily memorizable forms. The initial value of the controller gain ensures approximately the marginal aperiodic course. The use is shown in the example.

Abstrakt

Článek je věnován jednoduchému seřizování analogových a číslicových PI a PID regulátorů pro nekmitavé proporcionální regulované soustavy prvního a druhého řádu s dopravním zpoždění. Přístup je modifikací metody požadovaného modelu a využívá pro seřízení regulačního obvodu na požadovaný průběh přechodové charakteristiky změnu jediného parametru – zesílení regulátoru. Výsledné vztahy jsou uvedeny v jednoduchých snadno zapamatovatelných tvarech. Počáteční hodnota zesílení regulátoru zajišťuje přibližně mezní aperiodický průběh. Použití je ukázáno na příkladě.

1 INTRODUCTION

Controller tuning methods which use simple plant models are very popular among control engineering professionals e.g. the Ziegler-Nichols step response method, the Chien-Hrones-Reswick method and so on [Šulc & Vítečková 2004]. Recent tuning methods which came from the Dahlin or IMC controllers (λ-tuning) are used. Their advantage is the ability to obtain the desired step response course by changing the single tuning parameter.

This paper describes a similar conventional analog and digital PI and PID controller tuning method with single tuning parameter which is based on the desired model method (formerly known as inverse dynamics method) [Vítečková 1998, Šulc & Vítečková 2004, Švarc 2005] and on an approximation of the nonoscillatory proportional plant with the transfer function in the form

$$G_P(s) = \frac{k_1}{T_1 s + 1} e^{-T_{d1} s} \tag{1}$$

or

$$G_P(s) = \frac{k_1}{(T_2 s + 1)^2} e^{-T_{d2} s} \tag{2}$$

where k_1 is the plant gain, T_i – the time constant, T_{di} – the time delay, $i = 1, 2$.

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2 TRANSFER FUNCTIONS OF PLANTS AND CONTROLLERS

The plant transfer functions in the form (1) or (2) can be easily obtained on the basis of their step responses $h_P(t)$ in accordance with Fig. 1.

The time constant T_i and time delay T_{di} can be obtained from the formulas

$$\begin{aligned} T_1 &= 1.245(t_{0.7} - t_{0.33}) \\ T_{d1} &= 1.498 t_{0.33} - 0.498 t_{0.7} \end{aligned} \tag{3}$$

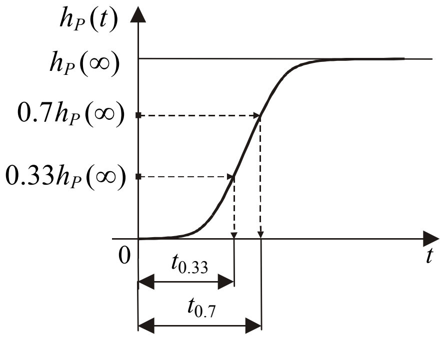

Fig. 1 Plant identification on the basis of its step response

or

$$\begin{aligned} T_2 &= 0.794(t_{0.7} - t_{0.33}) \\ T_{d2} &= 1.937 t_{0.33} - 0.937 t_{0.7} \end{aligned} \tag{4}$$

where times $t_{0.33}$ and $t_{0.7}$ are determined from the suitable smoothed plant step response (Fig. 1) [Vítečková 1998].

For the step of manipulated variable Δu the plant gain is given

$$k_1 = \frac{h_P(\infty)}{\Delta u} \tag{5}$$

Tab. 1 Transfer functions of conventional PI and PID controllers

TYPE	CONTROLLER	
	ANALOG	DIGITAL
PI	$k_P\left(1 + \frac{1}{T_I s}\right)$	$k_P\left(1 + \frac{T}{T_I}\frac{z}{z-1}\right)$
PID	$k_P\left(1 + \frac{1}{T_I s} + T_d s\right)$	$k_P\left(1 + \frac{T}{T_I}\frac{z}{z-1} + \frac{T_D}{T}\frac{z-1}{z}\right)$

The formulas (3) were obtained analytically and formulas (4) numerically on the basis of correspondence of approximated and real step responses of plant in the points $h_P(0)$, $h_P(t_{0.33}) = 0.33\, h_P(\infty)$, $h_P(t_{0.7}) = 0.7\, h_P(\infty)$, $h_P(\infty)$, see Fig. 1.

On the basis of formulas (3) and (4) it is possible to express the transfer function (1) in form (2) (the condition $T_1 < eT_{d1} \approx 2.718 T_{d1}$ must hold) and vice versa.

It is assumed that the controllers are conventional. Their transfer functions are given in Tab. 1.

3 CONTROLLER TUNING METHOD

The described controller tuning approach comes from the desired model method i.e. from the desired form of the closed control system transfer function G_{wy}, which for the control system with digital controller has the form (Fig. 2)

$$G_{wy}(z) = \frac{Y(z)}{W(z)} = \frac{T}{A(z-1)+Tz^{-d}} z^{-d}$$
$$d = \frac{T_d}{T} \tag{6}$$

and for the control system with analog controller with the form

$$G_{wy}(s) = \frac{Y(s)}{W(s)} = \lim_{T \to 0} G_{wy}(z)\Big|_{z=e^{sT}} = \frac{1}{As + e^{-T_d s}} e^{-T_d s} \tag{7}$$

where A is the varying parameter, T – the sampling period, T_d – the time delay, d – the relative discrete time delay (the integer for simplicity is supposed).

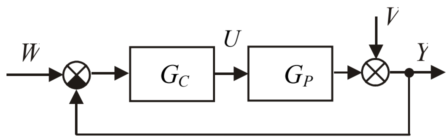

Fig. 2 Block diagram of control system

In the Fig. 2 the symbols W, U, V and Y are the transforms of desired, manipulated, disturbance and controlled variables, G_C and G_P – the controller and plant transfer functions. The independent variable s is considered for continuous control system with analog controller and the independent variable z is considered for discrete control system with digital controller.

On the assumption that zeros and non-dominant poles have negligible effect on the behaviour of the control process the characteristic polynomial of the control system with the digital controller has the form [see (6)]

$$N(z) = A(z-1) + Tz^{-d} \tag{8}$$

The conditions for the approximate marginal aperiodic control process are

$$N(z) = 0, \quad \frac{\mathrm{d}N(z)}{\mathrm{d}z} = 0 \tag{9}$$

From them the double stable real pole is obtained

$$z_2 = \frac{d}{d+1} \tag{10}$$

and the corresponding maximum (initial) value of the varying parameter

$$A_d = T(d+1)\left(\frac{d+1}{d}\right)^d \approx (4-\mathrm{e})T + \mathrm{e}T_d \tag{11}$$

For $d \geq 1$ the relative error of the approximation (11) is better then 1.6 %.

The maximum (initial) value of the varying parameter A_a for the control system with the analog controller on the basis of the relation (11) it is as well possible to obtain

$$A_a = \lim_{T \to 0} A_d = \mathrm{e}T_d \tag{12}$$

In the above mentioned relations time delay T_d must be considered, which is in approximate plant transfer function.

The controller transfer function on the basis of the synthesis equation

$$G_C = \frac{1}{G_P} \frac{G_{wy}}{1 - G_{wy}} \tag{13}$$

can be obtained, where the desired transfer function (6) for the control system with the digital controller and (7) for control system with the analog controller are considered.

For a control system with a digital controller the plant transfer function is given by the formula

$$G_P(z) = \left(1 - z^{-1}\right) Z \left\{ L^{-1} \left\{ \frac{G_P(s)}{s} \right\} \Bigg|_{t=kT} \right\} \tag{14}$$

Depending on the form of the plant transfer function (1) or (2) the transfer function of the PI or PID controller is obtained and at the same time as the relations for their adjustable parameters. The obtained relations for the digital controllers are simplified. For ease of memorizing some of the numerical values are substituted for well known constants (Tab. 2).

For a rough estimate of the sampling period T it is possible to use the relations

$$T = \left(\frac{1}{15} \div \frac{1}{6} \right) \left(4T_1 + T_{d1} \right) \tag{15}$$

or

$$T = \left(\frac{1}{15} \div \frac{1}{6} \right) \left(7T_2 + T_{d2} \right) \tag{16}$$

which come from the known relation

Tab. 2 Values of PI and PID controller adjustable parameters

CONTROLLED PLANT	CONTROLLER ANALOG $T = 0$ DIGITAL $T > 0$				INITIAL VALUE OF VARYING PARAMETER A	NOTE
	TYPE	T_I^*	T_D^*	k_P^*		
$\frac{k_1}{T_1 s + 1} \mathrm{e}^{-T_{d1} s}$	PI	$T_1 - \frac{T}{2}$	–	$\frac{T_I^*}{A k_1}$	$A \le (4 - \mathrm{e})T + \mathrm{e}T_{d1}$	
	PID	$(4 - \mathrm{e})T_1 - T$	$\frac{T_I^*}{4}$		$A \le (4 - \mathrm{e})T + \mathrm{e}T_{d1} - T_1$	$T_1 < eT_{d1}$
$\frac{k_1}{(T_2 s + 1)^2} \mathrm{e}^{-T_{d2} s}$	PI	$\frac{\pi}{2} T_2 - \frac{T}{2}$	–		$A \le (4 - \mathrm{e})T + \mathrm{e}T_{d2} + 1{,}5T_2$	
	PID	$2T_2 - T$	$\frac{T_I^*}{4}$		$A \le (4 - \mathrm{e})T + \mathrm{e}T_{d2}$	

$$T = \left(\frac{1}{15} \div \frac{1}{6}\right) t_{0.95} \qquad (17)$$

where $t_{0.95}$ is the time for reaching the value $0.95 h_P(\infty)$, see Fig. 1.

4 EXAMPLE

For the plant with the transfer function

$$G_P(s) = \frac{1.5}{(6s+1)(4s+1)(2s+1)} e^{-3s}$$

by identification, the three plant parameters were obtained: $k_1 = 1.5$, $t_{0.33} = 10.8$ s a $t_{0.7} = 17.4$ s. It is necessary to determine the PI and PID analog and digital adjustable parameters for the control process with the overshoot about 5 %.

Solution:

For plant approximate transfer function (1) and (2) in accordance with the relations (3) and (4) it is obtained: $T_1 \doteq 8.2\,\mathrm{s}, T_{d1} \doteq 17.4\,\mathrm{s}$ and $T_2 \doteq 5.2\,\mathrm{s}$, $T_{d2} \doteq 4.6\,\mathrm{s}$.

On the basis of the relation (15) and (16) the sampling period value was estimated $T \doteq 2.7 \div 6.8 \Rightarrow T = 4\,\mathrm{s}$.

Tab. 3 Values of PI and PID controller adjustable parameters – example

	k_P^* – initial	k_P^* – final	T_I^*	T_D^*	A_{max}
PI digital	0.16	0.206	6.2	–	25.51
PI analog	0.27	0.33	8.2	–	20.39
PID digital	0.25	0.237	6.51	1.63	17.31
PID analog	0.57	0.511	10.51	2.63	12.19

The step responses for desired overshoot about 5 % are shown in Fig. 3.

The obtained values of the controller adjustable parameters for the plant transfer function (1) and (2) are approximately equal. They are given in Tab. 3.

The initial overshoots for digital and analog PID controllers are produced by derivative implementation in the simulation programme Simulink.

CONCLUSIONS

The described controller tuning method is based on the desired model method. It is very simple and effective and it gives very good results for rough approximate transfer function of plants with high order or with time delay, which can be dominant.

This work was supported by research project GAČR 101/06/0491.

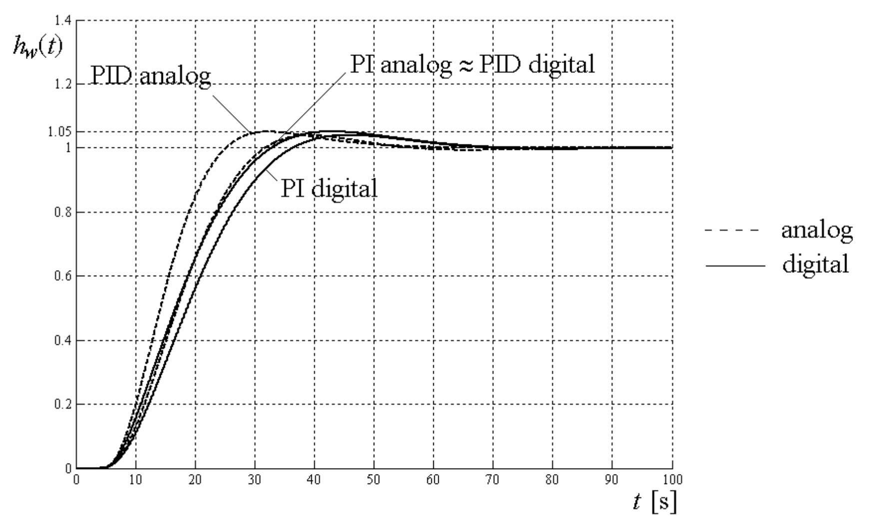

Fig. 3 Step responses of control system - example

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