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ON NONLINEAR CONTROL OF THE HYDRAULIC SERVOMECHANISMS

NELINEÁRNÍ ŘÍZENÍ HYDRAULICKÉHO SERVOMECHANISMU

Abstract

This paper deals with the design of a nonlinear control law for a hydraulic servomechanism. The servomechanism is in fact a double-acting double-ended piston actuator with a ¾ servovalve. In order to design the nonlinear controller, the mathematical model of the servomechanism is achieved. The nonlinear control law is designed using the feedback linearizing technique. Some numerical simulations are provided in order to reveal the performances and the behavior of the controlled system.

Abstrakt

Příspěvek popisuje návrh nelineárního řízení pro hydraulický servomechanismu. Servomechanismus je dvojčinný, s obousměrnou pístnicí a třípolohovým, čtyřcestným servoventilem. Pro návrh nelineárního regulátoru bylo nutno určit matematický model servomechanismu. Nelineární řízení bylo navrženo metodou exaktní linearizace. Pro určení výkonu a chování řízeného systému byly provedeny číslicové simulace.

1 INTRODUCTION

The classical methods of hydraulic control are based on a linearized description of the plant around a fixed reference position. In many practical applications, however, these linear controllers are sufficient in terms of accuracy and dynamic performance and hence are still very common in industry. But the hydraulic plants exhibit significant nonlinearities and therefore, an increase in the performance of the closed-loop can only be achieved by controllers that take into account the nonlinear nature of the system [Richard & Outbib 1995, Bobasu 2003]. In the literature, linear controllers either with an adaptation mechanism or robustly designed are often suggested as a means of coping with these nonlinearities. Apart from these design methods based on the linear model of the hydraulic system various works considering nonlinear control approaches have been proposed.

In the last years, significant advances have been made in the development of ideas such as feedback linearizing techniques. The problem of exact linearization via feedback and diffeomorphism consists in transforming a nonlinear system into a linear one using a state feedback and a coordinate transformation of the state [Fossard & Normand-Cyrot 1993, Isidori 1995].

In this paper, we will consider the basic configuration of a double-acting double-ended piston actuator with a ¾ servovalve. The servovalve is composed by a symmetrical double-nozzle and a torque-motor driven flapper for the first stage and a sliding spool for the second stage. The nonlinear mathematical model of the hydraulic servo-system is achieved, considering the case of neglecting valve's dynamics and taking into account the loss of the flow between chambers of the actuator. This model is represented by four differential equations. Next, by using the feedback linearizing technique, a nonlinear control law for the servomechanism is obtained. The control goal is the regulation of the

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position of the piston. This nonlinear control method provides an alternative solution to existing classical linear methods. For the implementation of the control law we suppose that all states are measurable. Some numerical simulation results for the controlled system are also presented.

2 THEORETICAL FUNDAMENTS

The nonlinear system that we consider is described in state space by equations of the following kind:

$$\begin{aligned} \dot{x} &= f(x) + \sum_{i=1}^{m} g_i(x) u_i \\ y_j &= h_j(x) \quad j = 1...m \end{aligned} \tag{1}$$

in which $f(x), g_1(x), g_2(x),....., g_m(x)$ are smooth vector fields.

The exact linearization via feedback and diffeomorphism consists in transforming the nonlinear system (1) into a linear one using a state feedback and a coordinate transformation of the systems state. We do not develop the details of input-output linearization techniques (for details see [Isidori 1995]) but directly show the application on the hydraulic servomechanism. This can be done introducing the Lie derivative of a function $h(x): R^n \to R$ along a vector field $f(x) = [f_1(x),...f_n(x)]$

$$L_f h(x) = \sum_{i=1}^{n} \frac{\partial h(x)}{\partial x_i} f_i(x) \tag{2}$$

Definition. [Isidori 1995]. A multivariable nonlinear system of the form (1) has a relative degree $\{r_1,..., r_m\}$ at a point x^0 if:

$$L_{g_j} L_f^k h_i(x) = 0 \tag{3}$$

for all $1 \le j \le m$, for all $1 \le i \le m$ for all $k < r_i - 1$, and for x in a neighborhood of x^0, the $m \times m$ matrix

$$A(x) = \begin{bmatrix} L_{g_1} L_f^{r_1-1} h_2(x) & . & . & L_{g_m} L_f^{r_1-1} h_1(x) \\ L_{g_1} L_f^{r_2-1} h_2(x) & . & . & L_{g_m} L_f^{r_2-1} h_2(x) \\ . & . & . & . \\ L_{g_1} L_f^{r_m-1} h_m(x) & . & . & L_{g_m} L_f^{r_m-1} h_m(x) \end{bmatrix} \tag{4}$$

is nonsingular at $x = x^0$.

Remark 1: For the single input – single output (SISO) nonlinear systems, the two conditions regarding the relative degree *r* from Definition 2.1 become:

1) $L_g L_f^k h(x) = 0$ for all $k < r - 1$, and for x in a neighborhood of x^0.

2) $L_g L_f^{r-1} h(x^0) \neq 0$.

Remark 2: Let be a SISO nonlinear system of the form (1), which has the relative degree *r* at a point x^0. The state feedback:

$$u = \frac{1}{L_g L_f^{r-1} h(x)} \left[-L_f^r h(x) + v \right]$$

transforms the nonlinear system into a system, whose input-output behaviour is the same with a linear system having the transfer function:

$$H(s) = \frac{1}{s^r}$$

Theorem. [Isidori 1995]. Let be the nonlinear system of the form (1). Suppose the matrix $g(x^0)$ has rank m. Then, the State Space Exact Linearization Problem is solvable if:

1) for each $0 \le i \le n-1$, the distribution G_i has constant dimension near x^0;
2) the distribution G_{n-1} has dimension n;
3) for each $0 \le i \le n-2$, the distribution G_i is involutive.

3 THE NONLINEAR MODEL AND CONTROL LAW DESIGN

The nonlinear mathematical model of the hydraulic servo-system (presented in Fig.1) is achieved, considering the case of neglecting valve's dynamics and taking into account the loss of the flow between chambers of the actuator. In this simplified model, the spool displacement is proportional to the input signal.

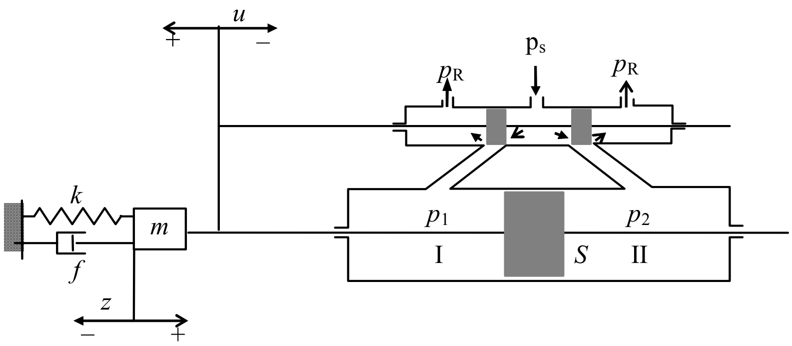

Fig. 1 The structure of the hydraulic servo-system

The nonlinear model is represented by next differential equations:

for $u > 0$

$$\dot{p}_1 = \frac{Bc_d\sqrt{2/\rho}k_u u}{V+Sz}\sqrt{p_s - p_1} - \frac{BSv}{V+Sz};\quad \dot{p}_2 = \frac{-Bc_d\sqrt{2/\rho}k_u u}{V-Sz}\sqrt{p_2} + \frac{BSv}{V-Sz}$$

for $u < 0$

$$\dot{p}_1 = \frac{Bc_d\sqrt{2/\rho}k_u u}{V+Sz}\sqrt{p_1} - \frac{BSv}{V+Sz};\quad \dot{p}_2 = \frac{-Bc_d\sqrt{2/\rho}k_u u}{V-Sz}\sqrt{p_s - p_2} + \frac{BSv}{V-Sz} \tag{5}$$

$$\dot{z} = v$$

$$\dot{v} = \frac{S}{m}(p_1 - p_2) - \frac{k}{m}z - \frac{f}{m}v - \frac{F}{m}$$

$$y = h(x) = z$$

where:
u - is the input voltage to servovalve [V],
c_d - nozzle flow coefficient [-],
B - bulk-modulus of fluid $[N/m^2]$,
p_s - supply pressure $[N/m^2]$,
ρ - is the mass density of the hydraulic fluid $[Kg/m^3]$,
p_1, p_2 - pressure in left and right cylinder chambers, respectively $[N/m^2]$,
V - enclosed volume on each side of actuator where $z = 0$ $[m^3]$,
S - effective area of double-ended piston $[m^2]$
m - piston mass [Kg],
f - equivalent viscous friction force coefficient $[Ns/m]$,
k - equivalent aerodynamic elastic force coefficient $[N/m]$,
F - disturbance force input on actuator [N],

k_u - is a proportional coefficient (spool displacement / input signal) $[m/V]$,
z - the position of the piston [m].

Choosing as vector of state variables $x^T = [p_1(t), p_2(t), z(t), v(t)]$, the state space representation (5) can be written as a linear-analytical form:

$$\dot{x}(t) = f(x) + g(x) \cdot u(t) \quad (6)$$

in which smooth vector fields *f(x)* and *g(x)* have the following expressions:

$$f(x) = \begin{pmatrix} -\dfrac{a_2 v}{V+Sz} \\ \dfrac{a_2 v}{V-Sz} \\ v \\ a_5(p_1 - p_2) - a_3 z - a_4 v \end{pmatrix} \quad (7)$$

$$g(x) = \begin{pmatrix} \dfrac{a_1\sqrt{p_s - p_1}}{V+Sz} \\ -\dfrac{a_1\sqrt{p_2}}{V-Sz} \\ 0 \\ 0 \end{pmatrix} \text{ for } u>0 \,;\, g(x) = \begin{pmatrix} \dfrac{a_1\sqrt{p_1}}{V+Sz} \\ -\dfrac{a_1\sqrt{p_s - p_2}}{V-Sz} \\ 0 \\ 0 \end{pmatrix} \text{ for } u<0 \quad (8)$$

where:

$$a_1 = Bc_d\left(\sqrt{2/\rho}\right)k_u;\, a_2 = BS;\, a_3 = k/m;\, a_4 = f/m;\, a_5 = S/m \quad (9)$$

Using the Lie derivatives we have

$$L_f^0 h(x) = y(x); L_g L_f^0 h(x) = 0 \;;\; L_f^1 h(x) = v; L_g L_f^1 h(x) = 0 \;;\; L_f^2 h(x) = a_5(p_1 - p_2) - a_3 z - a_4 v \quad (10)$$

- for $u > 0$:

$$L_g L_f^2 h(x) = a_1 a_5 \left(\frac{(V - Sz)\sqrt{p_s - p_1} + (V + Sz)\sqrt{p_2}}{V^2 - S^2 z^2} \right) \quad (11)$$

- respectively for $u < 0$:

$$L_g L_f^2 h(x) = a_1 a_5 \left(\frac{(V - Sz)\sqrt{p_1} + (V + Sz)\sqrt{p_s - p_2}}{V^2 - S^2 z^2} \right) \quad (12)$$

We denote:

$$L_f^3 h(x) = -\frac{2Va_2 a_5}{V^2 - S^2 z^2} + v\left(a_4^2 - a_3\right) + a_3 a_4 z - a_4 a_5 (p_1 - p_2) \quad (13)$$

$$a = L_g L_f^2 h(x) = \begin{cases} a^+ = a_1 a_5 \left(\dfrac{(V - Sz)\sqrt{p_s - p_1} + (V + Sz)\sqrt{p_2}}{V^2 - S^2 z^2} \right) & for\, u > 0 \\ a^- = a_1 a_5 \left(\dfrac{(V - Sz)\sqrt{p_1} + (V + Sz)\sqrt{p_s - p_2}}{V^2 - S^2 z^2} \right) & for\, u < 0 \end{cases} \quad (14)$$

and

$$b = L_f^3 h(x) \quad (15)$$

Then, the state feedback:

$$u = \alpha(x) + \beta(x) \cdot v \quad (16)$$

where

$$\alpha(x) = -\frac{b}{a} = \begin{cases} \alpha^+(x) \; for \;\; u > 0 \\ \alpha^-(x) \; for \;\; u < 0 \end{cases} \quad (17)$$

$$\alpha^{+} = -\frac{V^2 - S^2}{a_1 a_5\left[(V - Sz)\sqrt{p_s - p_1} + (V + Sz)\sqrt{p_2}\right]} \times \left[\frac{-2Va_2 a_5}{V^2 - S^2 z^2} + v\left(a_4^2 - a_3\right) + a_3 a_4 z - a_4 a_5 (p_1 - p_2)\right]$$

$$\alpha^{-} = -\frac{V^2 - S^2}{a_1 a_5\left[(V - Sz)\sqrt{p_1} + (V + Sz)\sqrt{p_s - p_2}\right]} \times \left[\frac{-2Va_2 a_5}{V^2 - S^2 z^2} + v\left(a_4^2 - a_3\right) + a_3 a_4 z - a_4 a_5 (p_1 - p_2)\right] \quad (18)$$

$$\beta(x) = \frac{V^2 - S^2 z^2}{a_1 a_5\left((V - Sz)\sqrt{p_s - p_1} + (V + Sz)\sqrt{p_2}\right)}$$

transforms the system (5) into a system whose input-output behavior is identical to that of a triple integrator

$$H(s) = \frac{Z(s)}{V(s)} = \frac{1}{s^3} \quad (19)$$

On the linear system thus obtained one impose a feedback control of the form:

$$\tilde{v} = c_0\left(\tilde{v}^r - L_f^0 h(x)\right) - c_1 L_f^1 h(x) - c_2 L_f^2 h(x) \quad (20)$$

then, the obtained system has a linear input-output behavior, described by the following transfer function

$$H(s) = \frac{Z(s)}{\tilde{V}(s)} = \frac{c_0}{s^3 + c_2 s^2 + c_1 s + c_0} \quad (21)$$

The design parameters are computed using a pole-placement design technique.

4 SIMULATION RESULTS

In order to test the performances and the behavior of the obtained nonlinear controller, extensive computer simulations were performed in Matlab / Simulink using the following hydraulic servomechanism parameters:

$m = 30\ Kg;\ f = 300\ Ns/m;\ p_s = 21.10^6\ N/m^2;\ c_d = 0{,}63;\ \rho = 850\ Kg/m^3;\ S = 10^{-3}\ m^2$

$V = 3.10^{-5}\ m^3;\ k = 10^5\ N/m;\ k_u = 1{,}7.10^{-4}\ m/V$

The simulations were performed using the nonlinear model (6), (7), (8) with the above values of the parameters. The nonlinear control law (16)-(18), (20) is implemented and a step reference profile is used for the position of the piston. The simulation results are presented in Fig. 2-5.

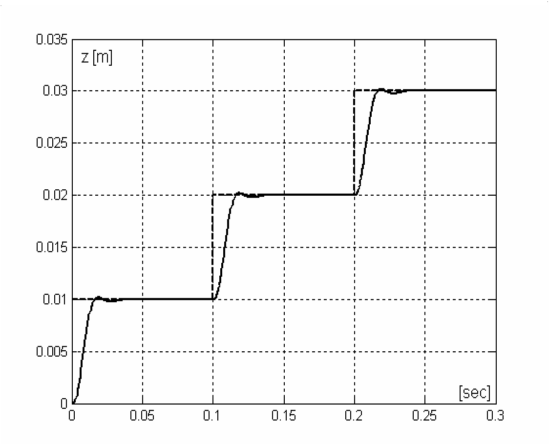

Fig. 2 Reference versus output ($u > 0$)

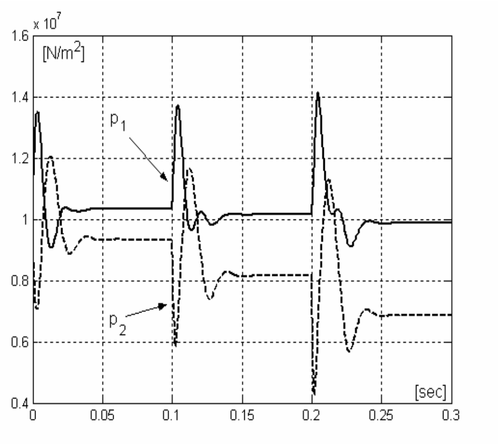

Fig. 3 Evolution of pressures ($u > 0$)

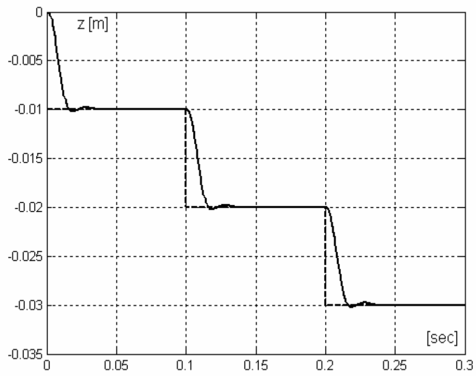

Fig. 4 Reference versus output ($u < 0$)

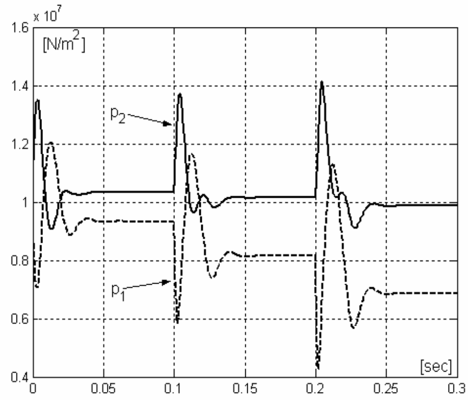

Fig. 5 Time profiles of pressures ($u < 0$)

Fig. 2 shows the time profiles of the reference and of the output (the position of the piston) for the case $u > 0$. From this figure it can be seen that the response of z is quite good (a small settling time and a very small overshoot). Fig. 3 depicts the evolution of the pressures (for $u > 0$).

For the case $u < 0$, the simulation results are represented in Fig. 4 and Fig. 5. In Fig. 4 the reference profile and the output z are shown. Fig. 5 depicts the time evolution of the pressures.

5 CONCLUDING REMARKS

In this paper a nonlinear linearizing control technique for hydraulic servomechanism was presented. The design of the control law uses the exact feedback input-output linearization [Fossard & Normand-Cyrot 1993, Isidori 1995]. The mathematical models of hydraulic servomechanism are studied in order to try the implementation of the nonlinear control laws. Using monovariable modelling and control design, exact linearizing controllers are obtained. Computer simulation is performed in order to test and validate the proposed nonlinear controllers. From the simulation results it can be seen a good behavior of the controlled systems. Further research will take into consideration the cases when some states and/or parameters are unknown.

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