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INCREMENTAL ROTARY ENCODERS ACCURACY

PŘESNOST MĚŘENÍ ÚHLU OTOČENÍ

Abstract

The paper deals with the accuracy of an incremental rotary encoder as a source of pulses, which determines the encoder rotation angle. The frequency of pulses, which are generated by the encoder attached to a shaft, is proportional to the shaft rotational speed. The non-uniformity in the pulse distribution with respect to the rotation angle results in an error in determining the shaft rotation angle. The main topic of the paper is focused at the comparison of the encoder pair intended to employ for detecting the difference between the rotation angles of two meshing gears. The method for evaluation of the rotation angle is based on the phase demodulation.

Abstrakt

Příspěvek pojednává o přesnosti měření úhlu otočení na základě rozložení impulsů z inkrementálního snímače otočení (encoder). Četnost pulsů, které jsou vygenerovány dekodéry napojenými na hřídele, je úměrná úhlové rychlosti hřídele. Nerovnoměrnost úhlové rychlosti má za následek nerovnoměrnost ve výstupních impulsech z dekodéru. Hlavním tématem příspěvku je vyhodnocení rozdílu úhlu otočení dvou hřídelů, spojených ozubeným převodem. Metoda pro vyhodnocení úhlu otočení je založená na fázové demodulaci.

1 INTRODUCTION

The main sources of rotating machine vibrations are unbalance of rotors, misalignment of shafts and non-uniform driving torque. All these excitations result in dynamic force affecting bearing supports. Bearing vibration excites vibration of the machine housing, which increase the noise level. This paper is focused at the shaft angular vibration as a consequence of the non-uniformity of driving torque.

Rotational speed is measured in terms of the number of revolutions per minute (RPM) while the angular vibration is measured in terms of the angle, angular velocity or angular acceleration. The uniform rotational speed at the constant RPM corresponds to growing up the shaft angle proportionally to the elapsed time. The angle time history, having the form of the sum of a term that is depending linearly on time and a term that is randomly or regularly varying in time around zero, results from angular vibration during rotation. The angular velocity is obtained as the first derivative of the angle while the angular acceleration is evaluated as the second derivative of the angle.

There are many possible approaches to measuring angular vibration during rotation

- Tangentially mounted accelerometers
- Laser torsional vibration meter based on the Doppler effect
- Incremental rotary encoders (several hundreds of pulses per revolution).

In practice, measurements based on the use of encoders dominate. Instantaneous angular velocity is proportional to the reciprocal value of the time interval, which is elapsed between

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consecutive impulses. The measurement methods for the length of the time interval measurements are as follows:

- Sample number & Interpolation
- High frequency oscillator (100 MHz) & Impulse counter
- Phase demodulation.

The simplest method for evaluation of the instantaneous rotational speed is the reciprocal value of the time interval between two consecutive pulses. If the impulse signal is sampled then the time interval between the adjacent impulses is determined by interpolation of some values 50 times more accurately than indicated by the actual sampling interval. The accuracy is satisfying for the RPM measurement based on only one pulse per shaft rotation. This method is not suitable if the large number of pulses per revolution is generated, which results in a few samples between impulses and the time interval length is impossible to estimate at satisfying accuracy. If the string of encoder impulses as an analogue signal controls a gate for the high frequency clock signal (10 GHz) that is an input of an impulse counter then this method works properly. This principle is implemented in the signal analyzers produced by Rotex. The primary output of these analyzers is angular velocity.

This paper is dealing with accuracy of the angular vibration measurements based on the impulse signal phase demodulation using the Hilbert transform.

2 REQUIREMENTS FOR ANGULAR VIBRATION MEASUREMENTS

Noise and vibration problems in gearing are mainly concerned with the smoothness of the drive. The parameter that is employed to measure smoothness is the Transmission Error [1] (TE). This parameter can be expressed as a linear displacement at a base circle radius defined by the difference of the output gear's position from where it would be if the gear teeth were perfect and infinitely stiff.

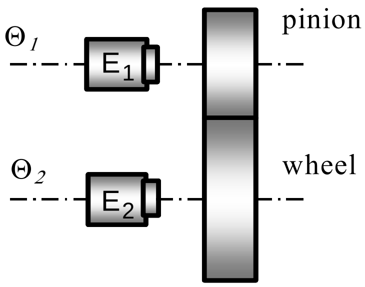

Fig. 1. Arrangement of TE measurements

The displacement varyies in micrones what results in accuracy of the angular vibrations in the value which is less then an angular second. The sketch of the gear set under test and attached incremental rotary encoders [2], designated by E1 and E2 is shown in Fig. 1. Both the encoders generate a string of pulses. As a consequence of Shannon's sampling theorem a few pulses must be recorded during each mesh cycle. It means that the number of pulses produced per encoder revolution must be a multiple of the tooth number. If five harmonics of toothmeshing frequency are required then the number of pulses per gear revolution must be at least ten times greater than the number of teeth. The encoder generating 500 pulses per revolution seems to be an optimum.

3 PRINCIPLE OF THE PHASE DEMODULATION

The angular vibration can be measured by using shaft encoders giving usually a train of pulses, rather then a sinusoid. As the impulse signal consists of several harmonics of the basic impulse frequency the first step in phase demodulation procedure is to separate the frequency band containing a carrier component and with sideband components by using a band-pass filter [3]. An example of the phase-modulated signal is shown in figure 2.

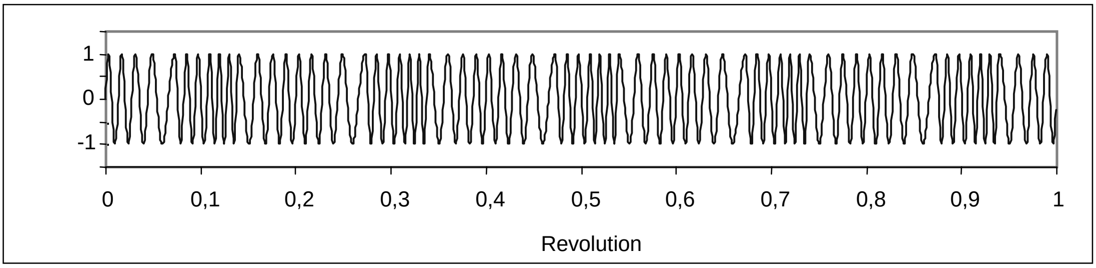

Fig. 2. Phase-modulated signal

Let the sampled signal is designated by $x_n, n = 0,1,...,N-1$. The relationship between the components $X_k, k = 0,1,...,N-1$ and $Y_k, k = 0,1,...,N-1$, which are corresponding to the Fourier transforms of the sampled signal x_k and its Hilbert transform y_k respectively, is given by the following formula

$$Y_k = j\, sign(N/2 - k) X_k, \quad k = 0,1,...,N-1 . \tag{1}$$

The inverse Fourier transform of $Y_k, k = 0,1,...,N-1$ results in $y_n, n = 0,1,...,N-1$.

The angle $\varphi_n = \mathrm{atan}(y_n / x_n)$ of the complex values $x_n + j\,y_n$ ranges from $-\pi$ to $+\pi$ and contains jumps at $-\pi$ at or $+\pi$ (see figure 3). The true phase φ_n of the analytical signal as the time function must be unwrapped. The unwrapping algorithm is based on the fact that the absolute value of the phase difference $\Delta\varphi_n = \varphi_n - \varphi_{n-1}$ between two consecutive samples of the angle is less than π

$$\Delta\varphi_n < -\pi \Rightarrow \varphi_n + 2\pi \to \varphi_n, \quad \Delta\varphi_n > +\pi \Rightarrow \varphi_n - 2\pi \to \varphi_n . \tag{2}$$

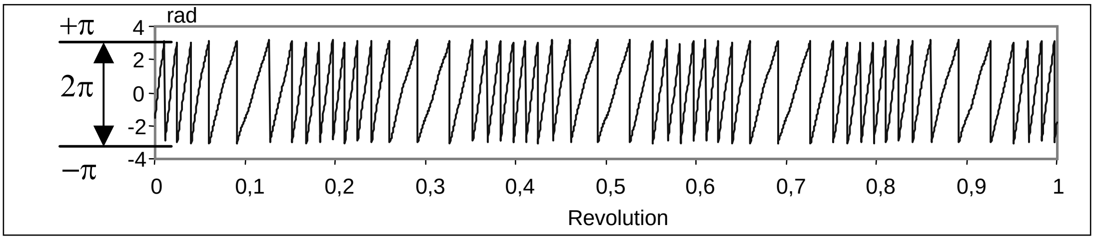

Fig. 3. Phase of analytical signal ranging from –π to +π

The result of phase unwrapping of a harmonic signal is shown on the left diagram in figure 4. The phase on this diagram is not a linear function of time, but adding a harmonic signal to the linear function influences it. The unwrapped phase change per one complete rotation is equal to the 2π multiplied by the number K of the impulses per rotation. To evaluate uniformity of rotation the phase normalization has to be performed

$$\varphi_n / K \to \varphi_n, \quad n = 0,1,...,N-1 . \tag{3}$$

The phase normalization results in the diagram phase scale that corresponds to an angle of the complete revolution. The corresponding angle of rotation is a function composed from a linear term and phase modulation signal. The linear term corresponds to the steady-state rotational speed. After removing the linear term the phase modulation signal is obtained and it is seen on the right diagram in figure 4.

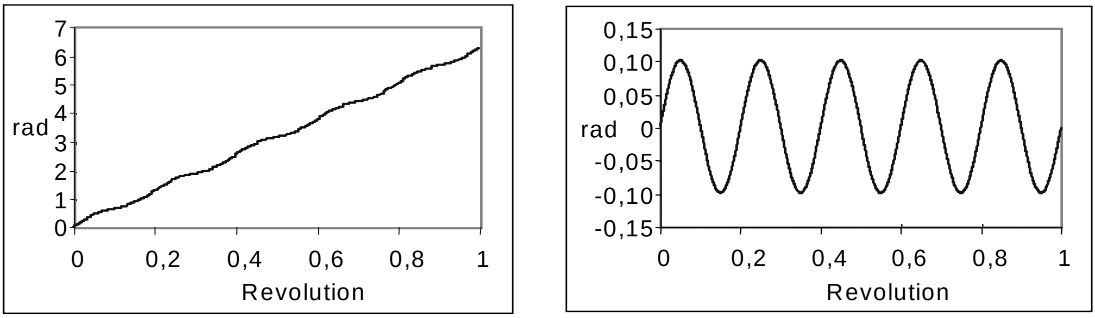

Fig. 4. Unwrapped and removed linear trend in phase of analytical signal

4 INCREMENTAL ROTARY ENCODER ACCURACY

Both the encoders under test shown in figure 5 are of Heidenhain origin, the ERN 460-500 type. To evaluate errors in pulses distribution against the angle of rotation, both the encoders were mounted on a shaft what ensured the same rotational speed of them (see figure 6). Accuracy was assessed at the rotational speed of 634 and 1040 RPM. The pulse string generated by encoders was sampled at the frequency of 65536 Hz for 1040 RPM and at the frequency of 16384 Hz for 634 RPM.

Fig. 5. Heidehain encoders of the ERN 460-500 type (500 pulses per revolution)

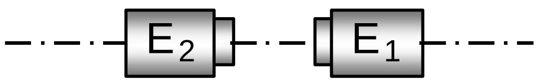

Fig. 6. Arrangement of encoders to be tested

As the running was not perfectly uniform from the point of the measurement method sensitivity, both the pulse signals were under influence of phase modulation. The encoder speed variation results in the phase modulation of the impulse signal base frequency. As noted above the phase-modulated signal contains sideband components around the carrying component. The frequency of the carrying component is equal to 500 orders as it is shown in figure 7.

Using the analytical method described above, the phase difference between modulation signals gives the error in pulse distribution. The time history of the mentioned error for 3 complete revolutions at 1040 RPM is shown in figure 8.

Synchronized averaging according to the rotational frequency is a signal processing tool, which results in the averaged time record of the phase difference, which is shown in figure 9.

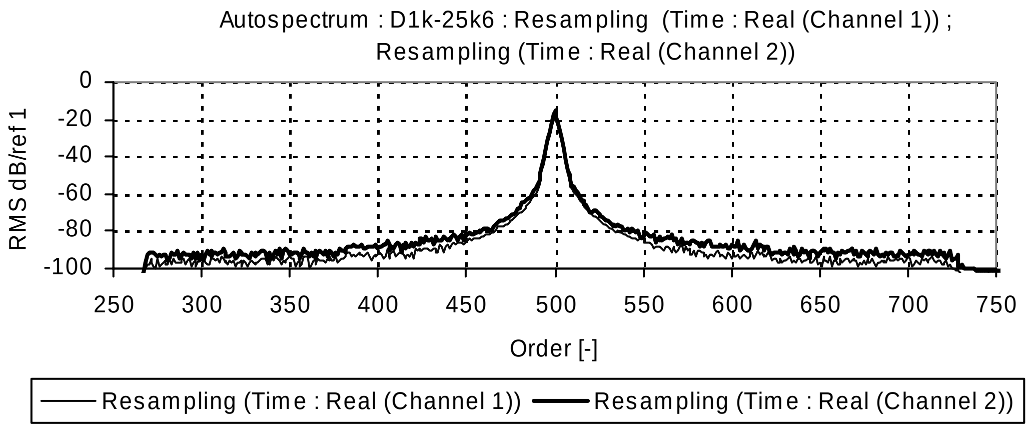

Fig. 7. Frequency spectrum of phase modulated signal generated by the E1 and E2 encoders

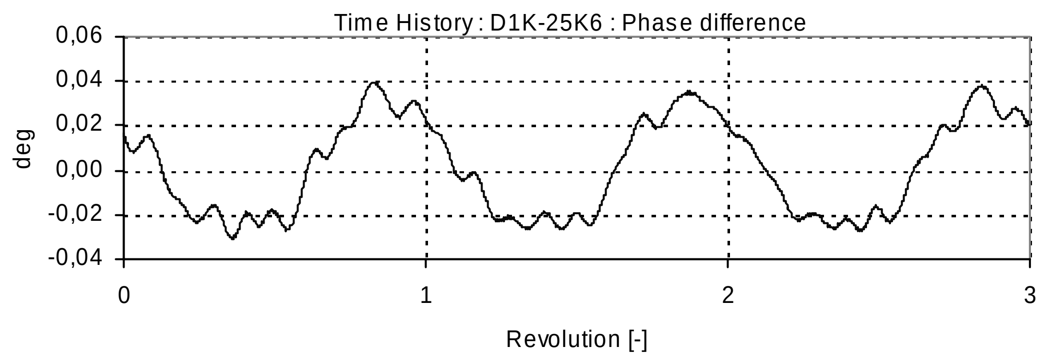

Fig. 8. Phase difference during 5 complete encoder revolutions

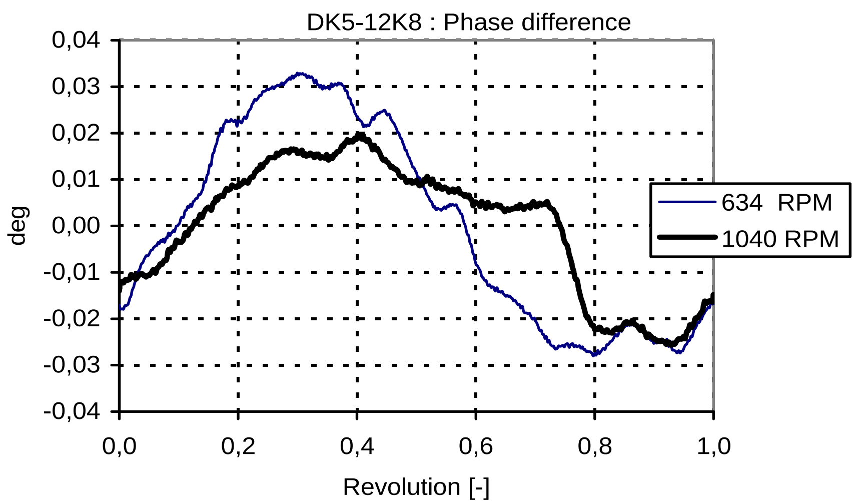

Fig. 9. Phase difference during a complete encoder revolution for 634 and 1040

The frequency spectrum of the resulting error is shown in figure 10. The frequency axis is in orders. The quantity "order" determines a part of a circle related to the error level. The error of determination the angle rotation over the arbitrary number of adjacent pulses is given by the sum of individual errors corresponding to the rotation by a pulse. The summation process results in the dependence of the error on the order with the roll-off corresponding to the error decade per the order decade. The error level at the distance corresponding to the tooth pitch of the adjacent teeth determines the final accuracy of the T.E. measurement. As it is evident the magnitude of an error at 20 through 200 orders is less than an angular second.

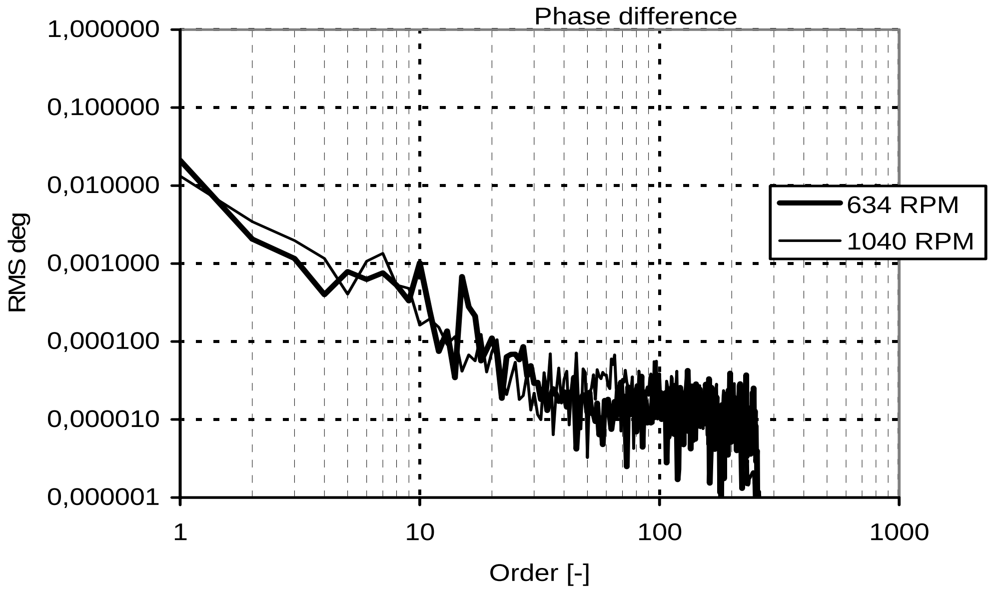

Fig. 10. Spectrum of the phase difference for 634 and 1040 RPM

5 CONCLUSIONS

Tests show that the errors in pulse distribution as a function of the angular displacement are not exceeding an angular second. Measurement accuracy for the transmission error is satisfactory.

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