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ON A METHOD OF MODELLING AND SIMULATION OF VIBRATION SURVEILLANCE DURING BALL END MILLING OF FLEXIBLE DETAILS

METODA MODELOVÁNÍ A SIMULACE SLEDOVÁNÍ VIBRACÍ BĚHEM OBRÁBĚNÍ PRUŽNÝCH SOUČÁSTÍ FRÉZOU S KULOVÝM ZAKONČENÍM

Abstract

The paper concerns vibration surveillance during ball end milling of flexible details and considering assurance of parameters of a modal subsystem. Dynamic analysis of a slender ball end milling process has been performed. Further is explained dynamic analysis of non-stationary vibrating system, from which are separated subsystems: modal, structural and connective. Minimising vibration level by matching the spindle speed to optimal phase shift has been employed. Modal model of the work piece whose parameters are determined either solving an eigenvalue problem of structural model, or using the methods of experimental modal analysis, allowed us to determine optimal value of the spindle speed. Results of suitable computer simulation, as well as – of experimental investigation on the Alcera Gambin 120 CR high speed milling machine mean to be in support. Hence, amplitude values for chosen optimal spindle speed became relatively smaller than for the other ones.

Abstrakt

Příspěvek se zabývá sledováním vibrací během obrábění pružných součástí frézou s kulovým zakončením a bere v úvahu parametry modálního subsystému. Byla provedena dynamická analýza obrábění štíhlou frézou s kulovým zakončením. Dále je vysvětlena dynamická analýza nestacionárního systému vibrací, ze kterého jsou vyčleněny subsystémy: modální, strukturální a spojovací. Minimalizace úrovně vibrací byla dosažena pomocí přizpůsobení rychlosti otáčení vřetena optimálnímu fázovému posunutí. Modální model obráběného kusu, jehož parametry jsou určovány buď vyřešením problému vlastních hodnot (módů) strukturálního modelu nebo použitím experimentálních metod modální analýzy, umožňuje určit optimální hodnotu rychlosti otáčení vřetena. Výsledky jsou podpořeny vhodnou počítačovou simulací a také experimentálně na vysokorychlostním frézovacím stroji Alcera Gambin 120 CR. Z tohoto důvodu optimální hodnoty rychlosti otáčení vřetena jsou relativně menší než v jiných případech.

1 INTRODUCTION

Ball end milling of flexible details is observed very frequently in case of modern machining centres. It is obvious that tool-workpiece relative vibration plays a principal role during cutting process. Due to certain conditions, it may lead to a loss of stability and cause a generation of self-excited *chatter* vibration [1]. One method of the *chatter* reduction depends upon the spindle speed optimal control [2, 3] and the spindle speed optimal-linear control [4]. Results of further research disclosed that milling flexible billets at changing spindle speed appears unsuccessful however, with

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respect to vibration surveillance. Thus, the paper proposes the other method of minimising vibration level, which is based on matching the spindle speed to the optimal phase shift [5].

2 CUTTING PROCESS DYNAMICS

Dynamic analysis of a slender ball end milling process has been performed, based upon following assumptions [1].

- The spindle together with the tool fixed in the holder, and the table with the workpiece, are separated from the machine.
- Here is considered flexibility of the tool and flexibility of the workpiece.
- Position between symmetry axis of the tool and the v_f feed speed, refers to the pulling milling. The latter prevents from cutting at contribution of the ball end mill top.
- Coupling elements (CEs) are applied for modelling the cutting process.
- An effect of first pass of the edge along cutting layer causes proportional feedback, but multiple passes cause delayed feedback additionally. An effect of first pass of the edge along cutting layer causes proportional feedback, but multiple passes cause delayed feedback additionally.

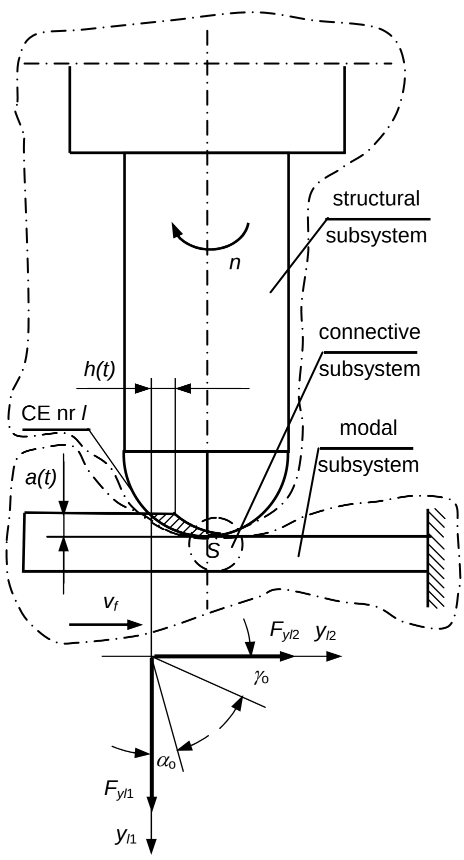

Fig. 1 A scheme of a slender ball end milling of one-side-supported flexible plate

As result of a milling process modelling, we get a hybrid system, in which are separated (Fig. 1):

– modal subsystem. It is a stationary model of one-side-supported flexible plate, which displaces itself with a desired feed speed v_f;

– structural subsystem, i.e. non-stationary discrete model of ball end mill (speed of revolution n) and cutting process;

– connective subsystem as conventional contact point S between tool and workpiece.

For instantaneous contact between chosen tool edge and the workpiece (idealised by CE no. l), proportional model of the cutting dynamics is included [1, 2]. As result of transformation of generalised displacements, we shall get non-stationary discrete system's dynamics, whose matrix equation has a form [1]:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{L}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{f} \ , \tag{1}$$

where:

$\mathbf{q}$	–	vector of generalised displacements of the system,
$\mathbf{M}$, $\mathbf{L}$, $\mathbf{K}$, $\mathbf{f}$	–	matrices of inertia, damping, stiffness, and of generalised forces of the system.

3 DYNAMICS OF MILLING FLEXIBLE DETAILS AS OF A HYBRID SYSTEM

Further consideration relates to the system decomposition into following ones.

1. Modal subsystem, which is described in generalised co-ordinates $\mathbf{q}_m$. Matrices of inertia, damping and stiffness are $\mathbf{M}_{mm}$, $\mathbf{L}_{mm}$, $\mathbf{K}_{mm}$, but vector of generalised forces is $\mathbf{f}_m$. Properties of that subsystem are defined by:

$$\mathbf{\Omega}_m = diag\left[\omega_{01} \quad \omega_{02} \quad \dots \quad \omega_{0mod}\right]$$

– undamped angular natural frequencies ω_{0k}, k=1, ..., mod,

$$\mathbf{\Psi}_m = \left[\mathbf{\Psi}_1 \quad \mathbf{\Psi}_2 \quad \dots \quad \mathbf{\Psi}_{mod}\right]$$

– normal modes $\mathbf{\Psi}_k$ corresponding to undamped angular frequencies of the system ω_{0k}, i=1, ..., mod,

$$\mathbf{Z}_m = diag\left[\zeta_1 \quad \zeta_2 \quad \dots \quad \zeta_{mod}\right]$$

– dimensionless damping coefficients corresponding to modes k=1, ..., mod, mod–number of modes being considered.

Thus, following conditions are fulfilled:

$$\mathbf{q}_m = \mathbf{\Psi}_m \mathbf{a}_m \, , \ \mathbf{\Psi}_m^T \mathbf{M}_{mm} \mathbf{\Psi}_m = \mathbf{I}_m \, , \ \mathbf{\Psi}_m^T \mathbf{L}_{mm} \mathbf{\Psi}_m = 2\mathbf{Z}_m \mathbf{\Omega}_m \, , \ \mathbf{\Psi}_m^T \mathbf{K}_{mm} \mathbf{\Psi}_m = \mathbf{\Omega}_m^2 \, . \tag{2}$$

2. Structural subsystem, described in generalised co-ordinates $\mathbf{q}_s$. Vector of referred generalised forces is $\mathbf{f}_s$.

3. Connective subsystem, whose generalised co-ordinates are $\mathbf{q}_c$.

It is assumed that rheonomic–holonomic constraints are between co-ordinates of modal subsystem q_m and connective subsystem q_c, i.e.:

$$\mathbf{W}_c \mathbf{q}_c = \mathbf{W}_m \mathbf{q}_m \, . \tag{3}$$

If we consider constraint reactions' equation, constraints' equations and their time derivations, we shall get description of dynamics of non-stationary system in hybrid co-ordinates $\boldsymbol{\xi}$, that is to say [6]:

$$\mathbf{M}_{\xi}\ddot{\boldsymbol{\xi}} + \mathbf{L}_{\xi}\dot{\boldsymbol{\xi}} + \mathbf{K}_{\xi}\boldsymbol{\xi} = \mathbf{f}_{\xi}\ , \tag{4}$$

where:

$$\mathbf{M}_{\xi} = \begin{bmatrix} \mathbf{I}_m + \boldsymbol{\Psi}_m^T\mathbf{W}^T\mathbf{M}_{cc}\mathbf{W}\boldsymbol{\Psi}_m & \boldsymbol{\Psi}_m^T\mathbf{W}^T\mathbf{M}_{cs} \\ \mathbf{M}_{sc}\mathbf{W}\boldsymbol{\Psi}_m & \mathbf{M}_{ss} \end{bmatrix},$$

$$\mathbf{L}_{\xi} = \begin{bmatrix} 2\mathbf{Z}_m\boldsymbol{\Omega}_m + 2\boldsymbol{\Psi}_m^T\mathbf{W}^T\mathbf{M}_{cc}\dot{\mathbf{W}}\boldsymbol{\Psi}_m + \boldsymbol{\Psi}_m^T\mathbf{W}^T\mathbf{L}_{cc}\mathbf{W}\boldsymbol{\Psi}_m & \boldsymbol{\Psi}_m^T\mathbf{W}^T\mathbf{L}_{cs} \\ 2\mathbf{M}_{sc}\dot{\mathbf{W}}\boldsymbol{\Psi}_m + \mathbf{L}_{sc}\mathbf{W}\boldsymbol{\Psi}_m & \mathbf{L}_{ss} \end{bmatrix},$$

$$\mathbf{K} = \begin{bmatrix} \boldsymbol{\Omega}_m^2 + \boldsymbol{\Psi}_m^T\mathbf{W}^T\mathbf{M}_{cc}\ddot{\mathbf{W}}\boldsymbol{\Psi}_m + \boldsymbol{\Psi}_m^T\mathbf{W}^T\mathbf{L}_{cc}\dot{\mathbf{W}}\boldsymbol{\Psi}_m + \\ + \boldsymbol{\Psi}_m^T\mathbf{W}^T\mathbf{K}_{cc}\mathbf{W}\boldsymbol{\Psi}_m & \boldsymbol{\Psi}_m^T\mathbf{W}^T\mathbf{K}_{cs} \\ \mathbf{M}_{sc}\ddot{\mathbf{W}}\boldsymbol{\Psi}_m + \mathbf{L}_{sc}\dot{\mathbf{W}}\boldsymbol{\Psi}_m + \mathbf{K}_{sc}\mathbf{W}\boldsymbol{\Psi}_m & \mathbf{K}_{ss} \end{bmatrix},$$

are respectively matrices of inertia, damping, stiffness in hybrid co-ordinates of the whole system,

$\boldsymbol{\xi} = \begin{bmatrix} \mathbf{a}_m \\ \mathbf{q}_s \end{bmatrix}$ – hybrid co-ordinates of the whole system,

$\mathbf{f}_{\xi} = \begin{bmatrix} \boldsymbol{\Psi}_m^T\left(\mathbf{f}_m + \mathbf{W}^T\mathbf{f}_c\right) \\ \mathbf{f}_s \end{bmatrix}$ – "hybrid" forces of the system,

Relationships (2) – (4) showed, that for a performance of dynamic analysis in hybrid co-ordinates, here is required matrix $\boldsymbol{\Omega}_m$ of angular natural frequencies and matrix $\boldsymbol{\Psi}_m$ of corresponding normal modes of modal subsystem. The latter are time-invariant, due to the modal subsystem being stationary. In order to determine them we can apply:

- computer software for calculation of eigenfrequencies and corresponding normal modes of systems idealised discretely;
- methods of experimental modal analysis.

Both the approaches above are recommended, with respect to necessity of mutual verification of the results obtained.

Example. For one-side-supported plate, dimensions 135×50×5, made of bronze BA1032, natural frequencies and corresponding normal modes have been calculated, using the MSC NASTRAN package. Normal modes referred to first four natural frequencies of the BA1032 plate are illustrated (Fig. 2).

Because only first normal mode is important, suitable modal experiment has been performed. Subsequently, modal assurance criterion (*MA*C) is assessed (Fig. 3) using the *FeGraph* package. Good agreement between results of calculation and experimental investigation has been confirmed.

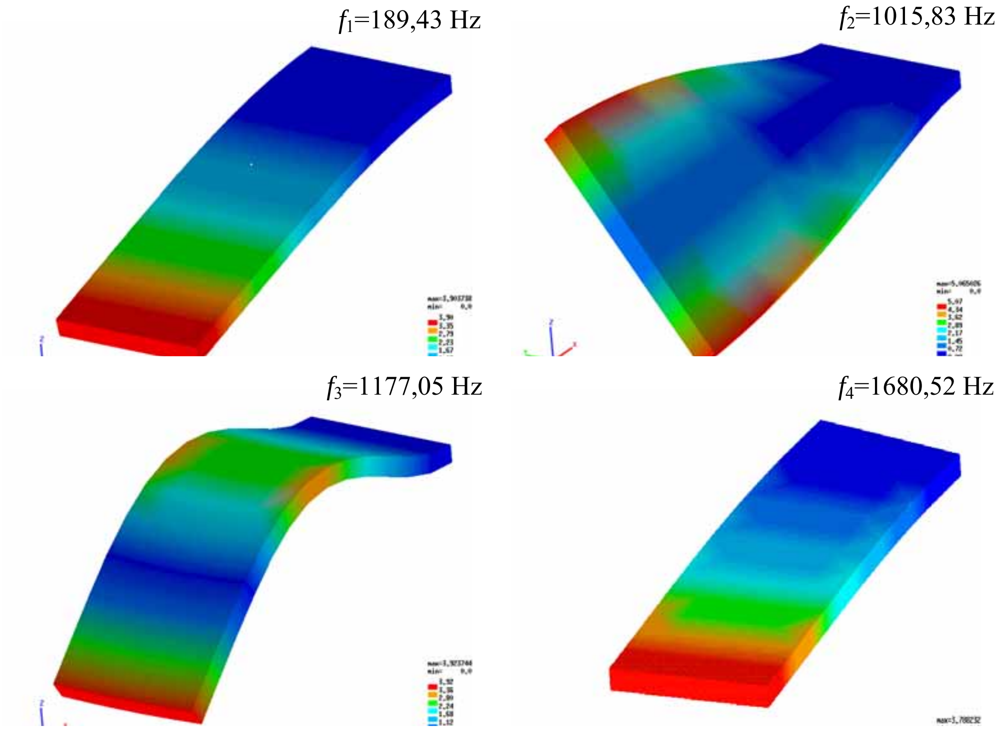

Fig. 2 Natural frequencies and normal modes of rectangle plate, material: bronze BA1032

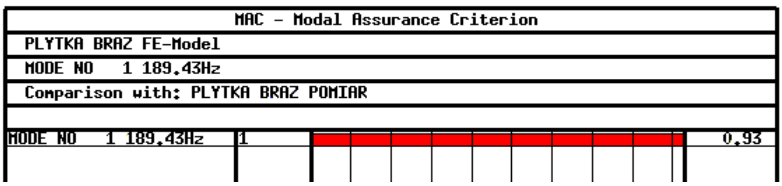

Fig. 3 Results of modal assurance criterion (*MAC*) calculation for the BA1032 plate

4 MATCHING A SPINDLE SPEED OF REVOLUTION TO OPTIMAL PHASE SHIFT

If *chatter* vibration with only one angular frequency $\omega_\alpha=2\pi f_\alpha$ has been observed during the cutting, some quantities in equation (1) may be written as harmonic functions. Then we calculate work of cutting forces of all CEs, which is done by the component with angular frequency ω_α during time of one vibration period.

It has been proved [2, 6] that the system is free off *chatter* vibration when the work done results minimal energy of vibration being stored in the system. The latter shall be accomplished when following condition is satisfied:

$$\omega_\alpha \tau_l = \frac{\pi}{2} + 2\pi m, \quad m = 0, 1, ... \tag{5}$$

If we rearrange equation (5), we derive condition of optimality in the form:

$$\frac{zn_\alpha}{60} = \frac{f_\alpha}{0.25 + m}, \tag{6}$$

where:

f_α – observed *chatter* frequency,

n_α – optimal spindle speed corresponding to vibration with frequency f_α,

z – number of cutting edges.

The approach described in this chapter generalises Liao-Young derivation [5] to a class of multi-degree-of-freedom systems and it converges with the results of vibrostability analysis with a use of the Nyquist frequency plots. However it gives ability of matching optimal spindle speed taking into consideration an influence of only one resonant peak *chatter*. Thus, application of this approach requires separation of a *chatter* resonance with frequency f_α, whose amplitude is dominant in the spectrum of displacements.

The approach provides ability of finding optimal spindle speed based on influence of exactly one resonant peak *chatter*. As result of considerations placed in this chapter, following procedure of vibration reduction is suggested.

- Time-domain modal investigation of a flexible billet fixed in the holder. As result, transient time response is obtained.
- Detection of *chatter* frequency by time-plot analysis.
- Matching required spindle speed to the *chatter* frequency being observed.

5 AN ILLUSTRATIVE EXAMPLE

Here is performed experimental investigation of vibration surveillance during ball end milling of straight grooves on the Alcera Gambin 120CR milling machine, maximum spindle speed n_{max}=35000 rev/min (Fig. 4). The billet was one-side-supported plate, dimensions 150×50×5, made of steel S235JR and sloped with 15° and 30°, with respect to the tool. Cutting parameters are as follows: ball end mill diameter D=16 mm, desired depth of cutting a_p=0,1 mm, feed per tooth f_z=0,01 mm.

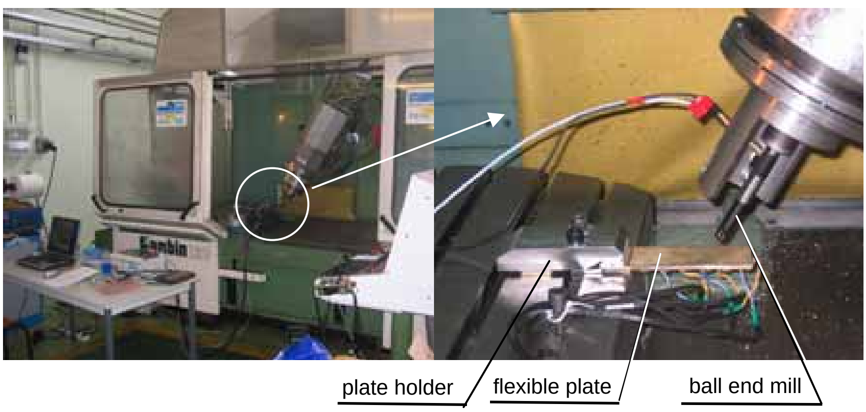

Fig. 4 View of a slender ball end milling of one-side-supported flexible plate.

Because only first normal mode is important, suitable modal experiment was performed. Then, first natural frequency resulted $f_n \cong 186$ Hz. Following that, sequence of optimal spindle speeds has been determined, considering that $f_\alpha = f_n$ and using relationship (6). Result of calculation is n_α = 22324 rev/min. Although the latter is much higher than in the previous experiment, however it lies in a range of allowable spindle speeds. Maximum vibration amplitudes q_{max} and corresponding frequencies for various values of spindle speeds considered in the experiment, are placed in table 1.

Tab. 1 Maximum vibration amplitudes q_{max} and corresponding frequencies f_{max}

No. of groove	n_α [rev/min]	Tilting angle	q_{max} [mm]	f_{max} [Hz]
1	22324	15°	0,0003	199
2	24324	15°	0,0127	189
3	20324	15°	0,0322	189
6	22324	30°	0,0004	189
7	24324	30°	0,0119	188
8	20324	30°	0,0220	188

4 CONCLUSIONS

Here is evidenced usefulness of the hybrid system control for vibration surveillance of non-stationary systems idealised discretely. A synthesis of control system of ball end milling of flexible details requires identification of parameters (i.e. natural frequencies and normal modes) of stationary modal subsystem. Because only first normal mode is important, suitable modal assurance criterion (*MAC*) has been assessed and confirmed good agreement between the results.

The method of vibration surveillance during machining flexible details by matching the spindle speed to optimal phase shift has been developed with success. Thanks to that, vibration surveillance appeared efficient. Modal model of the workpiece allowed to determine optimal value of the spindle speed. Results of experimental investigation on Alcera Gambin 120CR milling machine mean to be in support. Hence amplitude values for chosen optimal spindle speed became relatively smaller than for the other ones. Quality of machining is improved in significance as well (Fig. 5).

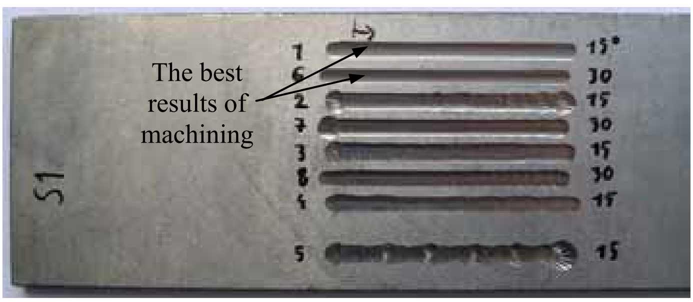

Fig. 5 Results of milling flexible billet of steel at various spindle speeds

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