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OPTIMAL DESIGN OF "HCR" GEARS IN TERMS OF A RELATION BETWEEN A TEETH SLOPE AND SPECIFIC SLIDINGS

OPTIMÁLNÍ NÁVRH „HCR“ SOUKOLÍ Z HLEDISKA POMĚRU SKLONU ZUBŮ A MĚRNÝCH SKLUZŮ

Abstract

Designer is obliged to choose different angle of a teeth slope β to adjust working centre distance for a demanded value when helical gear pairs are designed. He has to respect to the value of specific slidings at the same time. This article makes basis of a designer who is just solving this task.

Abstrakt

Při geometrickém návrhu ozubeného soukolí čelního se šikmými zuby je často výpočtář nucen volit úhel sklonu zubů β, tak aby upravil osovou vzdálenost na požadovanou hodnotu. Současně však musí brát zřetel na hodnotu měrných skluzů. Tento článek vytváří podklad pro konstruktéra, který právě tuto úlohu řeší.

Used symbols

a	pitch centre distance
a_w	working centre distance
d_b	base diameter
h^*_a	addendum coefficient
c^*	tip clearance coefficient
m_n	normal module
x_1	addendum modification coefficient for pinion
x_2	addendum modification coefficient for wheel
x_Σ	sum of addendum modification coefficients
z_1	number of teeth of pinion
z_2	number of teeth of wheel
α_n	basic rack flank angle
α_t	transverse rack flank angle
α_{tw}	working transverse pressure angle
β	helix angle at reference cylinder
ρ^*_f	root radius coefficient
ϑ_{A1}	specific sliding at pinion root

Introduction

The system of addendum modification coefficients for gaining of balanced specific slidings for V gearing is the most widespread modification system. For given working centre distance a_w and a sum of addendum modification coefficients x_Σ it is necessary to use equations (1) and (2) for the numerical solving of addendum modification coefficients for gaining of balanced specific slidings.

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$$\frac{1}{\sqrt{\left(\frac{a_w}{m_n}+h_{a1}^*-\frac{z_2}{2\cdot\cos\beta}-x_{2v}\right)^2\cdot\frac{4\cdot\left(\mathrm{tg}^2\alpha_n+\cos^2\beta\right)}{z_1^2}-1}}-$$

$$-\frac{\frac{z_2}{z_1}}{\sqrt{\left(\frac{a_w}{m_n}+h_{a2}^*-\frac{z_1}{2\cdot\cos\beta}+x_{2v}-x_\Sigma\right)^2\cdot\frac{4\cdot\left(\mathrm{tg}^2\alpha_n+\cos^2\beta\right)}{z_2^2}-1}}+ \tag{1}$$

$$+\frac{\frac{z_2}{z_1}-1}{\sqrt{\left(\frac{a_w}{m_n}\right)^2\cdot\frac{4\cdot\left(\mathrm{tg}^2\alpha_n+\cos^2\beta\right)}{\left(z_1+z_2\right)^2}-1}}=0$$

$$x_{1v}=x_\Sigma-x_{2v} \tag{2}$$

For the next progress it is suitable to split this problem to the two partial tasks. The first task is derived from the fixed sum of addendum modification coefficients x_Σ and the second one is coming-out from the condition of the fixed working transverse pressure angle. It is necessary to remind that when the angle β is changing, while the angle α_{tw} is fixed, the sum of addendum modification coefficients x_Σ must adjust itself and vice versa.

The first task

It is requested to find a relation between a teeth slope angle β and other parameters (a_w, x_Σ, specific slidings). Thus the constant parameter is the working transverse pressure angle - α_{tw} = const. It is possible to derive an equation for the working centre distance a_w as a function of the angle β (6).

$$a_w=a\cdot\frac{\cos\alpha_t}{\cos\alpha_{tw}} \tag{3}$$

$$a=\frac{\left(z_1+z_2\right)\cdot m_n}{2\cdot\cos\beta} \tag{4}$$

$$\alpha_t=\mathrm{arctg}\left(\frac{\mathrm{tg}\,\alpha_n}{\cos\beta}\right) \tag{5}$$

$$a_w=\frac{\left(z_1+z_2\right)\cdot m_n}{2\cdot\cos\beta}\cdot\frac{\cos\left(\mathrm{arctg}\left(\frac{\mathrm{tg}\,\alpha_n}{\cos\beta}\right)\right)}{\cos\alpha_{tw}} \tag{6}$$

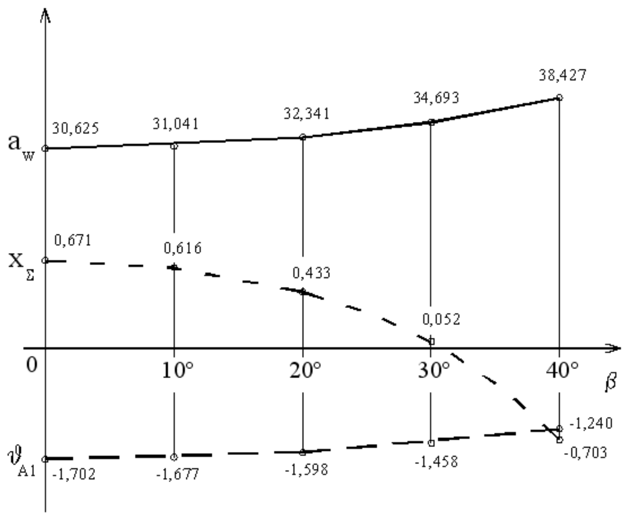
The next equation is derived from the starting equation for no tooth backlash of V gearing (7) using the equation (5). This time it is dependence between x_Σ and angle β again.

$$\mathrm{inv}\,\alpha_{tw}=\frac{2\cdot x_\Sigma}{z_1+z_2}\cdot\mathrm{tg}\,\alpha_n+\mathrm{inv}\,\alpha_t \tag{7}$$

$$x_\Sigma=\left(\mathrm{tg}\,\alpha_{tw}-\alpha_{tw}-\frac{\mathrm{tg}\,\alpha_n}{\cos\beta}+\mathrm{arctg}\left(\frac{\mathrm{tg}\,\alpha_n}{\cos\beta}\right)\right)\cdot\frac{z_1+z_2}{2\cdot\mathrm{tg}\,\alpha_n} \tag{8}$$

The third equation is a function between a size of balanced specific slidings at pinion root and angle β. Addendum modification coefficients for balanced specific slidings are computed using (1) and (2). A numeric value of the specific sliding at pinion root is determined by equation (9). There is necessary to express parameters a_w a d_{a2} as function of angle β there. A system of transcendental equations is formed. It must be solved numerically. All three equations are drawn in fig.1.

$$\vartheta_{A1} = 1 - \frac{z_1}{z_2} \cdot \frac{\sqrt{d_{a2}^2 - d_{b2}^2}}{2 \cdot a_w \cdot \sin\alpha_{tw} - \sqrt{d_{a2}^2 - d_{b2}^2}} \tag{9}$$



$z_1 = 20$
$z_2 = 40$
$m_n = 1$ [mm]
$\alpha_n = 20^\circ$
$h^*_{a1} = 1,2$
$h^*_{a2} = 1,2$
$c^*_1 = 0,244$
$c^*_2 = 0,244$
$\rho^*_{f1} = 0,3708$
$\rho^*_{f2} = 0,3708$

$\alpha_{tw} = 23,000^\circ$
(const.)

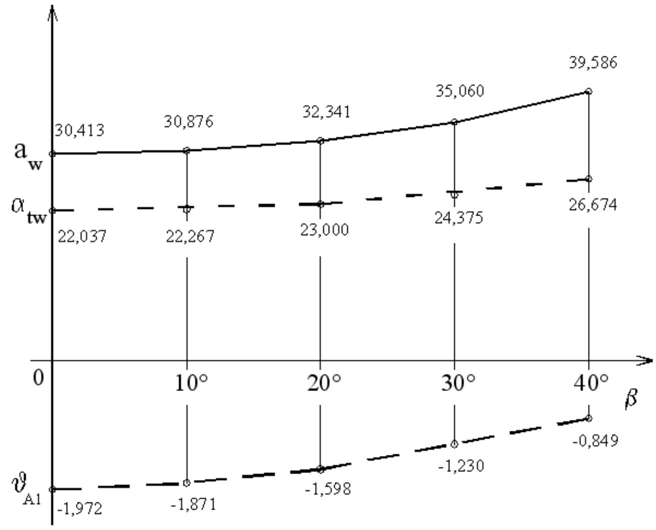
Fig.1

The second task

This task is similar like the first one but the constant parameter is the sum of addendum modification coefficients x_Σ. Now a dependence between tooth slope angle β and other parameters (a_w, α_{tw}, specific slidings) is finding. The equation (6) is used for solving relation between working centre distance a_w and angle β. For solving dependence between working transverse pressure angle α_{tw} and angle β is useful equation (7). Angle α_{tw} must be numerically computed from (10), to which the value inv α_{tw} from (7) is substituted during each computational loop.

$$\operatorname{tg}\alpha_{tw} - \alpha_{tw} - \operatorname{inv}\alpha_{tw} = 0 \tag{10}$$

For the third dependence – a relation between a value of specific sliding at pinion root and angle β is the same procedure like in the first task. For substituting of working centre distance a_w will be used the same method like in the previous paragraph again. Working transverse pressure angle is now fixed and a diameter d_{a2} is setting using usual procedure for the addendum modification coefficient x_{2v} resulting from (2). For this task all three equations are drawn in fig.2.



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$h^*_{a2} = 1{,}2$
$c^*_1 = 0{,}244$
$c^*_2 = 0{,}244$
$\rho^*_{f1} = 0{,}3708$
$\rho^*_{f2} = 0{,}3708$

$x_\Sigma = 0{,}433197$
(const.)

Fig.2

Conclusion

Two considerable conclusions result from both graphs. First it is good to notice that when angle β is changing so it is not true that for fixed x_Σ stay a_w and α_{tw} fixed too. Here the designer would take a false step. The main result however is that when a gear pair is designed, a designer may use also bigger values of angle β without a danger of rising of specific slidings. On the contrary its value is falling just with bigger values of angle β.

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Literature

[1] Němček,M. *Vybrané problémy geometrie čelních ozubených kol.* MONTANEX a.s. Ostrava, 2003 ISBN 80-7225-111-2.

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