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## POSITION CONTROL WITH ROBUST ALGORITHMS

### ŘÍZENÍ POLOHY POMOCÍ ROBUSTNÍCH ALGORITMŮ

#### Abstract

This contribution deals with a synthesis of non-linear control systems with a robust control algorithms, which are designed based on the aggregation method of the state variables. The method of aggregation of state variables enables designing a robust control algorithm with a high gain and robust algorithms work in a slide mode (three modifications). They are robust to changes of properties of the control sub-system. The accuracy of designed algorithms was verified both with a numerical simulation in MATLAB-Simulink program and on a real model.

#### Abstrakt

Príspevek se zabývá syntézou nelineárních systémů řízení pomocí robustních algoritmů řízení, které jsou navrženy na základě metody agregace stavových proměnných. Metoda agregace stavových proměnných umožňuje navrhnout robustní algoritmus řízení s vysokým zesílením a robustní algoritmy pracující v klouzavém režimu. Tyto algoritmy jsou robustní ke změnám vlastností řízeného pod systému. Správnost navržených algoritmů byla ověřena jak číslicovou simulací v programu MATLAB-Simulink, tak i na reálném modelu.

## 1 INTRODUCTION

Helicopter laboratory model is supplied by the HUMUSOFT Company. It is a non-linear system with three input ( $u_1, u_2, u_3$ ) and two output ( $y_\psi, y_\varphi$ ) variables. The scheme of a laboratory model is in Figure 1.

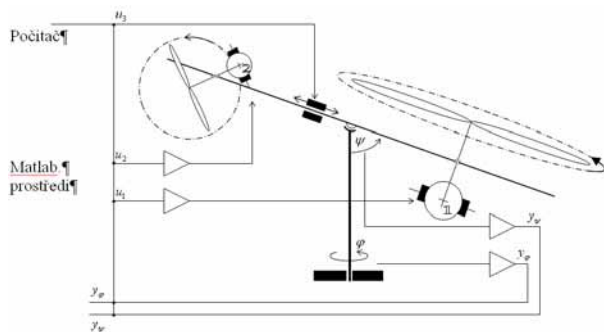


Fig. 1 Helicopter model

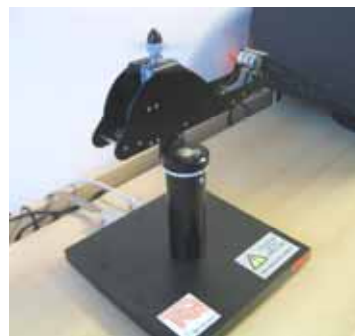


Fig. 2 Helicopter model

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This model is created with a body carrying two DC motors (1, 2), they drive propellers, and a solid stand. The body has two degrees of freedom. Rotation axes are mutually perpendicular, as well as the motors axes. Two angles (a tilt angle of the vertical axis and an angular displacement from the horizontal axis) are influenced at the same time by the propeller rotation, which is being measured with IRC sensors. A helicopter is equipped with the third motor, which changes the centre of gravity by drifting a small weight along the horizontal axis, it acts as a disturbance. Figure 2 shows a model's detail during regulation.

A laboratory model is connected to PC with a data acquisition card MF614. A system is control from PC with MATLAB program. It is possible to access all the measured signals with this software, therefore it is possible to do further post-processing.

## 2 DESIGN OF CONTROL ALGORITHMS

Since a mathematical model is non-linear, the positioning control is designed by using an aggregation method of state variables. With the help of this method it is possible to design several algorithms, which are robust for inaccuracy. The details of chosen synthesis method can be found in [Koudela, T., 2005], [Wagnerová, R., 1999], [Zítek&Víteček, 1995].

The mathematical model of chosen system is described with two non-linear; second order differential equation that is why the aggregation matrix  $D$  and some constant matrix  $T$  have presentation

$$D = \begin{bmatrix} \frac{1}{T_1} & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{T_2} & 1 \end{bmatrix}, T = \begin{bmatrix} T_1 & 0 \\ 0 & T_2 \end{bmatrix},$$

where:

$T_1, T_2$  – selected time variables describing required response of close-loop system (marginal aperiodic course).

When this method is applied, first a control algorithm design with a high gain is used [Zítek, Víteček, 1995]

$$\begin{aligned} u_1^x &= \Theta_1 \left[ \frac{2}{T_1} e_1 + \dot{e}_1 + \frac{1}{T_1^2} \int_0^t e_1 d\tau \right], \\ u_2^x &= \Theta_2 \left[ \frac{2}{T_2} e_3 + \dot{e}_3 + \frac{1}{T_2^2} \int_0^t e_3 d\tau \right], \end{aligned} \quad (2)$$

where:

$u_1^x$  – motor 1 control voltage,

$u_2^x$  – motor 2 control voltage,

$\Theta_1, \Theta_2$  – selected gain,

$e_1$  – difference between a required and a real tilt-angle,

$e_2 = \dot{e}_1$

$e_3$  – difference between a required and a real angular displacement,

$e_4 = \dot{e}_3$ .

A disadvantage of this control is quite high gain, which is caused by the algorithm's effort for compensating not known properties of a mathematical model. This disadvantage can have even negative impact on stability of the closed loop control system.

Another possibility is using sliding mode control, which can be described by following equations [VÍTEČEK&VÍTEČKOVÁ, 2004]

$$\left. \begin{aligned} u_1^{sl} &= U_1^m \operatorname{sgn} \left[ \frac{2}{T_1} e_1 + \dot{e}_1 + \frac{1}{T_1^2} \int_0^t e_1 d\tau \right], \\ u_2^{sl} &= U_2^m \operatorname{sgn} \left[ \frac{2}{T_2} e_3 + \dot{e}_3 + \frac{1}{T_2^2} \int_0^t e_3 d\tau \right] \end{aligned} \right\} \quad (3)$$

where:

$U_i^m$  - marginal value of control variables,

$\operatorname{sgn}$  - sign function.

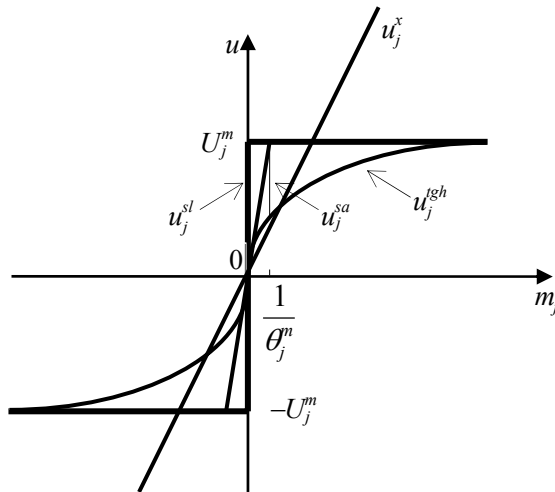
Control algorithm (3) is robust, but discontinuous. Its disadvantage is high activity of control variable; it means quick switching between marginal values. It can be removed; instead of sign function it is suitable to use a saturation function as a continuous substitution

$$\left. \begin{aligned} u_1^{sa} &= U_1^m \operatorname{sat} \left[ \Theta_1^m \left( \frac{2}{T_1} e_1 + \dot{e}_1 + \frac{1}{T_1^2} \int_0^t e_1 d\tau \right) \right], \\ u_2^{sa} &= U_2^m \operatorname{sat} \left[ \Theta_2^m \left( \frac{2}{T_2} e_3 + \dot{e}_3 + \frac{1}{T_2^2} \int_0^t e_3 d\tau \right) \right] \end{aligned} \right\} \quad (4)$$

or a hyperbolic tangent function as a smooth continuous substitution

$$\left. \begin{aligned} u_1^{tgh} &= U_1^m \operatorname{tgh} \left[ \Theta_1^m \left( \frac{2}{T_1} e_1 + \dot{e}_1 + \frac{1}{T_1^2} \int_0^t e_1 d\tau \right) \right], \\ u_2^{tgh} &= U_2^m \operatorname{tgh} \left[ \Theta_2^m \left( \frac{2}{T_2} e_3 + \dot{e}_3 + \frac{1}{T_2^2} \int_0^t e_3 d\tau \right) \right] \end{aligned} \right\} \quad (5)$$

A relation between each control algorithm can be seen in Figure 3.



**Fig. 3** The courses of control with a high gain  $u_j^x$ , sliding mode control  $u_j^{sl}$  and its continuous approximation  $u_j^{sa}$  and  $u_j^{tgh}$

### 3 CONTROL MODULE

During control environment programming for this laboratory model with the help of Matlab/GUI the Switched Board Programming method was used. It is based on using one function feature, the possibility of calling itself with different parameters. An advantage of such method is reaching the optimal source code and improvement of orientation in this code environment. A part of this source code for helicopter control can be seen in Figure 4.

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Fig. 4 Part of a source code

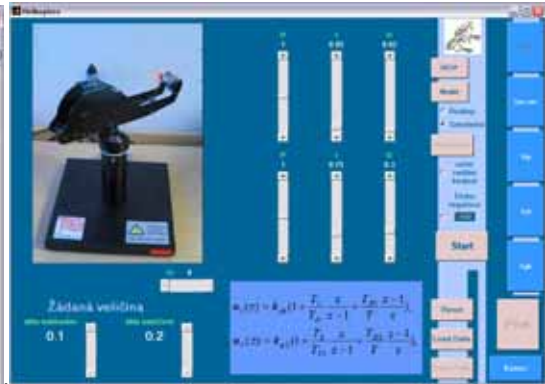


Fig. 5 Program environment

The fundamental is one selected working environment, in which certain rules are kept and conditions for control elements, whether they should be visible or not. An advantage of such setting is the simplification and clarity in a program for the user.

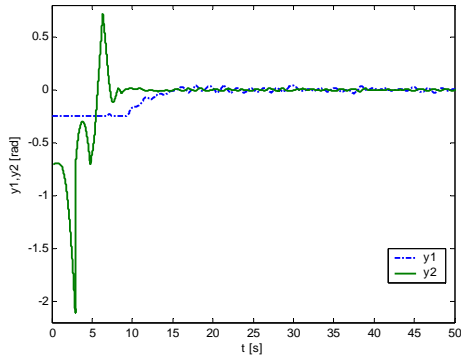
In this given control environment we can select one of the five control algorithms; PID controller, robust control with high gain, sliding mode controls with function signum, saturation or hyperbolic tangent. For all of them it is possible to select the numerical simulation or work with the real model (see Figure 5).

Values of following setting parameters can be changed for each control algorithm; time of control, required state, gain, etc. Furthermore, it displays a figure of a model and the equation of a chosen control algorithm.

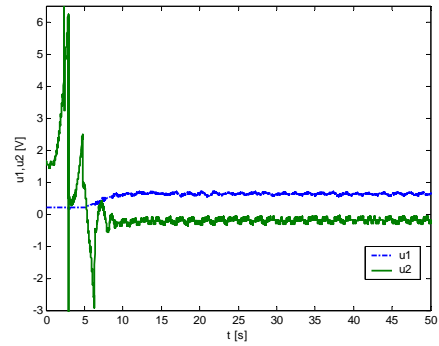
### 4 VERIFICATION OF DESIGNED ALGORITHMS

On figures 6, 7, 8 and 9 we can see courses of angular displacements, the tilt-angles and actuating variables. For the control algorithm with a high gain (see Figure 6 and 7) these adjustable parameters were set:  $T_1 = 1$  s,  $T_2 = 1.1$  s,  $\theta_1 = 0.4$ ,  $\theta_2 = 1.1$ . For slide mode control with hyperbolic tangent function (see Figures 7 and 8) these parameters were used:  $T_1 = 1$  s,  $T_2 = 1$  s,  $U_1^m = 0.5$ ,  $U_2^m = 2$ ,  $\theta_1^m = 1$ ,  $\theta_2^m = 1$ .

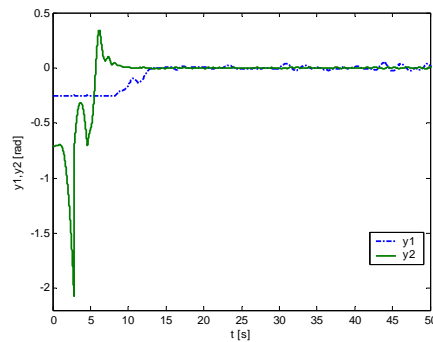
Outputs of the helicopter simulation model are marked in graphs: tilt-angle  $\psi = y_1$  and angular displacement  $\varphi = y_2$ .



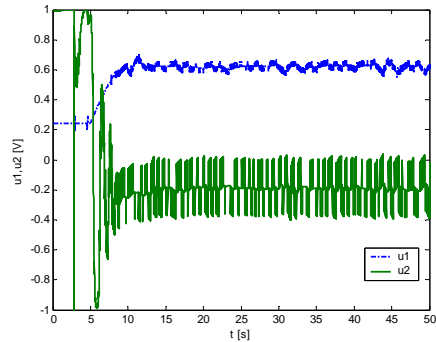
**Fig. 6** Angular displacement and declination for an algorithm with a high gain



**Fig. 7** Actuating variables for an algorithm with a high gain



**Fig. 8** Angular displacement and declination for an algorithm with a high gain



**Fig. 9** Actuating variables for an algorithm with a high gain

### 3 CONCLUSIONS

A laboratory model is a strongly non-linear system, therefore the most important while verifying designed algorithms was to select suitably all of the parameters ( $T_1$ ,  $T_2$ ,  $\Theta_1$ ,  $\Theta_2$ ,  $U_1^m$ ,  $U_2^m$ ). After simulation verification of all designed algorithms, results were tested on a real model. During this testing a design of a robust control system with a slide mode was not used (3), because this algorithm is not suitable for this particular model. During verification on a real model the selected constants had to be adjusted from a numerical simulation. All control algorithms (with high gain, sliding mode with saturation and hyperbolic tangent function) achieved required position of helicopter.

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