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STRUCTURAL PROPERTIES OF BOND GRAPH MODELS

STRUKTURÁLNÍ VLASTNOSTI MODELU VAZEBNÍHO GRAFU

Abstract

In modeling causality is a fundamental concept that presents the dependencies between elements. Structural analysis of dynamic systems can be performed using causal manipulations on the bond-graph model. The paper presents a study on the structural properties of bond graph models starting from causal bond graph model and using theorems and an illustrative physical example. Our research focused on the controllability and the observability of the system and we propose a series of structural controllability and observability properties of the systems represented by bond graphs. To test the controllability and the observability of these systems a matriceal method is also used.

Abstrakt

Při modelování kauzality je základní zásadou zobrazit závislost mezi elementy. Strukturální analýza dynamických systémů může být provedena za použití kauzálních manipulacích v modelu vazebního grafu. Tento příspěvek presentuje studii strukturálních vlastností modelu vazebního grafu začínajících od kauzálních vazebních grafů a používající teorémy a ilustrativní fyzické příklady. Náš výzkum se zaměřil na schopnost kontroly a pozorovatelnosti systému a navrhujeme sérii vlastností systémů reprezentovanými vazebními grafy. Pro testování kontroly a pozorovatelnosti těchto systémů se využívá maticová metoda.

1 INTRODUCTION

In engineering systems the processes can be generally classified as belonging to, for example, rigid and solid body mechanics, fluid mechanics, electricity and magnetism, semiconductors physics, thermo-dynamics, and so forth. Because each branch has its particular solving methodology it is natural to apply the methodology of the field in question, if the problem in question deals with a single physical domain, including any specialized computational methods that may be available. If the interactions between domains are weak the same approach can be applied even in multi-domain problems.

A well-known approach designed to deal with multi-domain engineering problems is the bond graph method developed by H. Paynter. He presented this methodology for the first time in the lecture “Ports, Energy, and Thermo-dynamic Systems” in 1959. Karnopp, Rosenberg, Thoma were the first to apply the Paynter’s bond-graph method [Karnopp, Rosenberg 1975]. To describe physical processes the method uses the effort-flow analogy. These processes are represented graphically in the form of elementary components - bond graph elements - with one or more ports representing the places where interactions with other processes take place. One-port elements include active elements and passive elements, two-port elements are transformers (TF) and gyrators (GY) and three-port or multi-port elements are 0-junction and 1-junction.

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All physical processes are described using several elementary components, or elements: sources of effort and flow (denoted as Se and Sf), elements of storage of energy (I and C), dissipation of power (R), transformers of power – transformers and gyrators (TF and GY), junction 0 or parallel junction, and junction 1 or series junction.

I, C, and R elements are passive elements because they convert the supplied energy into stored or dissipated energy. The half arrow is always pointing towards these elements.

Se and Sf elements are active elements because they supply power to the system and the half arrow is always pointed outwards these elements.

TF, GY, 0 and 1 junctions are junction elements that serve to connect I, C, R, and source elements and constitute the junction structure of bond graph model.

The concept of causality is an important concept embedded in bond-graph theory [Thoma 1990]. This refers to cause (input) and effect (output) relationship. Thus, as part of the bond-graph modelling process, a causality assignment is implicitly introduced. Causality is graphically represented by a short stroke, called causal stroke, placed perpendicular to the bond at one of its ends indicating the direction of the effort variable. Causal stroke assignment is independent of the power flow direction. This leads to the description of bond-graphs in the form of state – space equation.

The sources (Se and Sf) have fixed causality, the element of dissipation (R) has free causality depending on the causality of the other elements of bond graph, and the storage elements (I, C) have preferential causality, that is integral causality or derivative causality, but it is always desirable that C and I elements to be in integral causality.

Transformers, gyrators and junction elements have constrainedly causality. Thus on a 0-junction one effort pushes inward, all others outward and on a 1-junction one flow points inwards, all others point outwards.

The process seen at a port is described by a pair of variables, effort and flow. These are the power variables, and their product is power. In addition to the power variables, there also are internal variables that represent the accumulation of effort and flow over time. These variables are called generalized momentum and generalized displacement.

Bond graph models describe the dynamic behaviour of physical systems and they are based on the principle of conservation of power.

Bond graph method illustrates the energetic transfer in a system using bond lines. Bond lines are represented by a half arrow. The orientation of the arrow shows the direction in which power flows.

Table 1 presents the power and generalized variables together with their significations in several energetic domains.

Tab 1. Power variables and generalized variables

Domain	*Effort (e)*	*Flow (f)*	*Momentum (p)*	*Displacement (q)*
Electrical	Voltage	Current	Flux linkage	Charge
Mechanical translation	Force	Velocity	Momentum	Displacement
Mechanical rotation	Torque	Angular velocity	Angular momentum	Angle
Hydraulic	Pressure	Volume flow rate	Pressure momentum	Volume
Thermal	Temperature	Heat flow	-	Heat energy

After the assignation of causality of elements and junctions in a bond graph model it is obtained the causal bond graph, and one can write the characteristic equations of junctions and elements, and deduce the state equations of the system and state variables associated to the I and C elements. It is also possible the determination of causal paths, causal loops and chains of action, that illustrate the links between the elements of bond graph model and gives the possibility to proceed to structural analysis of the system.

2 CONTROLLABILITY AND OBSERVABILITY THEOREMS

Starting from causal bond graph model, through causal manipulations on the bond-graph model and using some theorems, it is possible to study the controllability and the observability of the system. Structural controllability and observability may be studied using the following theorems [Dauphin-Tanguy 2000].

Theorem 1. A bond graph model is structural state controllable if and only if both of the following conditions are satisfied:

1. in bond graph model in integral causality there is a causal path between all I and C dynamic elements in integral causality and an effort source or flow source (Se or Sf)
2. all dynamic I and C elements have derivative causality in the bond graph model in derivative causality.

Theorem 2. A bond graph model is structural state observable if and only if both of the following conditions are satisfied:

1. in bond graph model in integral causality there is a causal path between all I and C dynamic elements in integral causality and a detector (De or Df)
2. all dynamic I and C elements have derivative causality in the bond graph model in derivative causality.

In order to test the controllability and the observability of physical systems a matriceal method may be also used [Fossard 1993]. The great advantage of this method it that it uses the junction structure matrix *S*. This matrix describes the relations between efforts and flows in the junction structure of bond graph model.

A bond graph model can be organized as is presented in figure 1, where x^i and x^d are the state vectors composed of energy variables *p* and *q* associated to I and C elements in integral causality, and respectively in derivative causality, Z^i and Z^d are the complementary state vectors composed of efforts and flows, and D_{in} and D_{out} are the vectors composed of efforts and flows representing the inputs and the outputs of R-elements.

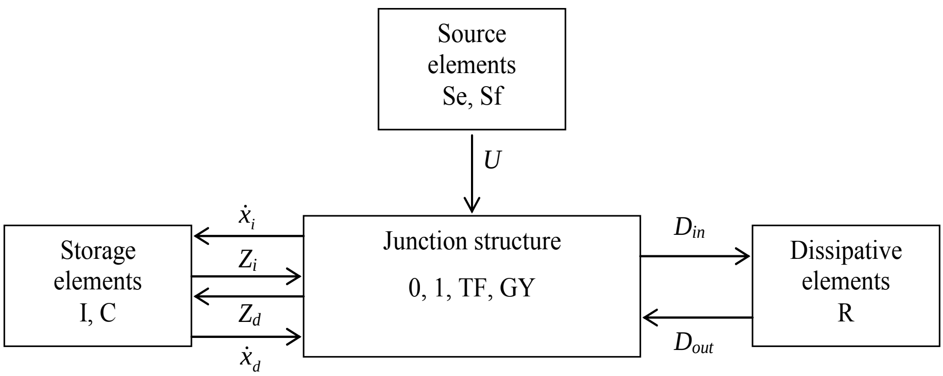

Fig 1. Vectorial representation of a bond graph model

The junction structure of a bond graph model is characterized through the matrix *S* composed of submatrices S_{ij} (*i*=1,2,3; *j*=1,2,3,4).

$$\begin{bmatrix} \dot{x}_i \\ D_{in} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \end{bmatrix} \cdot \begin{bmatrix} Z^i \\ \dot{x}^d \\ D_{out} \\ U \end{bmatrix} \tag{1}$$

$$y = \begin{bmatrix} S_{31} & S_{32} & S_{33} & S_{34} \end{bmatrix} \cdot \begin{bmatrix} Z^i \\ \dot{x}^d \\ D_{out} \\ U \end{bmatrix} \tag{2}$$

The junction structure matrix S is composed of 0, +1, -1, and m or 1/m, k or 1/k if the bond graph model has in its structure transformers and gyrators, where m is the transformer modulus and k is the gyrator modulus.

Controllability and observability are studied using the following theorems [Sueur, Dauphin-Tanguy 1989].

Theorem 3. A linear system is structurally controllable if both of the following conditions are satisfied:

1. there is at least one non-zero term in S_{14} or S_{24}
2. there is no linear combination between the rows of $(S_{11} \quad S_{13} \quad S_{14})$.

Theorem 4. A linear system is structurally observable if both of the following conditions are satisfied:

1. there is at least one non-zero term in $(S_{31} \quad S_{32} \quad S_{33})$
2. there is no linear combination between the columns of $(S_{11}^T \quad S_{21}^T \quad S_{31}^T)^T$.

3 A SIMPLE ELECTRIC CIRCUIT EXAMPLE

Using the theorems presented above we will study the controllability and observability of an electric circuit presented in figure 2.

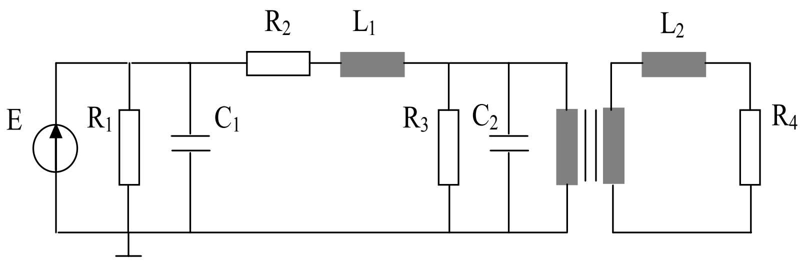

Fig 2. An electric circuit

The bond graph model associated to this electric circuit is given in figure 3.

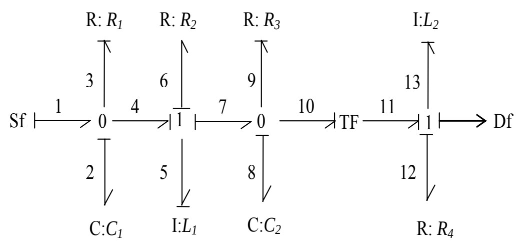

Fig 3. Bond graph model

The state variables corresponding to I and C elements in integral causality are $\begin{pmatrix} q_2 & p_5 & q_8 & p_{13} \end{pmatrix}$.

Thus:

$$x^i = \begin{pmatrix} q_2 \\ p_5 \\ q_8 \\ p_{13} \end{pmatrix}, Z^i = \begin{pmatrix} e_2 \\ f_5 \\ e_8 \\ f_{13} \end{pmatrix} \tag{3}$$

$$D_{in} = \begin{pmatrix} e_3 \\ f_6 \\ e_9 \\ f_{12} \end{pmatrix}, \; D_{out} = \begin{pmatrix} f_3 \\ e_6 \\ f_9 \\ e_{12} \end{pmatrix} \tag{4}$$

$$U = f_1 \tag{5}$$

In order to obtain the junction structure matrix S of the system it is necessary the establishment of relations that characterize the junction elements.

The relations that characterize 0 junctions, 1 junctions and the transformer are presented below. For 0 junction we have the following relations:

$$\begin{matrix} e_1 = e_2 = e_3 = e_4 \\ f_1 - f_2 - f_3 - f_4 = 0 \end{matrix} \quad \text{and} \quad \begin{matrix} e_7 = e_8 = e_9 = e_{10} \\ f_7 - f_8 - f_9 - f_{10} = 0 \end{matrix} \tag{6}$$

Taking into account that $\dot{q}_2 = f_2$ and $\dot{q}_8 = f_8$ we obtain:

$$\begin{matrix} \dot{q}_2 = f_1 - f_3 - f_4 \\ \dot{q}_8 = f_7 - f_9 - f_{10} \end{matrix} \tag{7}$$

The relations characterizing 1 junctions are:

$$\begin{matrix} e_4 - e_5 - e_6 - e_7 = 0 \\ f_4 = f_5 = f_6 = f_7 \end{matrix} \quad \text{and} \quad \begin{matrix} e_{11} - e_{12} - e_{13} = 0 \\ f_{11} = f_{12} = f_{13} \end{matrix} \tag{8}$$

Using $\dot{p}_5 = e_5$ and $\dot{p}_{13} = e_{13}$ we obtain:

$$\begin{matrix} \dot{p}_5 = e_4 - e_6 - e_7 \\ \dot{p}_{13} = e_{11} - e_{13} \end{matrix} \tag{9}$$

For TF element:

$$\begin{matrix} e_{11} = \frac{1}{m} e_{10} \\ f_{10} = \frac{1}{m} f_{11} \end{matrix} \tag{10}$$

Using the relations that characterize the junction elements one obtain the equation:

$$\begin{pmatrix} \dot{q}_2 \\ \dot{p}_5 \\ \dot{q}_8 \\ \dot{p}_{13} \\ e_3 \\ f_6 \\ e_9 \\ f_{12} \\ y \end{pmatrix} = \begin{pmatrix} 0 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 1 \\ 1 & 0 & -1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -\frac{1}{m} & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & \frac{1}{m} & 0 & 0 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} e_2 \\ f_5 \\ e_8 \\ f_{13} \\ f_3 \\ e_6 \\ f_9 \\ e_{12} \\ f_1 \end{pmatrix} \tag{11}$$

For the study of structural controllability we will verify if both of the conditions of theorem 3 are satisfied.

From the equation (11) it can be seen that there is one non-zero term in S_{14} and there is no linear combination between the rows of $\left(S_{11} \quad S_{13} \quad S_{14}\right)$. The system is then structurally controllable.

For observability, we use theorem 4 and we can see that there is a non-zero term in $\left(S_{31} \quad S_{32} \quad S_{33}\right)$, and there is no linear combination between the columns of $\left(S_{11}^T \quad S_{21}^T \quad S_{31}^T\right)^T$. Thus, the system is structural observable.

4 CONCLUSIONS

The research proved that bond graph method used in system modeling presents the advantage of both linear and nonlinear systems solving. In adition to that it uses only nine elements in the model structure to which we can assign the causality. After the causality assignation to elements and junctions, in the causal bond graph model it is possible to write the state equations of the system and state variables associated to the I and C elements. Using causal manipulations on the bond-graph model we can obtain information about system's structural properties.

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