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Jiří PODEŠVA*, Petr JANAS**

THE NON-LINEAR BENDING OF THE STEEL ARCH SUPPORT

NELINEÁRNÍ OHYB OCELOVÉ OBLOUKOVÉ VÝZTUŽE

Abstract

The calculation of the horizontal mine opening steel support can be performed by the special program, developed on the Department of civil mechanics. It can take into account the geometrical non-linearity (large displacement), but other effects, like plasticity and stiffness decrease due to change of the support section are not able to include into calculation. The paper describes the possibility to express these effects like the change of the beam stiffness E·J.

Abstrakt

Výpočty ocelové výztuže vodorovných důlních děl se mohou provádět speciálním programem, vyvinutým na katedře stavební mechaniky. Výpočet může zahrnovat geometrickou nelinearitu (velká posunutí), ale další efekty, jako plasticitu a snížení tuhosti v důsledku změny nosného profilu, není možné do výpočtu zahrnout. Příspěvek popisuje způsob jak tyto efekty interpretovat jako změnu tuhosti nosníku E·J.

1 INTRODUCTION

There are two reasons to perform the non-linear static analysis of the steel arch support of the horizontal mine opening (see Fig. 1).

During the process of maintenance of the horizontal underground mine openings the calculation must be done to verify the sustain of the static load capacity.

The probabilistic calculations of the structure using the Monte Carlo method require many times repeated static analysis.

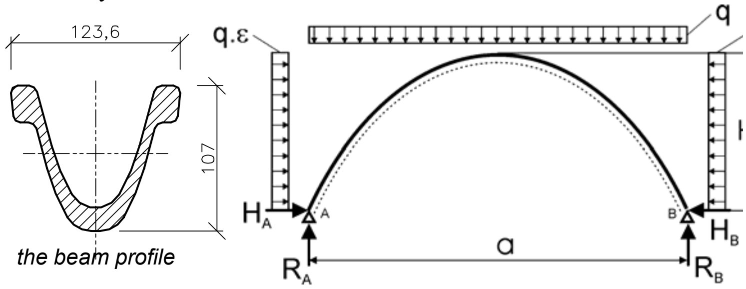
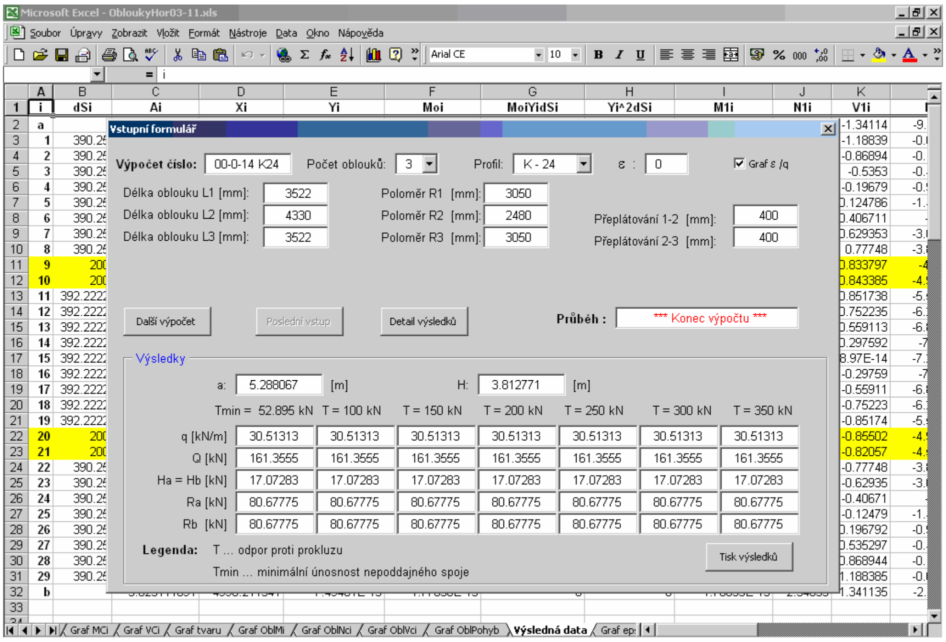


Fig. 1. The steel arch support.

For both reasons the simple program in the MS Excel / Visual Basic environment, based on the beam elements, can be used (see Fig. 2). Three features of non-linearity are present in the structure : Geometrical non-linearity (large displacement), material non-linearity (plasticity) and the change of the beam cross sectional area. First can be included in the calculation by the repeated iterations, second and third must be included different way.

* Doc. Ing. Jiří Podešva, Ph.D., Department of Mechanics, Faculty of Mechanical Engineering, VŠB - Technical University of Ostrava, 17. listopadu 15, Ostrava - Poruba, 708 33, tel. +420 597 324 350, e-mail jiri.podesva@vsb.cz

** Doc. Ing. Petr Janas, CSc, Department of Civil Mechanics, Faculty of Civil Engineering, VŠB - Technical University of Ostrava, Ludvíka Podéště 1875, Ostrava - Poruba, 708 00, tel. +420 597 321 308 , e-mail petr.janas@vsb.cz



The parameters (cross sectional area S, quadratic area moment of inertia J, Young modulus of the element material E and the element length ℓ) gives the element tension / pressure stiffness (k_{tp}) and bending stiffness (k_b).

$$k_{tp} = \frac{E \cdot S}{\ell} \qquad k_b = \frac{E \cdot J}{\ell^3}$$

The element stiffness matrix **K** represents the linear relationship between deformation and forces. But there are three features of non-linearity.

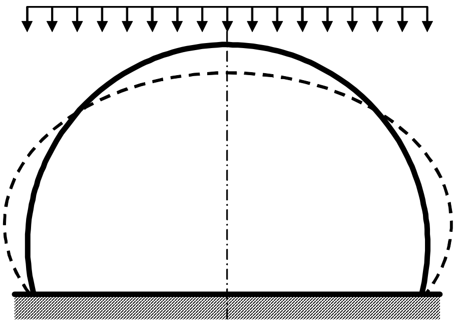


Fig. 3. The large deformation of the arch support.

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- ❑ The geometrical non-linearity. The large displacement of the arch points (see Fig. 3) can be included into the model by repeated, iterative calculations, with the stiffness matrix update between each iteration.
- ❑ The material non-linearity. Under higher bending the plastic strain appears (see Fig. 4). This feature can be included into model via stress-strain calculation individually in the number of thin layers, what can be satisfying. The assumption is that the normal cuts will stay planar after bending. This does not represent the fully plastic behavior.
- ❑ The stiffness loose. Finally under the higher bending moment the beam changes its cross sectional area (see Fig. 5). It becomes more flat. This changes the J parameter of the bending stiffness. This can lead to the buckling of the support structure - the loose of the stiffness.

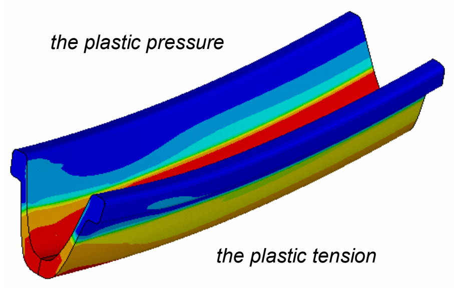


Fig. 4 The plastic deformation of the beam

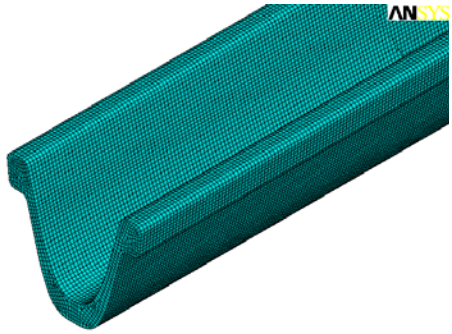


Fig. 6. The finite element model of the beam.

Fig. 6 The finite element model of the beam

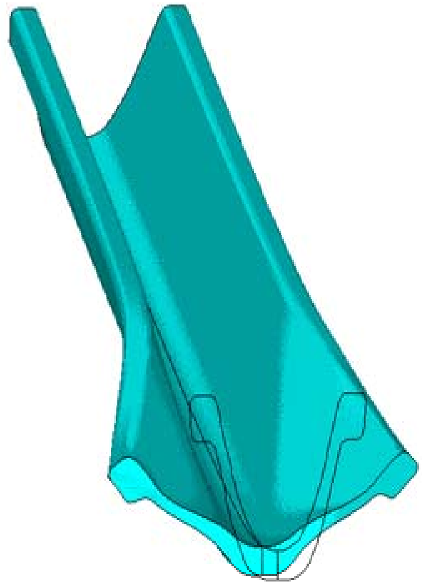


Fig. 5. The beam profile deformation.

Fig. 5 The beam profile deformation

2 THE NONLINEAR BENDING OF THE BEAM

The moment - deformation relationship can be the subject of the finite element based modeling. The 3D model, based on the brick elements (see Fig. 6) was built and all calculations were performed in the Ansys finite element based package.

The geometrical non-linearity is included the same way as in the MS Excel program - via iterative solution with the stiffness matrix update in each step.

To include the plastic behavior the tri-linear stress strain curve was defined (see Fig. 7).

The decrease of stiffness is also the feature of geometrical non-linearity. It is included into the model by the repeated update of the stiffness matrix during the iterative calculation, depending on the change of the model shape.

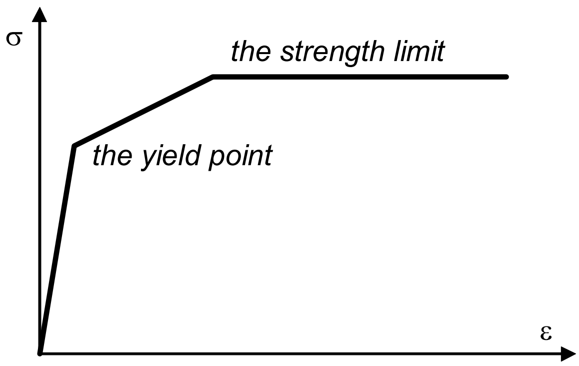


Fig. 7. The tri-linear stress-strain curve.

Fig. 7 The tri-linear stress-strain curve

The subject of modeling was the cantilever beam, fixed on one end, exposed to the bending moment M on the opposite end (see Fig. 8 and Fig. 9).

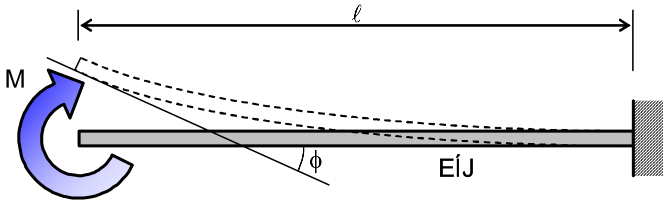


Fig. 8. The fixed beam, exposed to the bending moment.

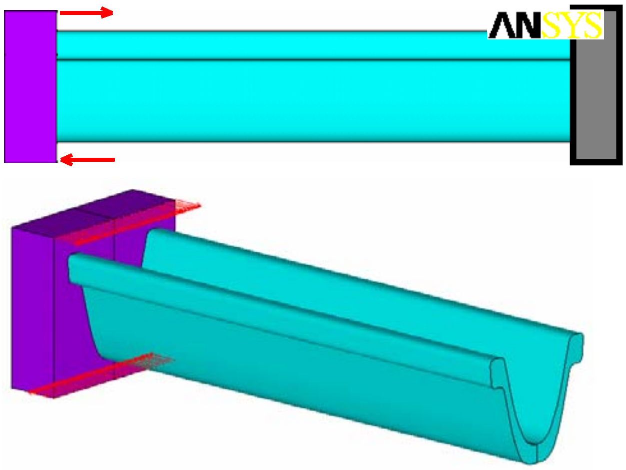


Fig. 9. The beam, exposed to the bending moment.

3 THE BENDING CHARACTERISTIC AND THE SUBSTITUTIONAL STIFFNESS

The increasing loading of the beam was divided into the number of load steps. Subsequently from the bending moment - bending angle pair data the bending characteristic can be designed (see Fig. 10).

Up to the bending angel about 2° the relation is linear and rather sheer. Then it becomes flat. That is clear that the bending moment about 23 kN·m at the bending angle 16° represents the buckling point. The structure looses the stability. The question is how to include this non-linear behavior into the beam-based model.

If the bending angle in the above described example is :

$$\phi_{(M)} = \frac{M \cdot \ell}{E \cdot J}$$

where

ϕ	is the bending angle,
M	is the bending moment,
ℓ	is the beam length,
E	is the Young modulus of the material,
J	is the quadratic area moment of inertia.

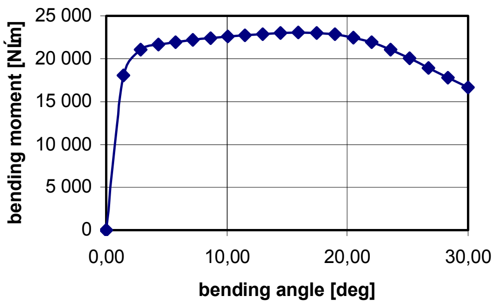


Fig. 10. The bending characteristic

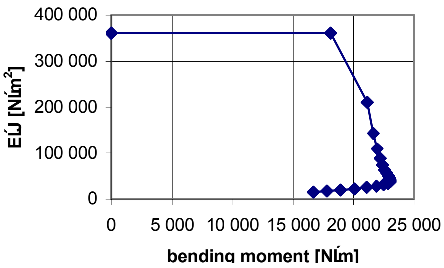


Fig. 11. The stiffness - moment curve

The relationship then can be interpreted like if the parameter E·J (the substitutional stiffness) was depending on the bending moment :

$$E \cdot J_{(M)} = \frac{M \cdot \ell}{\phi} \tag{1}$$

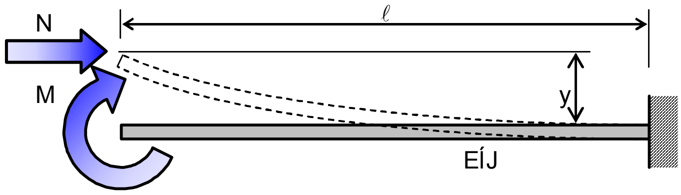
and the bending stiffness - bending moment curve can be designed (see Fig.11). The first part of the curve (up to the bending moment about 18 kN·m, E·J = *konst*) corresponds to the linear behavior. The decrease of the E·J represents the decrease of stiffness due to both plasticity and change of the cross sectional area. Finally the break point, after which the bending moment decreases (about 23 kN·m), represents the buckling. The structure collapses and is not more able to carry the load.

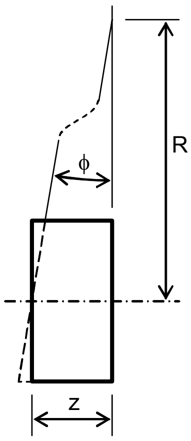
The E·J - M curve, expressing the non-linear behavior, can be easily included into the MS-Excel program. This represents the methodology how to include all non-linearities into the MS-Excel program.

4 THE BENDING UNDER THE AXIAL FORCE

The beam can be exposed, except of the bending moment M, also to axial force N (normal to cross sectional area, see Fig. 12).

In the expression of $\phi=\phi_{(M)}$ (see above) the constant bending moment M along the beam length is assumed. But under the axial force N corresponding bending moment N·y (where y is displacement normal to the beam axis) increases to the fixed end. In this case the relationship $\phi=\phi_{(M)}$ is much more complicated. It represents the II. order task (the bending moment depends on the deformation). In this case the different approach must be used.





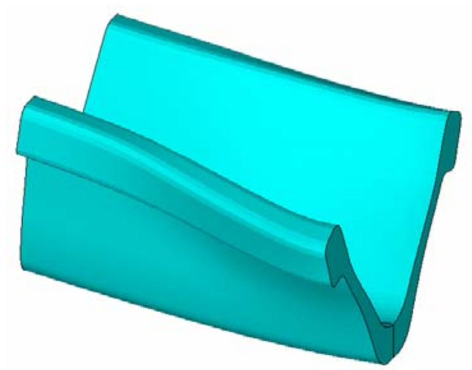


Fig. 15. The short part of the beam and itÁs deformation.
Fig. 15 The short part of the beam and it´s deformation

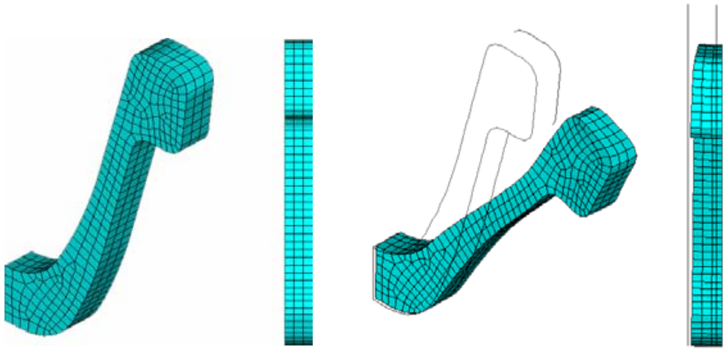


Fig. 14. The short part of the beam and it's deformation.

The deformation of not all the beam length ℓ, but only a short part z (see Fig. 13, 14 and 15) is taken into account. From it's deformation the bending angle ϕ is calculated. The radius of the bending curve is then

$$R = \frac{z}{\phi}$$

The bending moment M is calculated as the sum of moments of the nodal forces.

Finally, from the linear theory of beams is taken

$$E \cdot J_{(M)} = M \cdot R = \frac{M \cdot z}{\phi} \tag{2}$$

Surely the expression (2) is only the modification of the expression (1). But for the short length z the assumption of the constant moment along length is more acceptable than for all the length ℓ.

For the accurate calculation the z length should be as short as possible. But from the other point of view the z length must be long enough to cover the area of the 3D deformation (see Fig. 15). To determine the bending angle ϕ with acceptable small variation the cutting plane must keep planar after deformation (compare Fig. 13 and Fig. 15).

The above described calculation of the bending characteristic was performed for different values of the axial force (from 0 up to 1200 kN). This gives the series of the bending stiffness - bending moment (E·J - M) curves (see Fig. 16). The two-variable function $E \cdot J_{(M,\ N)}$ can be included into the MS Excel program.

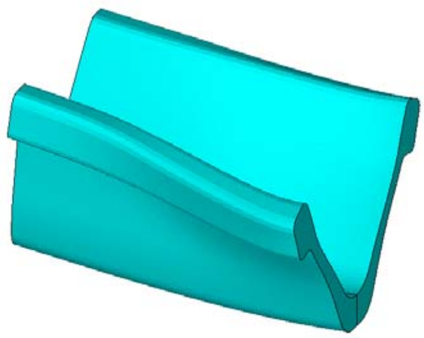


Fig. 15. The short part of the beam and it's deformation.
Fig. 15 The short part of the beam and it's deformation

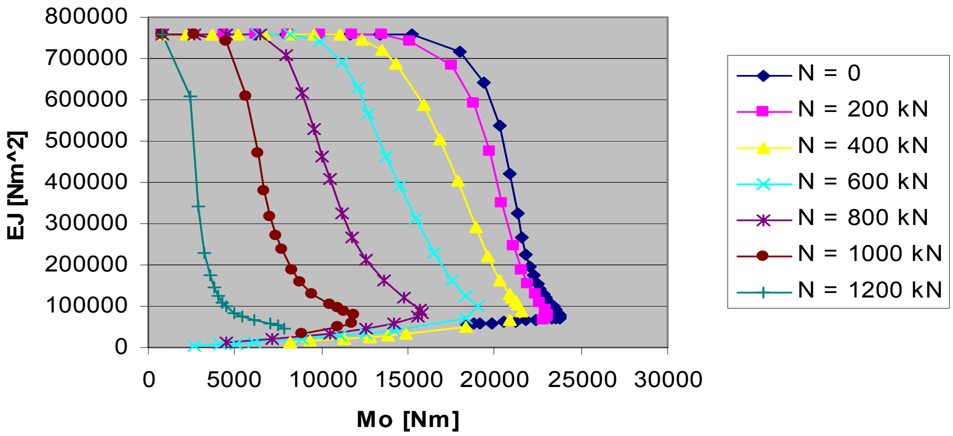


Fig. 16. The series of the stiffness - moment curves.

Fig. 16 The series of the stiffness - moment curves

5 CONCLUSION

The conclusions can be established :

- The calculation program for the analysis of strength and reliability of the steel arch support of the mine openings, worked out on the department of civil mechanics, Faculty of Civil Engineering, VŠB - Technical University of Ostrava, is a good tool both for design and maintenance of this kind of steel structures.
- The program allows including the geometric non-linearity ... the change of the structure geometry under load. The other features, plasticity and the change of the beam cross sectional area due to loading, are hard or impossible to take into account.
- The beam bending can be subject of the finite element modeling. The result is then the non-linear characteristic - the relation between the bending moment and the bending angle. Subsequently the various values of the substitutional stiffness E·J can be linked to various values of the bending moment. This substitutional stiffness gives the same deformation in the framework of the linear theory of beams as is the real deformation, calculated on the 3D FEM model. All these results were calculated and are different for different values of the axial force.
- This approach allows to include the material non-linearity and buckling effects into the beam based model in the same way as the geometrical non-linearity. The program searches the final solution in the number of iterations. In every iteration the stiffness of the structure is updated according to the deformation of the whole structure, the plasticity in the material and change of the beam cross sectional area.
- In both the moment - angle characteristic (M-ϕ) and stiffness - moment link (E·J-M) the following features can be observed.
  - Under low load the M-ϕ relation is linear, the stiffness E·J is constant.
  - With increasing load the M-ϕ characteristic becomes more flat, the E·J stiffness decreases.
  - When certain load is reached the break point in both curves appears. The M-ϕ curve begins to fall down. The E·J-M curve breaks to the left. This represent the buckling point. The beam looses the stiffness and is not more able to carry the load. At this point the structure collapses.

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