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SELF-VIBRATING CHAMBERS WITH CONTINUOUS PASSAGE OF LIQUID

KOMORY S VLASTNÍMI KMITY A KONTINUÁLNÍM PRŮTOKEM KAPALINY

Abstract

The generation of a modulated waterjet is the base for the creation of continual pulsing jets. The theoretical and experimental study is aimed at the formation of modulated jets by resonance inside a self-vibrating system that is assembled with chambers and tubes through which liquid flows. The significant parameters are identified and basic equations are submitted, derived from physical laws and analogies. Finally, a comparison of the theory here presented is supplied with an example of the up-to-date experiments.

Abstrakt

Generování modulovaného vodního paprsku je základem pro vytvoření kontinuálního pulzního paprsku. Teoretický a experimentální výzkum je zaměřen na vytvoření modulovaných paprsků rezonancí uvnitř systému s vlastními kmity, který je složen z komor a trubic protékaných kapalinou. Jsou určeny významné parametry a navrženy základní rovnice odvozené z fyzikálních zákonů a analogií. V závěru je prezentovaná teorie porovnána s příkladem dosavadních experimentů.

Keywords: liquid flow, self-vibration, modulated waterjet
Klíčová slova: tok kapaliny, vlastní kmitání, modulovaný paprsek

1 Introduction

The hydrodynamic acoustic vibrating system (hereafter HAVS) is considered to be such a system of chambers and tubes passed through by liquid that is able to generate a natural modulated or interrupted (pulsing) liquid jet. This type of jet is useful for material destruction in various technical applications [1]. The theoretical model presented in this paper is prepared for description, analyses and optimization of both the HAVS and any analogical system with known material and geometrical conditions, physical parameters and technical design. The configuration of our experimental device was inspired by theories and experiments published by Sami & Ansari [2] and Chahine & Conn [3].

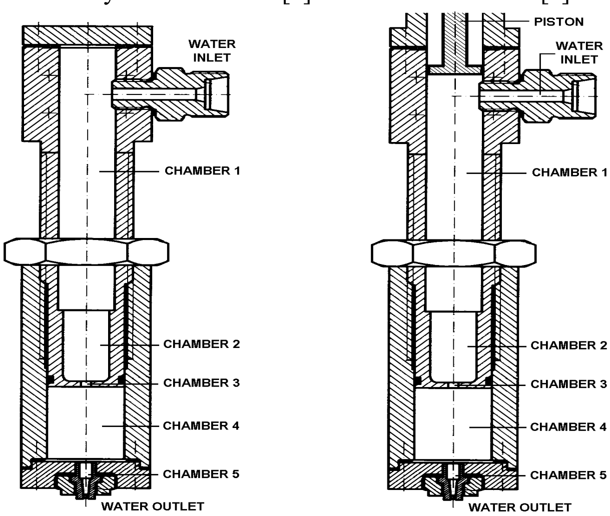

Fig.1 Diagrams of the experimental device designed for testing of the HAVS parameters.

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First of all, continuously distributed characteristic acoustic parameters are described. They affect the generation of mechanical waves inside the HAVS. The configuration of our model case, designed by Hlaváč et al. [4], is evident from Fig. 1.

The inflow tube is the first element of the system perpendicular to the initiatory chamber that is composed of an upper and a lower part (chamber 1 and 2 respectively). The subsequent connecting branch is marked chamber 3. An intrinsic resonation chamber (4) opens in both directions along flow. Finally, the fifth element - an outlet tube (chamber 5) - closes the system. In the process of searching for appropriate parameters and the derivation of useful theories we were inspired with analogical theories: the theory of acoustic circuits (valid for gaseous substances) and theory of electromagnetic RLC circuits and mechanical translation vibrating systems. Nevertheless, any analogy in theoretical area does not necessarily mean identical description and, therefore, its importance is mostly in the methodology of solution of a problem. Because some of the acoustic parameters are dominant in a certain part of a technical device while others are less important there, whilst elsewhere the situation is the opposite, the elements can be divided into two groups – tubes and chambers. Tubes are characterised primarily by acoustic mass and acoustic resistance, their acoustic plasticity is of minor importance. On the other hand, an acoustic plasticity is a characteristic attribute of chambers and their acoustic mass and acoustic resistance are of minor importance. The physical conditions determine which part of the system behaves as a tube and which as a chamber.

2 Physical description

The proposed theoretical model is based on the physical analysis of the HAVS functioning. Liquid in the tubes vibrates virtually as one rigid body whilst the substance in the chambers is compressed and diluted simultaneously in all points of volume under consideration. When the liquid passes through the system of cavities a turbulence produces quite high frequencies inside the flow. Those frequencies with low intensity can be intensified by an intrinsic resonation cavity. The waves generated then modulate a liquid outflow velocity. Nevertheless, the stationary waves can be generated in cavities under some conditions too, and similarly as in the open pipe, they can intensify the amplitudes of certain frequencies. Then the resonation state can be completely different from other common tuned states.

The harmonic variable deviations of vibrating particles of a liquid and their respective successive harmonic velocities depend on harmonic changes in the pump pressure. The effective pump pressure determines the velocities of liquid flow in any part of the HAVS, according to the law of continuity. The kinetic energy of the volume unit of flowing particles is directly linked to the energy of vibrations inside this volume unit. Nevertheless, the respective parameters usually describing a certain element of the chamber system need not represent a real behaviour of this element in any particular situation. It is especially the case when the element of the system under study is strongly influenced by actually tuned neighbouring elements. So the multi-elements system can be frequently reduced to a simpler one in resonation, i.e. with a reduced number of elements. The reduced number of elements is, as a rule, two; a tube and a chamber.

3 Acoustic mass

The relationship between the acoustic $\boldsymbol{m_a}$ and the inertial mass $\boldsymbol{m}$ in liquid acoustic systems can be expressed, as suggested Kušnerová [5], by Eq. (1)

$$m_a = \frac{m}{S^2} \,. \tag{1}$$

For the element with the cross-section $\boldsymbol{S_i}$ constant along the distance $\boldsymbol{L_i}$ filled by liquid with a density before vibration of $\boldsymbol{\rho_i}$ the Eq. (1) modifies into

$$m_{ai} = \frac{\rho_i \, L_i}{S_i} \,. \tag{2}$$

The total acoustic mass $\boldsymbol{m_{ac}}$ is a sum of the partial acoustic masses. Provided that elements of the HAVS are cylindrical with radii $\boldsymbol{R_i}$, lengths $\boldsymbol{L_i}$ and liquid densities $\boldsymbol{\rho_i}$ the total acoustic mass is calculated from the equation

$$m_{ac} = \sum_{i=1}^{n} \frac{\rho_i \, L_i}{\pi \, R_i^2} \,. \tag{3}$$

The velocities of particles making up the acoustic mass in the gaseous medium are considered to be constant throughout the channel cross-section. By contrast, velocities of particles in the liquid medium cannot be independent in any part of the cross-sections of the flow because they are significantly influenced by viscosity, turbulence and friction with the inner surface of the channel (tube). Therefore, the resistance force $\boldsymbol{F_R}$ is introduced into the equations of motion and a more exact relationship between the acoustic and inertial force is obtained

$$m_a = \frac{m \, p_a}{\left(F_R + p_a \, S\right) S} \,. \tag{4}$$

An acoustic pressure inside the tube is determined from the law of conservation of energy in a volume unit (determined by the effective pump pressure and kinetic energy of flowing liquid particles). Resistance forces $\boldsymbol{F_R}$ cannot be measured even indirectly and greatly depend on the flow conditions being classified by a Reynolds number value. The empirical Eq. (5), however, is frequently used for determination of the resistance force $\boldsymbol{F_R}$ in hydrodynamics

$$F_R = -k_r \rho \, S \, v^2 \,. \tag{5}$$

The total acoustic mass necessary for evaluation of the HAVS's self-oscillation frequency is then determined by equation

$$m_{ac} = \sum_{i=1}^{n} \frac{\rho_i \, L_i \left(p_P - \frac{1}{2} \rho_i v_i^2 \right)}{\pi \, R_i^2 \left(k_r \rho_i \, v_i^2 + p_P - \frac{1}{2} \rho_i v_i^2 \right)} \,. \tag{6}$$

It is supposed that the coefficient $\boldsymbol{k_r}$ has the same value for all parts of the HAVS and the acoustic masses inside the HAVS oscillate as one complex. So the velocity $\boldsymbol{v_i}$ is determined by the equation

$$v_i = \frac{\rho_0 R_N^2 v_0}{\rho_i R_i^2} \,. \tag{7}$$

4 Acoustic resistance

The relationship between the acoustic resistance $\boldsymbol{r_a}$ and the mechanical resistance $\boldsymbol{r}$ of the liquid inside the vibrating systems is expressed by the equation

$$r_a = \frac{r}{S^2} \,. \tag{8}$$

Using the formula of the resistance force for turbulent flow according to Kušnerová [6], the equation for acoustic resistance can be modified into

$$r_{ai} = \frac{k_a \, \rho_i \, u_i}{S_i} \,. \tag{9}$$

The coefficient of attenuation $\boldsymbol{k_a}$ characterises the resistance of the carrier medium to the motion of vibrating particles. It is determinable from experimental data.

The total acoustic resistance is obtained as a sum of partial acoustic resistances $\boldsymbol{r_{ai}}$ (provided that one general coefficient of attenuation is determinable for all parts of the system)

$$r_{ac} = \sum_{i=1}^{n} \frac{k_a \sqrt{2\rho_i \left(p_P - \frac{1}{2}\rho_i v_i^2 \right)}}{\pi \, R_i^2} \quad . \tag{10}$$

5 Acoustic plasticity

The relationship between acoustic plasticity $\boldsymbol{c_a}$ and mechanical plasticity $\boldsymbol{c}$ in vibrating systems is expressed by the equation

$$c_a = c \, S^2 \, . \tag{11}$$

The acoustic plasticity for a chamber with volume $\boldsymbol{V}$ filled by liquid with density $\boldsymbol{\rho}$ is normally determined from the phase velocity of the sound $\mathbf{v_c}$

$$c_a = \frac{V}{\rho \, v_c^2} \tag{12}$$

Nevertheless, the hydraulic acoustic plasticity inside vibrating systems like the HAVS is determined by the pressure of vibrations $\boldsymbol{p_{aj}}$

$$c_{aj} = \frac{V_j}{p_{aj}} \tag{13}$$

Simultaneously, the relationships for pressurised liquid in all parts of the HAVS can be simplified by the presumption $\rho_i \cong \rho_j \cong \rho \neq \rho_0$.

The total value of acoustic plasticity in a system with linear ordered elements with acoustic plasticities $\boldsymbol{c_{aj}}$ and volumes $V_j = \pi \, R_j^2 \, L_j$ is determined similarly to capacities in electrical circuits

$$\frac{1}{c_{ac}} = \sum_{j=1}^{m} \frac{p_P - \frac{1}{2}\rho_j v_j^2}{\pi \, R_j^2 \, L_j} \, . \tag{14}$$

6 Acoustic impedance

The complex resistance of any acoustic vibration system can be introduced analogically to the impedance of electrical circuits. Its real part is expressed by the acoustic resistance $\boldsymbol{r_a}$ and the imaginary parts (reactances) $\boldsymbol{X_{ma}}$ and $\boldsymbol{X_{ca}}$ are determined by acoustic mass and acoustic plasticity respectively. The acoustic impedance $\overline{\boldsymbol{Z}}_a$ can be calculated from the following equation

$$\overline{Z}_a = r_a + j \left(X_{ma} - X_{ca} \right) \tag{15}$$

where $X_{ma} = \omega \, m_a \;\; ; \;\; X_{ca} = \dfrac{1}{\omega \, c_a} \;\; ; \;\; \omega = 2 \, \pi \, f \, .$

The total acoustic impedance of the HAVS is determined through the total acoustic mass, the total acoustic resistance and the total acoustic plasticity. The magnitude of the impedance is calculated analogically to the impedance in electrical circuit from the equation

$$Z_{ac} = \sqrt{r_{ac}^2 + \left(\omega \, m_{ac} - \frac{1}{\omega \, c_{ac}} \right)^2} \, . \tag{16}$$

An example of a relationship between the theoretical and experimental results is presented in a graphic form in Fig. 2. Both the calculation and the experimental measurement were performed for initiatory chamber shortened by 10 mm regarding the basic state, the resonation chamber with the diameter 30 mm and the height 0.9 mm, the output nozzle with the diameter 1.8 mm and the length 20 mm.

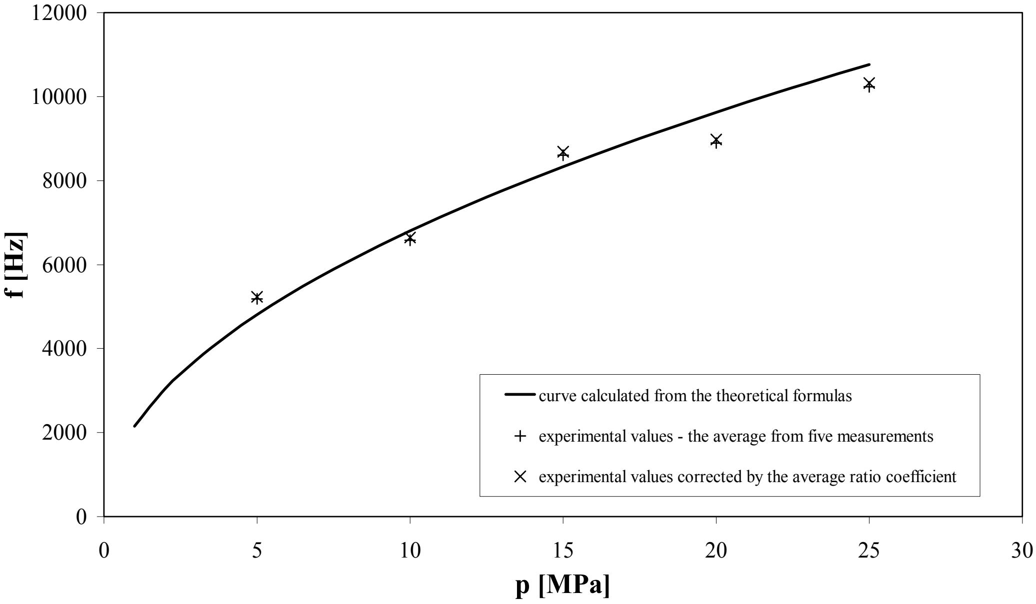

Fig. 2. The theoretical calculation and the experimental data - an example of a comparison.

7 Discussion

The theory presented here enables us to derive the self-frequency of oscillations in designed system of chambers. The resonation frequency can be specified from acoustic mass, acoustic plasticity and acoustic resistance

$$f_r = \frac{1}{2\pi} \cdot \sqrt{\frac{1}{m_{ac} \cdot c_{ac}} - 2\left(\frac{r_{ac}}{2\, m_{ac}}\right)^2} \; . \tag{17}$$

Nevertheless, the Eq. (17) can be modified if the presumption is introduced that a coefficient of theory to practice conversion $\boldsymbol{k}$ exists whilst the coefficient of attenuation $\boldsymbol{k_a}$ approaches zero and the coefficient $\boldsymbol{k_r}$ approaches one. The equation for resonation frequency is then modified to the form

$$f_r = \frac{k}{2\pi} \cdot \sqrt{\frac{1}{m_{ac} \cdot c_{ac}}} \tag{18}$$

The comparison of resonation frequencies determined from the suggested theory $\boldsymbol{f_r}$ with the measured frequencies $\boldsymbol{f_m}$ is presented here (an example of frequency spectrum obtained from measured time dependence of jet force is presented in Fig. 3). The results of comparison are summarized in Tab. 1. The coefficient $\boldsymbol{k}$ was determined by mathematical adjusting of the theory to the measured data. The comparison of theory and experiment is carried out through the relative percentage difference ρ_r.

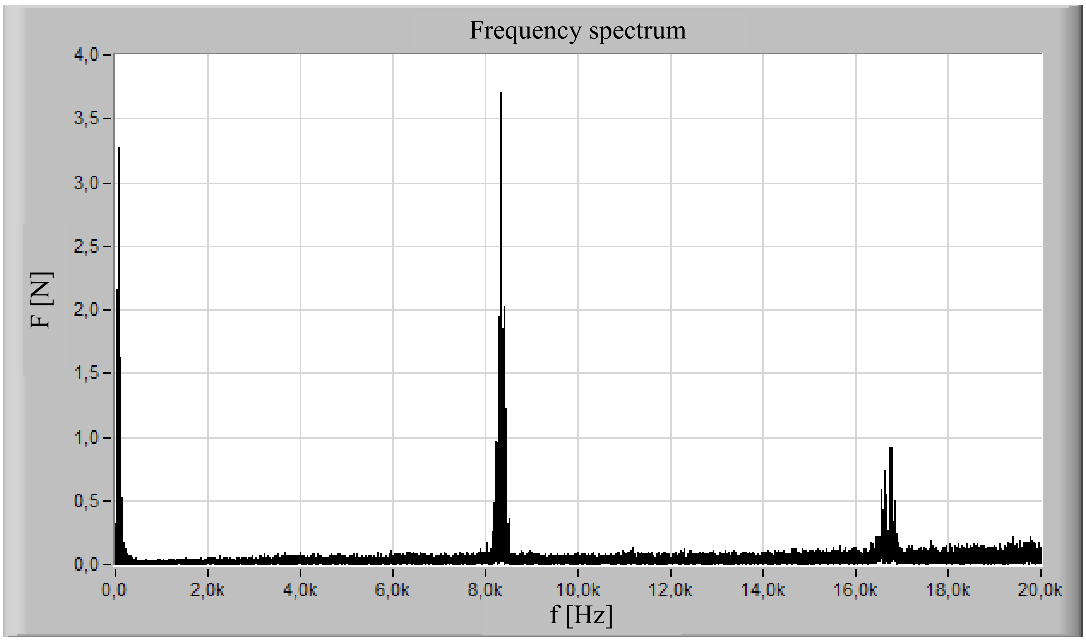

Fig. 3. Frequency spectrum of the tuned system in the basic geometrical configuration by the pump pressure 15 MPa.

Table 1. Comparison of theoretically and experimentally determined values.

	water pressure	5 MPa	10 Mpa	15 MPa	20 MPa	25 MPa
measured frequency	f_m [Hz]	5102	6316	8287	8975	10987
calculated frequency	f_r [Hz]	4863	6793	8223	9391	10388
percentage difference	ρ_r [%]	4.69	-7.55	0.76	-4.63	5.45

The results presented in Tab. 1 show that the average percentage difference between theory and measured data is close to zero ($\rho_r = -0.26$ %) and the maximum percentage difference is lower than 10%. Therefore, we consider the presented theory as proved. Now we are preparing analyses of further sets of experiments with the aim of developing the best algorithm for optimisation of parameters of the system of vibrating chambers. The aim is to prepare the model for determination of the system geometry according to the requirements of practice.

8 Conclusions

This contribution is the first of a series considering liquid filled self-vibrating system of tubes and chambers. It points out the basic characteristic parameters of such systems. The basic analogies and derivations are presented. More detailed papers are in the stage of preparation now, considering the energy of the system, resonation frequency and analysis of physical conditions for usage of acoustic parameters presented here. Nevertheless, in spite of the fact that this paper is just an introduction to the problem, some preliminary theoretical results based on the geometrical description of the self-vibrating system of tubes and chambers, concerning especially the resonation frequency which is presented here, bears a very close correlation between theory and preliminary experimental results with an average value of relative difference about 4.6%.

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Nomenclature

c	mechanical plasticity …[m.N^{-1}]
c_a	acoustic plasticity …[m^5.N^{-1}]
c_{ac}	total acoustic plasticity …[m^5.N^{-1}]
f	frequency ...[Hz]
f_m	measured resonation frequency …[Hz]
f_r	theoretical resonation frequency …[Hz]
F_R	resistance force …[N]
i	index of parameters inside the i-th part of the system (usually a tube) …[-]
j	index of parameters inside the j-th part of the system (usually a chamber) …[-]
k	coefficient of theory to practice conversion …[-]
k_a	coefficient of attenuation of oscillations in the carrier substance …[-]
k_r	coefficient of resistance in turbulent flow …[-]
L	length …[m]
m	inertial mass …[kg]
m_a	acoustic mass …[kg.m^{-4}]
m_{ac}	total acoustic mass …[kg.m^{-4}]
p_a	acoustic pressure of vibrations …[Pa]
p_P	pump pressure …[Pa]
ρ	liquid density …[kg.m^{-3}]
ρ_0	liquid density in a non-compressed state …[kg.m^{-3}]
ρ_r	relative percentage difference …[-]
r	resistance …[kg.s^{-1}]
r_a	acoustic resistance …[N.s.m^{-5}]
r_{ac}	total acoustic resistance …[N.s.m^{-5}]
R	radius of the tube or the chamber …[m]
R_N	radius of the nozzle …[m]
S	cross-section area …[m^2]
u	velocity of oscillating particle …[m.s^{-1}]
v	velocity …[m.s^{-1}]
v_0	outlet velocity of liquid jet …[m.s^{-1}]
v_c	phase velocity …[m.s^{-1}]
V	volume …[m^3]
ω	circular frequency …[s^{-1}]
X_{ca}	reactation of the acoustic plasticity …[N.s.m^{-5}]
X_{ma}	reactation of the acoustic mass …[N.s.m^{-5}]
Z_a	acoustic impedance …[N.s.m^{-5}]
Z_{ac}	total acoustic impedance …[N.s.m^{-5}]

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