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TIME-VARYING SWITCHING LINES FOR VSC OF ROBOT MANIPULATORS

ČASOVĚ PROMĚNNÉ PŘEPÍNACÍ KŘIVKY PRO VSC ROBOTICKÝCH MANIPULÁTORŮ

Abstract

The paper presents new sliding mode algorithms for control of rigid robot manipulators. We consider a variable structure scheme which ensures the reaching phase elimination, guarantees insensitivity of the manipulator with respect to its model uncertainty and external disturbance from the very beginning of the proposed control action, and assures fast, monotonic error convergence to zero. We explicitly consider the velocity and acceleration constraints of each link. The proposed control algorithms employ the time-varying switching lines that pass through the origin of the error state space. The lines are selected in such a way that the ITAE index is minimised.

Abstrakt

Příspěvek prezentuje nové algoritmy pracující v klouzavých módech pro řízení tuhých robotických manipulátorů. Je uvažováno řízení s proměnnou strukturou, které eliminuje počáteční fázi pohybu k přepínací křivce, zaručuje necitlivost manipulátoru s ohledem na neurčitost modelu a externí poruchy generováním akčního zásahu, který od počátku zaručuje rychlou monotónní konvergenci odchylky k nule. Jsou explicitně uvažována omezení na rychlost a zrychlení každého členu. Navrhovaný řídicí algoritmus využívá časově proměnné přepínací křivky, které procházejí počátkem stavového prostoru pro odchylku. Přepínací křivky jsou zvoleny tak, aby bylo minimalizováno kritérium ITAE.

1 INTRODUCTION

In recent years much of the research in the area of control systems theory focused on the design of a discontinuous feedback which switches the structure of the system according to the evolution of its state vector. This technique, usually called sliding mode control, provides an effective and robust means of controlling nonlinear plants [3], [4], [6]. The main advantage of this technique is that once the system state reaches a sliding surface, the system dynamics remain insensitive to a class of parameter variations and disturbances.

However, robust tracking is assured only after the system state hits the sliding surface, i.e. the robustness is not guaranteed during the reaching phase. Provided a conventional time-invariant sliding plane is considered, the advantage of the sliding mode control, namely the desired dynamic behaviour of the system, is not obtained for some time from the beginning of its motion. Furthermore, usually for the given initial conditions there is an essential trade-off between the short reaching phase and the fast system response in the sliding phase. In order to overcome these problems the idea of the time-varying switching lines applied to the sliding mode control of the second order time-varying, nonlinear systems has been introduced.

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In this paper, a new sliding mode algorithms for control of rigid robot manipulators are presented. The velocity and acceleration constraints of each link are considered. The proposed control algorithms employ the time-varying switching lines that pass through the origin of the error state space. The lines are characterized by the parameters representing their velocities of rotation. The main contribution of this work is the procedure for the optimal, in the sense of the ITAE index, selection of these parameters. The switching lines are designed to eliminate the reaching phase, assure insensitivity of the system to the external disturbance and parameter uncertainties from the very beginning of control action, and provide fast, monotonic error convergence to zero.

2 PROBLEM FORMULATION

Let us consider the following dynamic equation of an n – link rigid robot manipulator

$$\boldsymbol{D}(\boldsymbol{q})\ddot{\boldsymbol{q}}+\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}+\boldsymbol{G}(\boldsymbol{q})+\boldsymbol{d}(t)=\boldsymbol{u}(t), \tag{1}$$

where $\boldsymbol{q}(t)=[q_1(t)\ q_2(t)\ \dots\ q_n(t)]^{\mathrm{T}}$ is the vector of n joint positions, $\boldsymbol{u}(t)$ is the $n\times 1$ vector of joint control inputs, $\boldsymbol{D}(\boldsymbol{q})$ is the $n\times n$ symmetric, positive definite inertia matrix, $\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})$ is the $n\times n$ matrix of Coriolis and centripetal coefficients, defined such that $\dot{\boldsymbol{D}}(\boldsymbol{q})-2\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})$ is a skew–symmetric matrix, $\boldsymbol{G}(\boldsymbol{q})$ is the $n\times 1$ vector of gravitational torques, $\boldsymbol{d}(t)$ is a $n\times 1$ disturbance vector. It is assumed that for all $t\geq t_0$ disturbances are bounded $|d_i(t)|<d_{i\,\max}$, where $d_i(t)$ is the ith element of the vector $\boldsymbol{d}(t)$, and $d_{i\,\max}$ is a positive constant ($i=1,2,\dots,n$). The robot arm (1) is supposed to reach the final state $\boldsymbol{q}_{\mathrm{d}}=[q_{\mathrm{d}1}\ q_{\mathrm{d}2}\dots q_{\mathrm{d}n}]^{\mathrm{T}}$. The system error is defined by the following vector $\boldsymbol{e}(t)=[e_1(t)\ e_2(t)\ \dots\ e_n(t)]^{\mathrm{T}}=\boldsymbol{q}(t)-\boldsymbol{q}_{\mathrm{d}}$. Hence, we have $e_i(t)=q_i(t)-q_{\mathrm{d}\,i}$ ($i=1,2,\dots,n$). In this paper it is assumed that at the initial time $t=t_0$, the error and the error derivative of each link

$$e_i(t_0)=\mathrm{const}\neq 0;\quad \dot{e}_i(t_0)=0. \tag{2}$$

Clearly, the initial error equal to zero is a trivial case and it will not be discussed.

Consider the time-varying straight switching lines that pass through the origin of the phase plane. Originally, the lines rotate with the constant velocities around the origin of the phase plane, then stop at the time instants $t_{\mathrm{f}i}$ ($i=1,2,\dots,n$) when their slopes reach the predetermined values and remain fixed. Consequently, for $t\geq 0$ the switching lines are described by the following equation

$$\boldsymbol{s}(\boldsymbol{e},t)=\boldsymbol{0}\quad\text{where}\quad \boldsymbol{s}(\boldsymbol{e},t)=\dot{\boldsymbol{e}}(t)+\boldsymbol{c}(t)\boldsymbol{e}(t), \tag{3}$$

where $\boldsymbol{c}(t)$ is a diagonal matrix such that $\boldsymbol{c}(t)=\mathrm{diag}\{c_1(t),c_2(t),\dots,c_n(t)\}$ and

$$c_i(t)=\begin{cases}A_i t & \text{for } t\in\langle 0;t_{\mathrm{f}i});\\ A_i t_{\mathrm{f}i}=c_{\mathrm{f}i} & \text{for } t\in\langle t_{\mathrm{f}i};\infty),\end{cases} \tag{4}$$

where A_i are positive constants. Selection of these constants will be considered in the next section.

Taking into account the Lyapunov function $\boldsymbol{V}=\tfrac{1}{2}\,\boldsymbol{s}^{\mathrm{T}}\boldsymbol{D}\boldsymbol{s}$ the control law of the following structure can be derived

$$\boldsymbol{u}(t)=\boldsymbol{C}\dot{\boldsymbol{q}}+\boldsymbol{G}-\boldsymbol{D}\left[\boldsymbol{c}(t)\dot{\boldsymbol{e}}+\dot{\boldsymbol{c}}(t)\boldsymbol{e}\right]-\boldsymbol{C}\boldsymbol{s}-\boldsymbol{\gamma}\,\mathrm{sgn}(\boldsymbol{s}), \tag{5}$$

where $\boldsymbol{\gamma}=\mathrm{diag}\{\gamma_1,\gamma_2,\dots,\gamma_n\}$ and the condition $\gamma_i\geq d_{i\,\max}$ is satisfied to ensure the stability of the sliding motion on the ith switching line (3). Another Lyapunov function which can be considered here is $\boldsymbol{V}=\tfrac{1}{2}\,\boldsymbol{s}^{\mathrm{T}}\boldsymbol{s}$. Then the control law is obtained in the form

$$\boldsymbol{u}(t)=\boldsymbol{C}\dot{\boldsymbol{q}}+\boldsymbol{G}-\boldsymbol{D}\left[\boldsymbol{c}(t)\dot{\boldsymbol{e}}+\dot{\boldsymbol{c}}(t)\boldsymbol{e}\right]-\boldsymbol{D}\boldsymbol{\zeta}\,\mathrm{sgn}(\boldsymbol{s}), \tag{6}$$

where $\boldsymbol{\zeta}=\mathrm{diag}\{\zeta_1,\zeta_2,\dots,\zeta_n\}$ with condition $\zeta_i\geq\sum_{k=1}^{n}\left|b_{ik\,\max}\right|d_{k\,\max}$, where $b_{ik\,\max}$ is the greatest value of i,kth element of the inverse matrix of $\boldsymbol{D}$.

If at $t = t_0$ the representative point of ith link belongs to the ith switching line and control signal (5) is applied, then for any time $t \in \langle 0; t_{fi})$ the system dynamics is described by (3) with the initial conditions given in (2). Assuming, for the sake of clarity, that $t_0 = 0$ we get the errors and their derivatives for the time interval $t \in \langle 0; t_{fi})$

$$e_i(t) = e_i(0)\,\mathrm{e}^{-A_i t^2/2}, \tag{7}$$

$$\dot{e}_i(t) = -A_i t\, e_i(0)\,\mathrm{e}^{-A_i t^2/2}. \tag{8}$$

In the second phase of the robot manipulator motion control, i.e. when the switching lines do not move, for any time $t \geq t_{fi}$ we get the errors and their derivatives expressed by relations

$$e_i(t) = e_i(0)\,\mathrm{e}^{c_{fi}(c_{fi} - 2A_i t)/2A_i}, \tag{9}$$

$$\dot{e}_i(t) = -c_{fi}\, e_i(0)\,\mathrm{e}^{c_{fi}(c_{fi} - 2A_i t)/2A_i}. \tag{10}$$

Notice that the errors described by (7) and (9) do not exhibit any overshoots.

3 SWITCHING LINES DESIGN

In order to select the optimal switching line parameters A_i we will consider the following integral control quality criterion

$$J_i = \int_{t_0}^{\infty} t \left| e_i(t) \right| dt. \tag{11}$$

Substituting (7) and (9) into this expression we obtain the criterion in the form

$$J_i(A_i) = \frac{|e_i(0)|}{A_i}\left(1 + \frac{A_i}{c_{fi}^2}\,\mathrm{e}^{-c_{fi}^2/2A_i}\right). \tag{12}$$

Moreover, calculating the derivatives dJ_i/dA_i one can notice that the right-hand side of criterion (12) decreases with increasing A_i. Further in the paper, the considered criterion with velocity and acceleration constraints will be minimised.

3.1 Velocity constraints

Let us consider the velocity constraint of each link given by the inequality

$$\left| \dot{e}_i(t) \right| \leq v_{i\max}, \tag{13}$$

where $v_{i\max}$ are arbitrary positive constants. The extreme values of the joint velocities expressed by (8) are achieved at the time instants $t_{mi} = 1/\sqrt{A_i}$ and they are equal to $\dot{e}_i(t_{mi}) = -e_i(0)\sqrt{A_i/\mathrm{e}}$. Further, we consider two cases: one when $t_{mi} < t_{fi}$ and the other when $t_{fi} \leq t_{mi}$. Some straightforward calculations lead to the conclusion that in the first case the maximum absolute value of the ith joint velocity is achieved at the t_{mi}. Then the solution of the considered optimisation problem is

$$A_{i\,\mathrm{opt}}^{\mathrm{v1}} = \mathrm{e} v_{i\max}^2 / e_i^2(0), \tag{14}$$

$$J_{i\,\mathrm{opt}}^{\mathrm{v1}} = \frac{|e_i^3(0)|}{\mathrm{e} v_{i\max}^2}\left\{1 + \frac{v_{i\max}^2}{e_i^2(0)\, c_{fi}^2}\,\mathrm{e}^{1 - \left[e_i^2(0) c_{fi}^2 / 2\mathrm{e} v_{i\max}^2\right]}\right\} \tag{15}$$

assuming $|e_i(0)| > \sqrt{\mathrm{e}}\, v_{i\max}/c_{fi}$. In the second case, i.e. when $t_{fi} \leq t_{mi}$, for each link we get that the greatest joint velocity is given by the absolute value of $e_i(t_{fi}) = -c_{fi}\, e_i(0)\,\mathrm{e}^{-c_{fi}^2/2A_i}$. Since this expression contains the exponential function, another two sub-cases should be considered. The first one takes place if $\ln(v_{i\max}/c_{fi}|e_i(0)|) \geq 0$. This is equivalent to $|e_i(0)| \leq v_{i\max}/c_{fi}$ and then we get the optimal parameters $A_{i\text{opt}}^{v2} \to \infty$ and the minimum value of criterion (12) is $J_{i\text{opt}}^{v2} = \left|e_i(0)\right|/c_{\text{f}i}^2$.

The other sub-case, i.e. $\ln\left(v_{i\max}/c_{\text{f}i}\left|e_i(0)\right|\right) < 0$ yields $A_i \leq -c_{\text{f}i}^2/2\ln\left(v_{i\max}/c_{\text{f}i}\left|e_i(0)\right|\right)$. Hence, in the interval $\left|e_i(0)\right| \in \left(v_{i\max}/c_{\text{f}i};\ \sqrt{\text{e}}v_{i\max}/c_{\text{f}i}\right\rangle$ the optimal solution

$$A_{i\text{opt}}^{v3} = -c_{\text{f}i}^2/2\ln\left(v_{i\max}/c_{\text{f}i}\left|e_i(0)\right|\right), \tag{16}$$

$$J_{i\text{opt}}^{v3} = -2\frac{\left|e_i(0)\right|}{c_{\text{f}i}^2}\left\{1 - \frac{v_{i\max}}{2c_{\text{f}i}\left|e_i(0)\right|\ln\left[v_{i\max}/c_{\text{f}i}\left|e_i(0)\right|\right]}\right\}\ln\frac{v_{i\max}}{c_{\text{f}i}\left|e_i(0)\right|}. \tag{17}$$

3.2 Acceleration constraints

In this section the following acceleration constraints are taken into account

$$\left|\ddot{e}_i(t)\right| \leq a_{i\max}, \tag{18}$$

where $a_{i\max}$ are arbitrary positive constants. It can be easily verified that for any time $t \geq 0$ the maximum values of the absolute $\ddot{e}_i(t)$ are achieved at the initial time $t = 0$ and they are equal to $A_i\,|e_i(0)|$. Then the solutions of the optimisation problem are

$$A_{i\text{opt}}^{a} = a_{i\max}/\left|e_i(0)\right|, \tag{19}$$

$$J_{i\text{opt}}^{a} = \frac{e_i^2(0)}{a_{i\max}}\left(1 + \frac{a_{i\max}}{\left|e_i(0)\right|c_{\text{f}i}^2}\,\text{e}^{-\left|e_i(0)\right|c_{\text{f}i}^2/2a_{i\max}}\right). \tag{20}$$

3.3 Minimisation of criterion J_i subject to velocity and acceleration constraints

Now let us consider the situation when both the joint velocities and accelerations are limited simultaneously. The ith velocity cannot be greater than $v_{i\max}$ and the maximum admissible accelerations are $a_{i\max}$. In order to select the optimal solutions of the minimisation of criterion (12) the results of sections 3.1 and 3.2 are used. Furthermore, two additional cases should be considered. First of them takes place when $c_{\text{f}i} \geq a_{i\max}/\sqrt{\text{e}}\,v_{i\max}$. Then the optimal values of the parameters are

$$A_{i\text{opt}} = \begin{cases} A_{i\text{opt}}^{a} & \text{for} \quad \left|e_i(0)\right| \leq \dfrac{\text{e}v_{i\max}^2}{a_{i\max}}; \\ A_{i\text{opt}}^{v1} & \text{for} \quad \left|e_i(0)\right| > \dfrac{\text{e}v_{i\max}^2}{a_{i\max}}. \end{cases} \tag{21}$$

Consequently, when $\left|e_i(0)\right| \leq \text{e}v_{i\max}^2/a_{i\max}$ the minimum of criterion (12) is equal to $\min\limits_{A_i} J_i(A_i) = J_i\left(A_{i\text{opt}}^{a}\right) = J_{i\text{opt}}^{a}$, otherwise, i.e. for $\left|e_i(0)\right| > \text{e}v_{i\max}^2/a_{i\max}$, $\min\limits_{A_i} J_i(A_i) = J_{i\text{opt}}^{v1}$.

On the other hand, if $c_{\text{f}i} < a_{i\max}/\sqrt{\text{e}}\,v_{i\max}$, then the optimal value of the ith switching line rate of rotation and further criterion (12) can be expressed as follows

$$A_{i\text{opt}} = \begin{cases} A_{i\text{opt}}^{a} & \text{for} \quad \left|e_i(0)\right| \leq e_i^B; \\ A_{i\text{opt}}^{v3} & \text{for} \quad e_i^B < \left|e_i(0)\right| \leq \dfrac{\sqrt{\text{e}}\,v_{i\max}}{c_{\text{f}i}}; \\ A_{i\text{opt}}^{v1} & \text{for} \quad \left|e_i(0)\right| > \dfrac{\sqrt{\text{e}}\,v_{i\max}}{c_{\text{f}i}}; \end{cases} \tag{22}$$

$$J_{i\min}(A_i)=\min_{A_i} J_i(A_i)=\begin{cases} J_{i\,\mathrm{opt}}^{a} & \text{for } |e_i(0)|\le e_i^B; \\ J_{i\,\mathrm{opt}}^{v3} & \text{for } e_i^B<|e_i(0)|\le \dfrac{\sqrt{\mathrm{e}}\,v_{i\max}}{c_{\mathrm{f}i}}; \\ J_{i\,\mathrm{opt}}^{v1} & \text{for } |e_i(0)|> \dfrac{\sqrt{\mathrm{e}}\,v_{i\max}}{c_{\mathrm{f}i}}, \end{cases} \tag{23}$$

where $e_i^B\in\left(v_{i\max}/c_{\mathrm{f}i};\sqrt{\mathrm{e}}\,v_{i\max}/c_{\mathrm{f}i}\right)$ are solutions of the equations $f_i[e_i(0)] = A_{i\,\mathrm{opt}}^{a}-A_{i\,\mathrm{opt}}^{v3}=0$. They can be easily found using any standard numerical procedure (e.g. bisection method or *regula falsi*).

The parameters A_i determined in this way ensure the optimal performance of the controlled robot arm described by (1) satisfying velocity and acceleration constraints given by (13) and (18).

4 SIMULATION EXAMPLE

In order to verify the performance of the proposed sliding mode control algorithm a simulation test on the two degree-of-freedom planar robot with two revolute joints (see Fig.1) is conducted. The robot arm is described by the following dynamic equations

$$\begin{aligned} u_1(t) &= D_{11}(q_2)\ddot{q}_1+D_{12}(q_2)\ddot{q}_2-m_2 l_1 l_2\sin(q_2)\left(\dot{q}_2^2+2\dot{q}_1\dot{q}_2\right)+ \\ &\qquad +g\left[(m_1+m_2)l_1\cos(q_1)+m_2 l_2\cos(q_1+q_2)\right]+d_1(t) \\ u_2(t) &= D_{12}(q_2)\ddot{q}_1+m_2 l_2^2\ddot{q}_2+m_2 l_1 l_2\dot{q}_1^2\sin(q_2)+g\,m_2 l_2\cos(q_1+q_2)+d_2(t) \end{aligned} \tag{24}$$

where

$$D_{11}(q_2)=(m_1+m_2)l_1^2+m_2 l_2^2+2m_2 l_1 l_2\cos(q_2),$$

$$D_{12}(q_2)=m_2 l_2^2+m_2 l_1 l_2\cos(q_2).$$

We assume that the lengths of the first and second links of the arm are l_1 = 0.4 m, l_2 = 0.3 m, and their masses are $m_1 = m_2$ = 1 kg, respectively. The robot arm is subject to the external disturbances $d_1(t)$ = 0.3 cos(10t) Nm and $d_2(t)$ = 0.4 cos(5t) Nm. The initial position is determined by $e_1(0)$ = π rad, and $e_2(0)$ = π/3 rad. The arm should reach its final position $\boldsymbol{q}_{\mathrm{d}}$ =[π/4 rad π/7 rad]$^{\mathrm{T}}$. As the threshold values of velocity and acceleration of each link we take $v_{i\max}$ = 2 rad/s and $a_{i\max}$ = 4 rad/s^2 (i = 1,2).

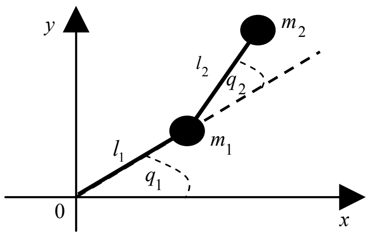

Fig. 1. Two degree-of-freedom robot arm

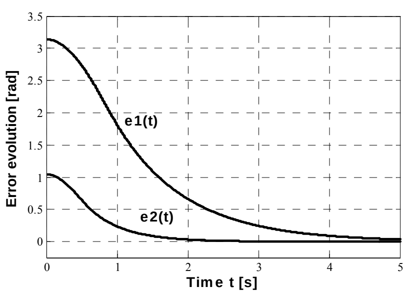

Fig. 2. Error of the both links

Figures 2, 3 and 4 show the error and its first and second derivatives of each link of the arm, respectively, under control law (5) with $\boldsymbol{\gamma}=\mathrm{diag}\{0.3\text{ Nm}, 0.4\text{ Nm}\}$. Then due to proposed control method the optimal switching line parameters are $A_{1\,\mathrm{opt}}\approx 1.1072$ s^{-2}, $A_{2\,\mathrm{opt}}\approx 3.8197$ s^{-2} and the lines stop rotating at the moments $t_{\mathrm{f}1}\approx 0.9032$ s, $t_{\mathrm{f}2}\approx 0.5236$ s. It can be seen from the figures that constraints $v_{i\max}$ = 2 rad/s, $a_{i\max}$ = 4 rad/s^2 (i = 1, 2) are satisfied, and the errors converge to zero monotonically.

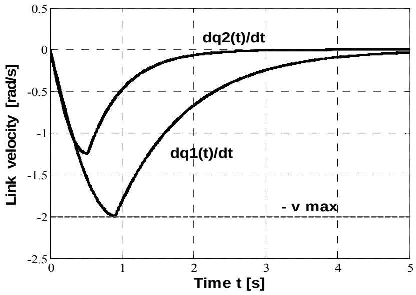

Fig. 3. Velocity of the links

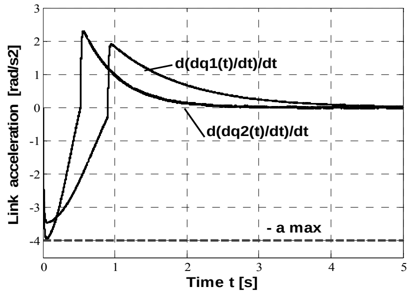

Fig. 4. Acceleration of the links

5 CONCLUSIONS

In this paper new sliding mode algorithms for control of rigid robot manipulators have been proposed. The algorithms employ the time-varying switching lines which rotate with constant velocities around the origin of the error state space. The designed switching lines guarantee that the reaching phase is eliminated, the joint errors converge to zero monotonically and the insensitivity of the manipulator with respect to its model uncertainty and external disturbance from the very beginning of the proposed control action are ensured. The switching lines rate of rotation is chosen in such a way that the ITAE index is minimised and the velocity and acceleration constraints of each link are satisfied. To verify the performance of the proposed control algorithms a simulation test on the two degree-of-freedom planar robot with two revolute joints has been conducted.

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