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STABILITY OF A ROTOR VIBRATION IN JOURNAL BEARINGS

STABILITA VIBRACÍ ROTORU V KLUZNÉM LOŽISKU

Abstract

The paper deals with stability of a rotor vibration in a journal bearing. The first part of the paper shows the lumped parameter mathematical model of a rotor motion in a fluid lubricant. The second part of the paper is focused at the problem of stability of the rotor system, which gives the explanation of the fluid induced instabilities using the Nyquist stability criterion.

Abstrakt

Článek pojednává o stabilitě vibrací čepu v kluzném ložisku. První část článku popisuje pohybovou rovnici rotoru se soustředěnými parametry. Čep se otáčí v mazivu. Druhá část článku se zaměřuje na problém stability pohybu motoru pro vysvětlení vzniku nestabilit způsobených tekutým mazivem užitím Nyquistova kritéria stability.

1 INTRODUCTION

The topic of this paper is focused at the motion of rotors in journal bearings. Measurement instrumentation is shown in figure 1. Proximity probes are a non-contacting device, which measures the displacement motion and position of an observed rotor surface relative to the probe mounting location. Typically, proximity probes, used for rotating machinery measurements, operate on the eddy current principle, and measure rotor displacement motion and position relative to the machine bearings or housing.

The first research work dealing with the stability of the rotor vibration in journal bearings was published by Newkirk in 1924 as it is mentioned by Tondl (1965). The problem of the fluid-induced vibration was investigated by Muszynska (1986) and Bently (1989) at Bently Rotor Dynamics Research Corporation. One of the latest reviews of the work, which was done in this branch of science, is a paper published by Ecker and Tondl (2005).

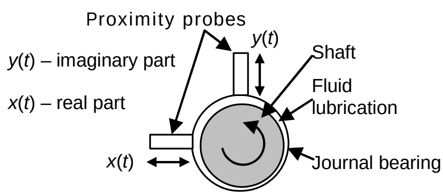

Fig. 1 Instrumentation arrangement

The topic of the mentioned paper is focused at solving the problem how to prevent the onset of the instability by control the bearing mount stiffness while increasing the rotor rotational speed. The goal of the submitted paper is drawing attention to the methods used in the control theory.

2 MODEL OF THE ROTOR SYSTEM

There are many ways how to model a rotor system, but this paper prefers an aproach, which is based on the concept developed by Muszynska [2] and Bently Rotor Dynamics Research Corporation

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[3]. The reason for this is that this concept offers an effective way to understanding the rotor instability problem. Another approach can be based on the lubricant flow prediction using a FE method for Reynolds equation solution, see [9] for instance. This approach does not allow benefiting of the dynamic system stability theory.

Let the rotor angular velocity is designated by Ω. It is assumed that the stator is fixed while rotor is rotating at the mentioned angular velocity. The rotor drags the fluid in a space between two cylinders into motion and acts as a pump. It is easy to understand that the fluid velocity is varying across the gap as a consequence of the fluid viscosity. If the fluid average angular velocity is designated after Muszynska by v_{avg} then the fluid circumferential velocity ratio λ is given by the formula

$$\lambda = \frac{v_{avg}}{\Omega}, \tag{1}$$

The value of this dimensionless ratio is slightly less than 0.5 due to the profile of the fluid velocity in the space between the rotor and journal, see figure 2.

This paper proposes to use complex variables to describe motion of a rotor. The position of the journal centre in the complex plane, which is perpendicular to the journal center-line, is designated by a position vector $\mathbf{r}$. The fluid pressure wedge is the actual source of the fluid film stiffness in a journal bearing and maintains the rotor in equilibrium. These bearing forces can be modeled as a rotating spring and damper system at the angular frequency $\lambda\Omega$.

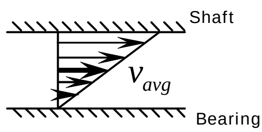

Fig. 2 Average fluid velocity

Fluid forces, which are acting on the rotor in coordinates rotating at the same angular frequency as the spring and damper system, are given by the formula

$$\mathbf{F}_{rot} = K\,\mathbf{r}_{rot} + D\,\dot{\mathbf{r}}_{rot}, \tag{2}$$

where the parameters, K and D, are specifying proportionality of stiffness and damping to the rotor centre-line displacement vector $\mathbf{r}_{rot}$ and velocity vector $\dot{\mathbf{r}}_{rot}$, respectively. The spring force acts opposite to the displacement vector.

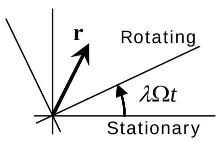

Fig. 3 Transform to stationary coordinates

To model the rotor system, the fluid forces have to be expressed in the stationary coordinate system, in which the rotor centre-line displacement and velocity vectors are designated by $\mathbf{r}$ and $\dot{\mathbf{r}}$, respectively. Conversion the complex rotating vector $\mathbf{r}_{rot}$ to the stationary coordinate system can be done by multiplication this vector by $\exp(j\lambda\Omega t)$, which is the same as multiplying the vector in the stationary coordinates by $\exp(-j\lambda\Omega t)$, see figure 3.

The relationship between the mentioned vectors in rotating and stationary coordinates are given by the formulas

$$\begin{aligned} \mathbf{r}_{rot} &= \mathbf{r}\exp(-j\lambda\Omega t) \\ \dot{\mathbf{r}}_{rot} &= (\dot{\mathbf{r}}_{rot} - j\lambda\Omega\,\mathbf{r})\exp(-j\lambda\Omega t) \end{aligned} \tag{3}$$

Substitution into the fluid force equation (2) results in the following formula

$$\mathbf{F} = K\,\mathbf{r} + D\dot{\mathbf{r}} - jD\lambda\Omega\,\mathbf{r} \tag{4}$$

where the complex term $jD\lambda\Omega\,\mathbf{r}$ has the meaning of the force acting in the perpendicular direction to the vector $\mathbf{r}$, this force is called tangential. As the rotor angular velocity increases, this force can become very strong and can cause instability of the rotor behavior.

As it was mentioned, the rotor is under influence of the external forces, for instance produced by unbalance mass or simply by gravity. To obtain general solution this external perturbation force, resulting from unbalance, is assumed to be rotating at the angular velocity Ω , which is considered to be completely independent of the rotor angular velocity . The unbalance force, which is produced by unbalance mass m mounted at a radius r_u , acts in the radial direction and has a phase δ at time $t=0$

$$\mathbf{F}_{Perturbation} = m r_u \omega^2 \exp(j(\omega t + \delta)) \tag{5}$$

The equation of motion for a rigid rotor operating in a small, localized region in the journal bearing, is as follows

$$M\ddot{\mathbf{r}} + D\dot{\mathbf{r}} + (K - jD\lambda\Omega)\,\mathbf{r} = m r_u \omega^2 \exp(j(\omega t + \delta)) \tag{6}$$

3 ROTOR SYSTEM AS A SERVOMECHANISM

The rotor/fluid wedge bearing/system can be demonstrated as a servomechanism working in the closed loop, which is shown in figure 4. The direct and quadrature dynamic stiffness is introduced according to the acting force direction. To obtain the Laplace transform of the motion equation, the imaginary variable $j\omega$ is replaced by a complex variable s

$$K_{Direct}(s) = K + Ds + Ms^2 \tag{7}$$

$$K_{Quadrature}(s) = -j\lambda\Omega D \tag{8}$$

and the equation of motion (6) takes the form

$$\mathbf{r} = \left(\mathbf{F}_{Perturbation} - K_{Quadrature}(s)\mathbf{r}\right)/K_{Direct}(s) \tag{9}$$

The individual transfer function $1/K_{Direct}(s)$ (direct dynamic compliance) is stable. The feedback path in the closed-loop system acts as a positive feedback and introduces instability for the closed-loop system. The gain of the positive feedback depends on the angular velocity Ω. The closed-loop system is stable for the low rotor rotational speed. But there is a margin for the stable behavior. If the gain of the positive feedback crosses over a limit value then the closed-loop becomes unstable. The properties of the unstable behavior can be analyzed using the servomechanism in figure 4.

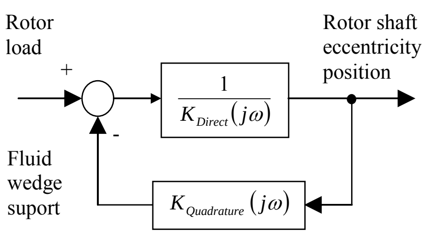

Fig. 4 Shaft/fluid wedge bearing/system as a servomechanism

The stability of the closed-loop dynamic system is depending on the open-loop frequency transfer function for $s = j\omega$

$$G_0(j\omega) = \frac{K_{Quadrature}(j\omega)}{K_{Direct}(j\omega)} = \frac{-\lambda\Omega D}{\omega D - j(K - M\omega^2)} \tag{10}$$

As it is known the closed-loop dynamic system is stable according to the Nyquist stability criterion if, and only if, the locus $G_0(j\omega)$ of the function in the complex plane does not enclose the (-1,0) point as ω is varied from zero to infinity (Burns, 2001).

Enclosing the (-1,0) point is interpreted as passing to the left of the mentioned point. The $G_0(j\omega)$ locus for three different values of the rotor angular velocity Ω is shown in Nyquist diagram in figure 5, witch is plotted as an illustrating example for K/D = 100 rad/s. All the contour plots are of the same shape. They are differing only in a scale and correspond to the stable, steady-state and unstable vibration. When the steady-state vibration occurs, the stability margin is achieved. The locus of the $G_0(j\omega)$ function, describing the steady-state vibration, meets the (-1,0) point, therefore

$$G_0(j\omega_{Crit}) = -1 \,. \tag{11}$$

An angular frequency, at which a system can oscillate without damping, is designated by ω_{Crit}. Substitution (11) into (10) results in formulas for the oscillating frequency

$$\omega_{Crit}^2 = K/M \text{ and } \omega_{Crit} = \lambda\Omega \tag{12}$$

It can be concluded that the frequency of the rotor subharmonic oscillation is the same as the fluid average circumferential angular velocity. The measurement shows that the value of the parameter λ is equal to 0.475, see a multispectrum of the rotor vibration, which is plotted versus the rotor RPM in figure 6. This result confirms the introductory assumption about the fluid forces acting on the rotor. The stability margin corresponds to the mechanical resonances of the rigid rotor mass supported by the oil spring. It can be noted that the frequency ω_{Crit} is not equal to the rotor critical speed when the vibration is excited by the rotor unbalance.

If the system were linear then the unstable rotor vibration would spiral out to infinity when the rotor angular frequency crosses the so-called Bently-Muszynska threshold

$$\Omega_{Crit} = \frac{\sqrt{K/M}}{\lambda} \tag{13}$$

The Bently-Muszinska threshold is inversely proportional to the ratio λ. The anti-swirl technique is focused at decreasing λ.

As it is experimentally verified (figure 6 [8]) the frequency spectrum of the fluid-induced vibration contains the single dominating component as it would be a solution of the second order linear differential equation without damping.

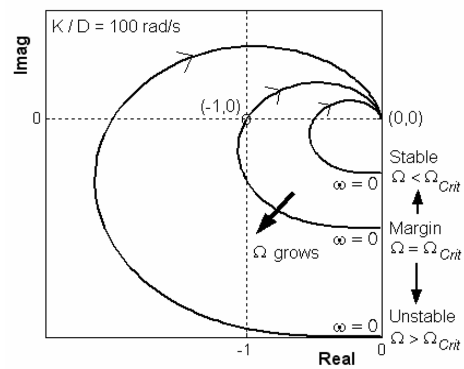

Fig. 5 Nyquist diagram showing stable, margin and unstable locus

The proportionality between ω and Ω is maintained for a wide range of Ω, which is greater than the threshold Ω_{Crit}.

The rotor lateral oscillations are limited by the journal bearing surface. Stiffness and damping coefficients are non-linear function of the eccentricity ratio, especially when the rotor is approaching to the journal wall. If the magnitude of vibration is increasing then the oil-film stiffness and damping is increasing as well and the relationship (12) is maintained adapting stiffness to the value of $M\omega^2$. A new balance forms a steady-state limit cycle of the rotor orbital motion.

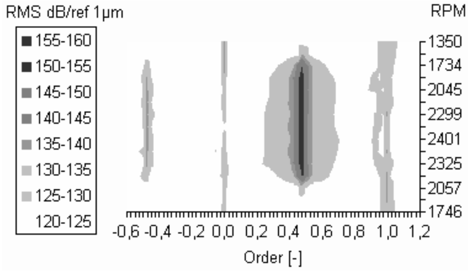

Fig. 6 Full spectrum of the fluid induced vibration

A fluid-induced instability, commonly referred to as oil whirl, is the special resonance vibration with the frequency that is proportional to the rotational speed. The rotor precession is self-excited by fluid induced instability and it is called whirl vibration. The whirl vibration is always forward precession and starts at the rotor rotational frequency given by (13). The orbit shape is nearly circular for whirl vibration (Bently and Muszynska, 1988).

3 CONCLUSIONS

The lumped parameter model of the rotor motion in the journal bearing gives explanation of the stability margin and the onset of the self-excited vibration. The analysis of the rotor behaviour is based on using the Nyquist stability criterion for linear dynamic systems.

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