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Dan POPESCU*, Emil PETRE*, Eugen BOBAŞU*

ON THE STABILITY ROBUSTIFICATION OF THE EXACT LINEARIZATION METHOD

ZVÝŠENÍ ROBUSTNOSTI METODY EXAKTNÍ LINEARIZACE

Abstract

The exact linearization method via feedback consists in transforming a nonlinear system into a linear one using a state feedback. The linearized system obtained is in a non-robust form whose dynamics is completely different from that of the original system. So, the use of feedback linearization requires the complete knowledge of the nonlinear system. It is possible that the controlled system become unstable in the presence of significant model uncertainties. To improve robustness, it may be necessary to modify the exact linearization controller. In this paper, some robustification techniques for the exact linearization method are discussed and an example is presented, also.

Abstrakt

Metoda zpětnovazební exaktní linearizace spočívá v transformaci nelineárního systému na lineární systém použitím stavové zpětné vazby. Získaný linearizovaný systém je nerobustní, jehož dynamika je naprosto odlišná od původního systému. Proto využití zpětnovazební linearizace vyžaduje plnou znalost vlastností nelineárního systému. Existence významných neurčitostí v modelu může způsobit, že řízený systém bude nestabilní. Pro zlepšení robustnosti může být nezbytné upravit regulátor exaktní linearizace. Tento příspěvek se zabývá metodami pro zvýšení robustnosti při exaktní linearizaci s uvedením příkladů.

1 INTRODUCTION

In the last years, significant advances have been made in the development of ideas such as feedback linearizing techniques. The problem of exact linearization via feedback and diffeomorphism consists in transforming a nonlinear system into a linear one using a state feedback and a coordinate transformation of the state [Isidori 1995].

Practical implementation of such controllers requires consideration of various sources of uncertainties such as: modelling errors, computation errors, unknown payloads, measurement noise, etc. It is possible that the controlled system become unstable in the presence of significant model uncertainties. To improve robustness, it may be necessary to modify the exact linearization controller to guarantee its robustness.

Several techniques from linear and nonlinear control theory have been applied to the problem of robust feedback linearization: Lyapunov redesign method, sliding modes, the $\mathbf{H}_\infty$ approach, etc. Here, we present two techniques that can be applied to obtain a robust controller for the feedback linearization. First, Glover-McFarlane $\mathbf{H}_\infty$ design is presented with the goal of increasing robustness of existing controllers without significantly compromising performance. The second approach is the

* Professor, Department of Automatic Control, Faculty of Automation, Computers and Electronics, University of Craiova, A.I. Cuza 13, 200585 Craiova, Romania. Tel. (+40) 251 438 198, e-mail: dpopescu@automation.ucv.ro, epetre@automation.ucv.ro, ebobasu@automation.ucv.ro.

two-degree of freedom controller design. In these methodologies, it is possible to separate the designing task of meeting performance specifications and robustness into two modular steps.

The paper it is organized as follows: in the next section the feedback linearizing technique is shortly presented. Then, the application of the two robustifying methods it is approached. Finally, some numerical simulation results for a handling crane model are presented.

2 THE FEEDBACK LINEARIZING METHOD

The nonlinear system that we consider is described by equations of the following kind:

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x) \end{aligned} \tag{1}$$

in which *f(x)*, $g(x)$ are smooth vector fields.

The exact linearization via feedback and diffeomorphism consists in transforming the nonlinear system (1) into a linear one using a state feedback and a coordinate transformation of the systems state. This can be done introducing the Lie derivative of a function $h(x): R^n \to R$ along a vector field $f(x) = [f_1(x), \ldots f_n(x)]$

$$L_f h(x) = \sum_{i=1}^{n} \frac{\partial h(x)}{\partial x_i} f_i(x) \tag{2}$$

Definition [Isidori 1995]. A nonlinear system of the form (1) has a relative degree r at a point x^0 if:

$$L_g L_f^k h(x) = 0 \tag{3}$$

for all $k < r - 1$ and for all x in a neighborhood of x^0, and $L_g L_f^{r-1} h(x^0) \neq 0$.

Proposition. Let be the nonlinear system of the form (1) having the relative degree r at a point x^0. Then, the state feedback

$$u = \frac{1}{b(x)}\left[-a(x) + v\right] = \frac{1}{L_g L_f^{r-1} h(x)}\left[-L_f^r h(x) + v\right] \tag{4}$$

transform the nonlinear system in a system having the input-output behaviour identical with those of a linear one with the transfer function (see Fig. 1):

$$H(s) = \frac{Y(s)}{V(s)} = \frac{1}{s^r} \tag{5}$$

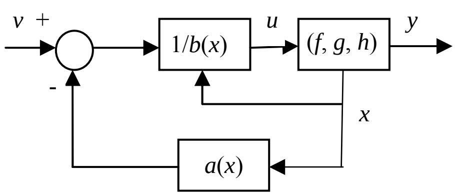

Fig. 1 The linearized model (L.M.)

On the linear system thus obtained one impose a feedback control of the form:

$$v = c_0\left(y_r - y\right) - c_1\dot{y} - \cdots - c_{r-1}y^{(r-1)} \tag{6}$$

then, the obtained system has a linear input-output behavior, described by the following transfer function

$$H(s)=\frac{Y(s)}{Y_r(s)}=\frac{c_0}{s^r+c_{r-1}s^{r-1}+\cdots+c_0} \tag{7}$$

The design parameters are computed using a pole-placement design technique.

3 GLOVER-MCFARLANE CONTROL DESIGN

We consider the structure of the control loop shown in Fig. 2, where is implemented the control law (6) and K_r is the robustifying controller (G_s is the nominal shaped plant).

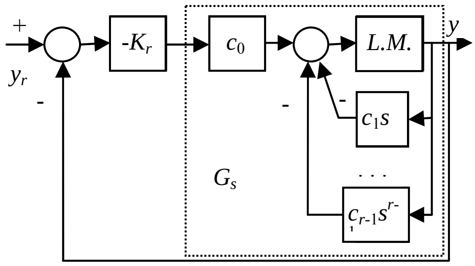

Fig. 2 The control loop

In this design, the model uncertainties are included as perturbations to the nominal model, and robustness is guaranteed by ensuring that the stability specifications are satisfied for the *worst-case* uncertainty.

Let $G_s = N/M$ be the normalized coprime factorization of the nominal shaped plant.

The normalized coprime factor uncertainty characterization is given by

$$\left\{\frac{N+\Delta_N}{M+\Delta_M} : \left\| \left[\Delta_N \ \Delta_M\right] \right\| \le \varepsilon \right\} \tag{8}$$

The following steps yield the optimal controller that assumes a state-space (A, B, C) available for the transfer function G_s:

1. Obtain Z by solving the algebraic Riccati equation (ARE)

$$AZ + ZA - ZC^T CZ + BB^T = 0 \tag{9}$$

2. Obtain X by solving the ARE

$$AX + XA - XBB^T X + C^T C = 0 \tag{10}$$

3. Compute the maximum possible ε for the given nominal shaped plant

$$\varepsilon_{\max} = (1+\rho(XZ))^{-1/2} \tag{11}$$

where ρ denotes the spectral radius. Hence, in this design scheme there is no need for an explicit characterization of uncertainty. The method detects and solves for the worst-case scenario.

4. The robustness margin ε is chosen to be slightly less than $\varepsilon_{\max}$. Let $\gamma = 1/\varepsilon$.

5. The state-space realization of the robustifying controller K_r is given by

$$\begin{bmatrix} A+BF+\gamma^2(L^T)^{-1}ZC^T C & \gamma^2(L^T)^{-1}ZC^T \\ B^T X & 0 \end{bmatrix} \tag{12}$$

where $F = -B^T X$ and $L = (1-\gamma^2)I + XZ$.

An important feature of this algorithm [McFarlane and Glover 1992] is that the loop transfer functions before and after robustification are not significantly different.

4 TWO-DEGREE OF FREEDOM CONTROLLER

Now, we consider the overall control system represented by the configuration of Fig.3, with a two-parameter compensator (*R*, *S*, *T*). Our design objective is to specify the two-parameter compensator to achieve the following two aims:

1. The compensator can robustly stabilize nominal model $G_0(s)$ against the uncertainty ΔG by specifying $R(s)$ and $S(s)$.
2. The transfer function from *r* to *y* is as close to the desired model $M(s)$ as possible via an adequately chosen $T(s)$.

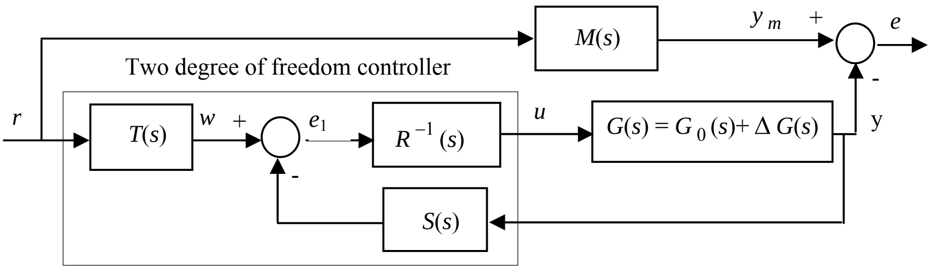

Fig. 3 The overall control system

Here, the nominal model $G_0(s)$ can be chosen as the transfer function of the linearized model (5).

The algorithm for designing the controller parameters (*R*, *S*, *T*) can be found in [Popescu and Bobasu 2001].

Remark: In this method it is necessary to evaluate the norm of the uncertainty.

5 A WORKING EXAMPLE

The Glover McFarlane control design it is applied for a handling crane model (Fig. 4).

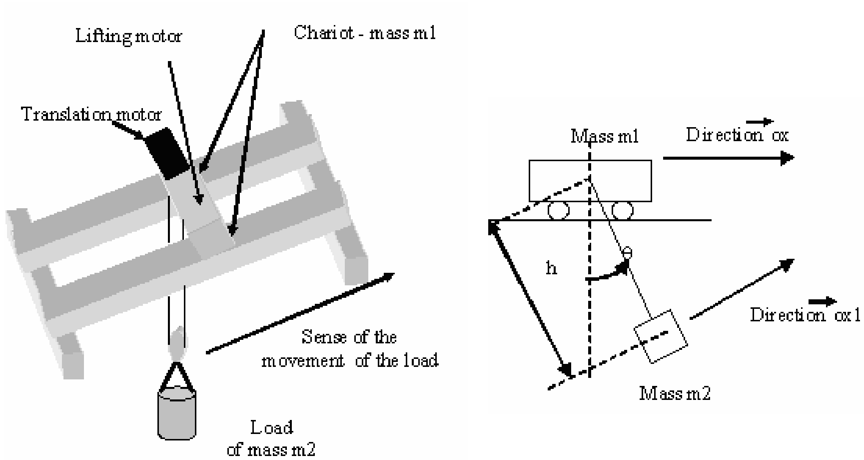

Fig. 4 The schema of the handling crane

The dynamical model consists of two nonlinear differential equations, both of order two [Bobasu *et al.* 2005]:

$$\ddot{\theta}=\frac{h\left[-bm_2\dot{x}\cos\theta+c\dot{\theta}\left(m_1+m_2-m_2\cos^2\theta\right)\right]}{h^2m_2^2\cos^2\theta-\left(m_1+m_2\right)\left(J+m_2h^2\right)}+\frac{h\left[hm_2^2\dot{\theta}^2\sin\theta\cos\theta+hm_2\left(F\cos\theta+g\sin\theta\left(m_1+m_2\right)\right)\right]}{h^2m_2^2\cos^2\theta-\left(m_1+m_2\right)\left(J+m_2h^2\right)}$$
$$\ddot{x}=\frac{-h^2m_2\cos\theta\left(c\dot{\theta}+gm_2\sin\theta\right)}{h^2m_2^2\cos^2\theta-\left(m_1+m_2\right)\left(J+m_2h^2\right)}-\frac{\left(J+m_2h^2\right)\left(F-b\dot{x}-c\dot{\theta}\cos\theta+hm_2\dot{\theta}^2\sin\theta\right)}{h^2m_2^2\cos^2\theta-\left(m_1+m_2\right)\left(J+m_2h^2\right)} \tag{13}$$

The control purpose is the regulation of the output:

$$y=x+h\sin\theta \tag{14}$$

In equations (13), (14) we have: F – the force developed by the translation motor, m_1 – the mass of chariot, m_2 – the mass of the load; b – the viscosity friction coefficient for the chariot, c – the friction coefficient opposing to the oscillation of the load, h – the height, g – the acceleration due to the gravity, J – the inertia moment, θ - the angular position, x – the position of the chariot, y – the position of the load.

The chariot is displaced using an induction motor and a reduction gear. The force F will be considered the input variable for the nonlinear model (13). Choosing,

$$x^T=\left[\theta(t),\dot{\theta}(t),x(t),\dot{x}(t)\right]\ ,\ u=F \tag{15}$$

the mathematical model (13), (14) is described in the state space by the following equations

$$\dot{x}(t)=f(x)+g(x)u \tag{16}$$

$$y=h(x)=x+h\sin\theta \tag{17}$$

where the expressions for $f(x)$ and $g(x)$ are given in [Bobasu *et al.* 2005].

The system has relative degree $r=2$. In this situation, the state feedback:

$$u=\frac{1}{L_gL_f^1h(x)}(-L_f^2h(x)+v) \tag{18}$$

transforms the system (16), (17) into a system whose input-output behavior is identical to that of a double integrator.

On the linear system thus obtained one impose a feedback control of the form:

$$v=c_0(y_{ref}-y)-c_1\dot{y} \tag{19}$$

then, the obtained system has a linear input-output behavior, described by the following transfer function

$$H(s)=\frac{c_0}{s^2+c_1s+c_0} \tag{20}$$

In order to test the performances of the obtained nonlinear controller, the following nominal handling crane parameters are used:

$$m_1=200\,kg,\ m_2=500\ kg,\ g=9.81\,m/s^2$$
$$b=1000\,Ns/m,\ c=10\,Ns/rad,\ h=6\,m$$

The design parameters are computed using a pole-placement design technique:

$$c_0=0.0625, c_1=0.375$$

Fig. 5 presents the load position evolution for the nonlinear controlled system.

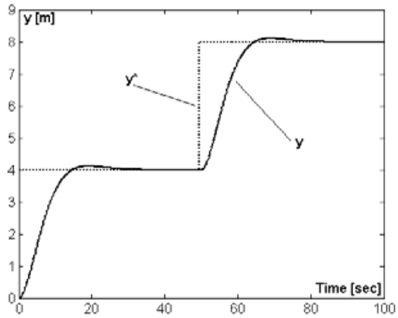

Fig. 5 Time evolution of the load position

Then, applying the Glover-McFarlane algorithm described in Section 3, the controller K_r was computed and the load position evolution is presented in Fig. 6:

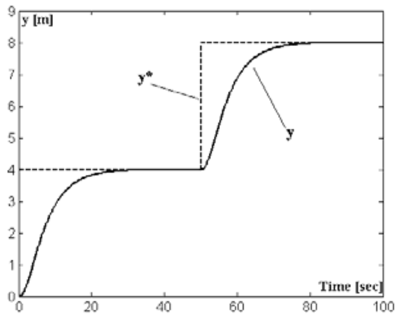

Fig. 6 Time evolution of the load position

6 CONCLUSIONS

In this paper, two robustification techniques for the exact linearization method were discussed. First, Glover-McFarlane $\mathbf{H}_\infty$ design was presented with the goal of increasing robustness of existing controllers without significantly compromising performance. The second approach was the two-degree of freedom controller design that allows separating the designing task of meeting performance specifications and robustness into two modular steps. Finally, an example was presented.

REFERENCES

[1] BOBASU, E. Nonlinear Algorithms for the Control of a Handling Crane. In *Proceedings of The International Carpathian Control Conference*. Miskolc, Hungary, 2005, pp.107-112. ISBN 963-661-644-2.

[2] ISIDORI, A. *Nonlinear Control System.* 3rd ed. New York: Springer-Verlag, 1995. 549 pp. ISBN 3-540-19916-0.

[3] McFARLANE, D., GLOVER, K. A Loop Shaping Design Procedure Using $\mathbf{H}_\infty$ Synthesis, *IEEE Transactions on Automatic Control*. 1992, **37**, no. 6, pp. 759-769. ISSN 0018-9286.

[4] POPESCU, D., BOBASU, D. On the Controller Design with Robust Stability Degree Assignment for Flexible Beams, In *Proceedings of The International Carpathian Control Conference*. Krynica, Poland, 2001, pp. 503-508. ISBN 83-91340-07-4.

Reviewer: prof. Antonín Víteček, CSc., Dr.h.c., VŠB-Technical University of Ostrava