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PHYSICAL AND MATHEMATICAL MODELLING OF LIQUID FLOW DYNAMICS IN PIPE (HYDRAULIC SHOCK)

FYZIKÁLNÍ A MATEMATICKÉ MODELOVÁNÍ DYNAMIKY PROUDĚNÍ KAPALINY V POTRUBÍ (HYDRAULICKÉHO RÁZU)

Abstrakt

Matematické modelování dynamiky proudění je problém, který vyžaduje verifikaci výsledků s fyzikálním experimentem. Na pracovišti byl sestaven fyzikální model, který je možno doplňovat hydraulickými prvky a na kterém je možno simulovat přechodové charakteristiky. V článku je popsána metoda charakteristik a metoda konečných objemů pro simulaci hydraulického rázu, jsou aplikovány software Flowmaster a Fluent. Systém Fluent je nutno doplnit o aproximaci hustoty závislé na tlaku. V závěru jsou zhodnoceny přístupy modelování a numerické výsledky porovnány s fyzikálním experimentem.

Abstract

Mathematical modelling of fluid dynamics is the problem, that requires results verification with the physical experiment. At the department of hydromechanics the physical model, which can be supplemented by hydraulic elements and where the transient characteristics can be measured, was assembled. In this paper the method of characteristics and method of finite volume for hydraulic shock waves simulation is described, the software Flowmaster and Fluent are applied. System Fluent is extended in approximation of density depending on pressure. In conclusion the approaches of modelling are evaluated and numerical results are compared with physical experiment.

1 Introduction

There exist many problems, which come under field of hydraulic circuit dynamics, e.g. hydraulic shock waves, pressure pulsations in system, changes of hydraulic resistances and losses of energy, turbulence, noisiness of systems and many other factors ruling trouble-free running of mechanisms. In new mechanism design it is inevitable to pay attention on development expenses. Using simulation programmes it is possible to detect the behaviour of new prototypes and to remedy faults.

The ground of theory for solving of dynamics is given by methods of classical hydromechanics. Mathematical apparatus was formulated during the previous period by excellent scientists, as Bernoulli, d´Alambert, Euler, Laplace, Stokes, Navier, Newton, Reynolds and many others. Mathematical models of complicated dynamic cases by using the theoretical hydromechanical approaches become very complicated, they can be described by the systems of differential equations. Therefore some simplified computational techniques were developed, e.g. analytical and graphical computational methods (Allievi, Schnyder), but their accuracy was poor. In the last years these methods are supplanted by numerical ones using powerful computers. Correctness of these computational approaches must be verified by physical experiment.

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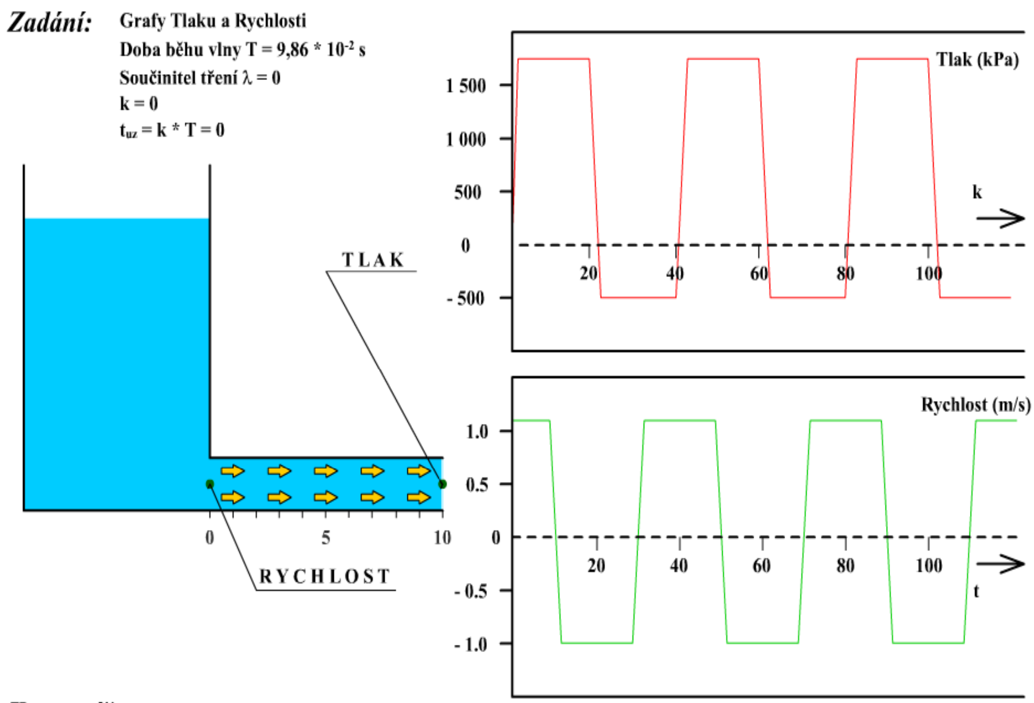
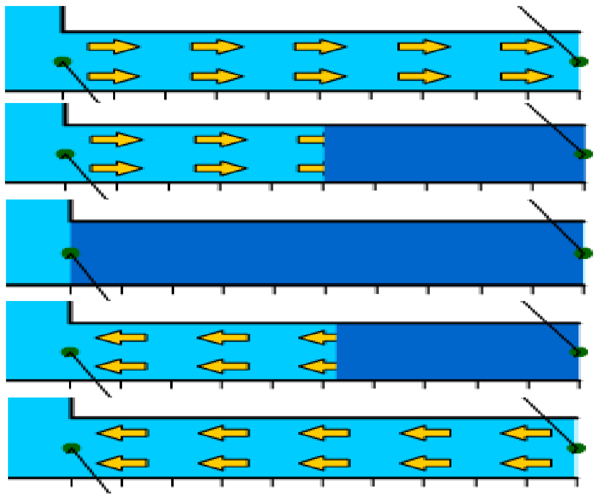
2 Unsteady pipe flow

Unsteady state of the flow in the pipe is the transient state from the origin one to new equilibrium state and it is characterized by time dependent velocity and pressure changes, when the acceleration or delay of the flow can be observed [1]. This unsteady state can be caused in simplest case by supposing of the steady pipe flow from the source (e.g. reservoir) and then quick closing the valve at the end of the pipe. Before the valve the periodic changes of pressure and near the reservoir the periodic changes of velocity discover, see Fig 1, where “Tlak” is pressure, “Rychlost” is velocity, “Doba běhu vlny T” is run time wave, “Součinitel tření λ k” is coefficient using in closing time, “t_{uz}” is both close-up armatures, “t” is time.

Supposing the real fluid flow with friction the pressure changes will be dumped.





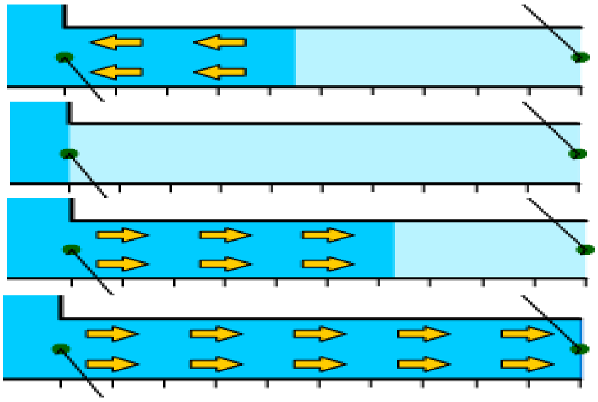


Fig.1 Scheme of unsteady flow in the pipe without friction – pressure

Mathematical methods for estimation of the pressure increase in case of hydraulic shock in the pipe are:

- algebraic relation - Žukovskij equation (named Rankin-Hugoniot equation too)
- wave equation in one-dimensional space – method of characteristics
- Navier-Stokes equation in three-dimensional space – method of finite volume

2.1 Algebraic relation

Supposing the high pressure changes in the hydraulic circuit it is necessary to consider the fluid compressibility. The unsteady flow is solved as hydraulic shock, when the kinetic energy of fluid is changed to deformation work. From this equilibrium the relation of pressure increasing Δp is derived (Žukovskij - 1898):

$$\Delta p = \rho . a . \Delta \upsilon$$

ρ $\Delta \upsilon$ is the difference of the origin and ending velocity, a is the real velocity of the pressure wave propagation (sound velocity in the pipe), which depends on elasticity modulus of fluid eventually pipe wall and is given in form:

$$a = \kappa \sqrt{\frac{K}{\rho}}, \quad \text{where} \quad \kappa = \frac{1}{\sqrt{1 + \frac{d}{s}\frac{K}{E}}} \qquad (2)$$

where K *s elastcty modulus of flud* κ

T *s the tme of wae runnn n the ppe of the lenth* l from the valve to the reservoir:

$$T = \frac{2l}{a} \qquad (3)$$

2.2 Wave equation of the unsteady compressible fluid flow

Žukovskij equation (Rankin-Hugoniot equation (1)) defines the pressure increasing, but does not provide the information about time depending pressure wave form at the end of the pipe or at another places of the pipe [6], [1]. Basely Fig. 2 it is possible to suppose the significant pressure and velocity changes depending only on space variable x *and tme arale* t *.e.* $p = p(x,t)$, $\upsilon = \upsilon(x,t)$.

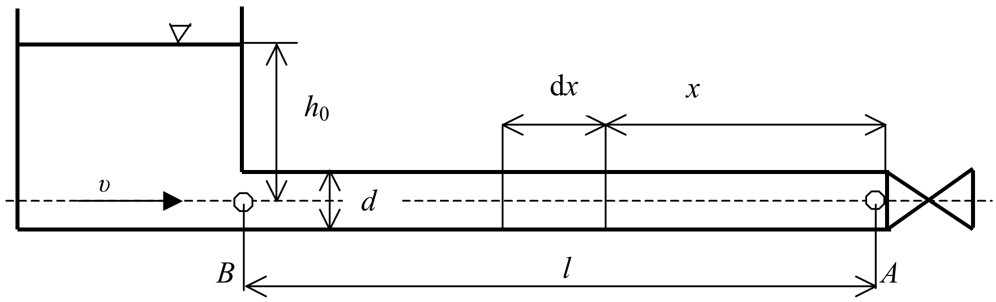


Fig. 2 Scheme for calculation of unsteady flow state in the pipe

Basic equations of unsteady compressible fluid flow in the pipe (one-dimensional case) are derived from following conditions using marking by scheme on Fig. 2.2.

- equilibrium of inertial and pressure forces on element of inviscid compressible fluid $d\vec{F}_s = d\vec{F}_p$ (Euler equation)
- mass equilibrium $\frac{d}{dt}\Delta m = 0$ (continuity equation)

Euler equation in one-dimensional form is

$$a_x = -\frac{1}{\rho}\frac{\partial p}{\partial x} \tag{4}$$

where the acceleration is $a_x = \frac{d\upsilon}{dt} = \frac{\partial \upsilon}{\partial t} + \frac{\partial \upsilon}{\partial x}\frac{\partial x}{\partial t} = \frac{\partial \upsilon}{\partial t} + \frac{\partial \upsilon}{\partial x}\upsilon$. Because of very small velocity changes on coordinate x *in comparison with velocity changes on time* t *it is possible to write Euler equation in form*

$$\frac{\partial \upsilon}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} \tag{5}$$

Second equation is derived from mass momentum, i.e. continuity equation applied on pipe flow

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho \upsilon)}{\partial x} = 0 \tag{6}$$

It is supposed, that density depends only on pressure changes and using elasticity modulus the decrease of volume caused by compressing fulfils the equation

$$K = -V\frac{dp}{dV}$$

By the flow compressing the mass is constant, $m = \rho V =$ const. By differencing it follows $\rho.dV + V.d\rho = 0 \Rightarrow \frac{dV}{V} = -\frac{d\rho}{\rho}$. Elasticity modulus of fluid and derived density are:

$$K = \rho\frac{dp}{d\rho} \Rightarrow d\rho = \frac{\rho}{K}dp \text{ respectively } \frac{\partial \rho}{\partial t} = \frac{\rho}{K}\frac{\partial p}{\partial t}$$

Using modification of equation (6) and $\frac{\partial(\rho \upsilon)}{\partial x} = \rho\frac{\partial \upsilon}{\partial x}$ is

$$\frac{\rho}{K}\frac{\partial p}{\partial t}+\rho\frac{\partial \upsilon}{\partial x}=0 \quad\Rightarrow\quad \frac{1}{a^2}\frac{\partial p}{\partial t}+\rho\frac{\partial \upsilon}{\partial x}=0 \tag{7}$$

It is evident, that in this equation the sound velocity is included.

Continuity and Euler equations constitute the system of partial differential equations of first order:

$$\begin{aligned}\frac{\partial \upsilon}{\partial t}&=-\frac{1}{\rho}\frac{\partial p}{\partial x}\\ \frac{\partial \upsilon}{\partial x}&=\frac{1}{\rho a^2}\frac{\partial p}{\partial t}\end{aligned} \tag{8}$$

The results of this system there are two functions, $\upsilon=\upsilon(x,t)$ $\quad p=p(x,t)$

Using the cross differencing the system of simultaneous differential equations of first order can be transfer to typical wave equations for unknown variables pressure and velocity

$$\frac{\partial^2 \upsilon}{\partial x \partial t}+\frac{1}{\rho a^2}\frac{\partial^2 p}{\partial t^2}=0$$

$$\frac{\partial^2 \upsilon}{\partial t \partial x}+\frac{1}{\rho}\frac{\partial^2 p}{\partial x^2}=0$$

By comparison the same terms in previous equations it gives

$$\frac{\partial^2 p}{\partial t^2}=a^2\frac{\partial^2 p}{\partial x^2} \quad \text{or} \quad \frac{\partial^2 \upsilon}{\partial t^2}=a^2\frac{\partial^2 \upsilon}{\partial x^2} \tag{9}$$

Last two equations are the partial differential equations of second order designed as wave ones. Their solution depends on boundary and initial conditions, i.e. on time velocity changes, e.g. in place near the valve and near the reservoir.

2.3 Method of characteristics

This method is the numerical method solving the wave or hyperbolic equations. It follows from momentum and continuity equation [5], [6]:

$$\begin{aligned}\frac{\partial \upsilon}{\partial t}&=-\frac{1}{\rho}\frac{\partial p}{\partial x}\\ \frac{\partial \upsilon}{\partial x}&=\frac{1}{\rho a^2}\frac{\partial p}{\partial t}\end{aligned} \tag{10}$$

It is possible to determine two real characteristic straight line of the system. These lines are connected physically and mathematically with moving coordinate system and define the dependence of time and space coordinates in form

$$x \pm at = const \tag{11}$$

So we can set the derivations $dx \pm adt = 0 \;\Rightarrow\; dt=\mp\frac{1}{a}dx$ and include it into the momentum equation

$$\frac{1}{\rho}\frac{dp}{dx}\pm a\frac{d\upsilon}{dx}=0 \quad\Rightarrow\quad dp=\pm\rho a d\upsilon \tag{12}$$

The continuity equation is satisfied identically. Using this step (moving along the characteristic lines, see Fig 3) the equation (10), see [1], is changed to the simpler ordinary differential equation and can be solved by method of finite difference.

Suppose the friction influence along the pipe, the equation must include the resistance terms. Into the equation (8) the term respecting the hydraulic losses will be inserted

$$\frac{\partial v}{\partial t}+\frac{2\lambda}{d}v|v|+\frac{1}{\rho}\frac{\partial p}{\partial x}=0 \tag{13}$$

where λ

$$\frac{\partial v}{\partial x}+\frac{}{\rho a}\frac{\partial p}{\partial t}= \tag{14}$$

The system (13) and (14) is solved again by the same approach of method of characteristics.

This method is basically difference method, only the grid is not typically rectangular, but it is modified by characteristic lines, see Fig. 3.

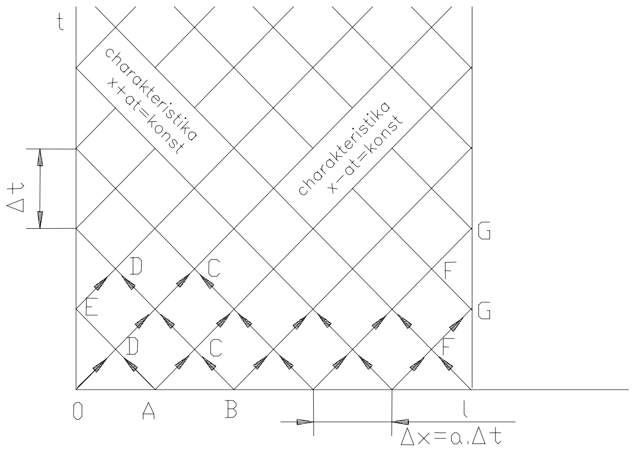


Fig. 3 Grid with characteristic lines.

2.4 Navier-Stokes equation of the unsteady compressible fluid flow

The basic equations of unsteady compressible fluid flow in general region are derived from following assumptions [10]

- equilibrium of inertial, pressure and viscous forces on element of viscous compressible fluid $d\vec{F}_p + d\vec{F}_t = d\vec{F}_s$ (Navier-Stokes equation)
- equilibrium of mass $\frac{d}{dt}\Delta m = 0$ (continuity equation)

Navier-Stokes equation is given in the form:

$$\begin{aligned}
&\frac{\partial(\rho u)}{\partial t}+\frac{\partial(\rho uu)}{\partial x}+\frac{\partial(\rho uv)}{\partial y}+\frac{\partial(\rho uw)}{\partial z}=-\frac{\partial p}{\partial x}+\eta\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)\\
&\frac{\partial(\rho v)}{\partial t}+\frac{\partial(\rho vu)}{\partial x}+\frac{\partial(\rho vv)}{\partial y}+\frac{\partial(\rho vw)}{\partial z}=-\frac{\partial p}{\partial y}+\eta\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)\\
&\frac{\partial(\rho w)}{\partial t}+\frac{\partial(\rho wu)}{\partial x}+\frac{\partial(\rho wv)}{\partial y}+\frac{\partial(\rho ww)}{\partial z}=-\frac{\partial p}{\partial z}+\eta\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)
\end{aligned} \tag{15}$$

and continuity equation is

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0 \tag{16}$$

The density depends on pressure changes using elasticity modulus of fluid as above. The system (15) and (16) generally describes the fluid flow and is useful in many engineering applications. It must be supplemented by boundary and initial conditions and than can be solved using method of finite volume.

3 Evaluation of experimental measurement

At the Fig. 4 it is shown the hydraulic circuit, where it is possible to measure the transient characteristics of hydraulic elements [2], [3], [4]. The hydraulic long pipe is part and parcel of circuit and it allows generate and measure so called hydraulic shock waves.

Fig. 4 View on the measured device

3.1 Scheme and description of hydraulic circuit

Control hydrogenerator HG (PPAR 2-63 10 AP, TOS Vrchlabí) supplies constant flow *Q*. Mineral oil flows through the pipe T, which is coiled in to the spiral with average diameter 1,831 m, continues through distributor R in to the tank N. By step closing of flow *Q* by distributor R the pressure response is invoked. Accumulator A is prepared for dumping the hydraulic shock. After closing the distributor R liquid flows through relief valve RV (ATOS ARAM-20/350) to the tank N. For measuring the universal measuring equipment M 5000 and 5050 Hydrotechnik is used.

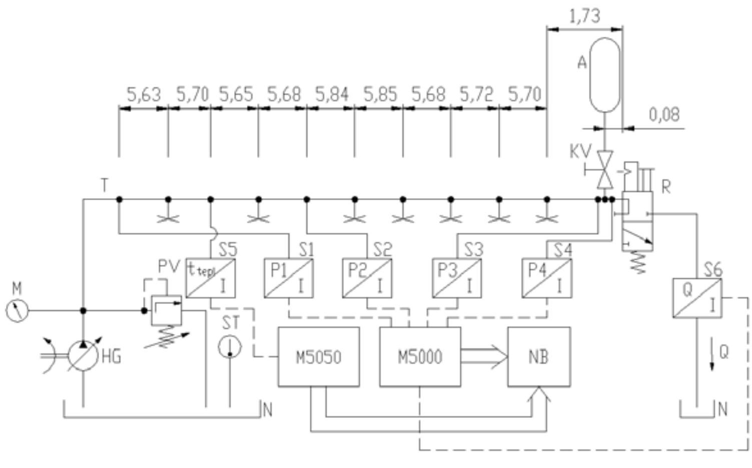


Fig. 5 Scheme of hydraulic circuit (the lengths are in m)

Specification of used hydraulic elements and devices

- control hydrogenerator HG: PPAR 2-63 10 AP, TOS Vrchlabí
- relief valve PV: ATOS, ARAM-20/350
- manometer M
- distributor R, closing time cca 0,003 s
- manometers S1,S2: PR 15 Hydrotechnik, (0 - 200) bar
- manometers S3,S4: PR 15 Hydrotechnik, (0 - 600) bar
- flowmeter S6
- temperature sensor S5
- system M5000, 5050: universal measuring equipment Hydrotechnik
- notebook NB
- long pipe T: diameter d = 12 mm, wall s = 2 mm, length l = 59,18 m, elasticity modulus E = 2,1.10 11 Pa
- physical properties of oil OH-HM-46: density $\rho = 870$

  $\nu =$

3.1.1 Measurement technique:

Using control hydrogenerator HG the value of oil flow rate is set. The steady flow rate flows through the pipe to the tank N. The physical properties of the oil are defined above. The pressure is measured in ten positions placed along the pipe. After closing the flow rate Q using distributor the oil flows through relief valve into the tank N. Dynamic curves of pressures are measured and evaluated using universal device Multi-system 5000 and 5050. Measuring plugs Minimess can help in position changes of the manometers without fluid leakage. Measured values are transformed into computer and can be evaluated using software HYDROcomsys/win or can be converted into *.*txt* file and Excel [7].

4 Results

4.1 Measurement evaluation and calculation of basic parameters of hydraulic shock

On the experimental circuit the parameters of the flow were set up and the measured values were written. These values defined the boundary conditions for numerical simulation.

- steady state, the valve is open: flow rate before closing was $Q = 15$ $\mathrm{l.min^{-1}}$
- steady state, the valve is closed: pressure on the relief valve was set to $p_{PV} = 67$ bar
- fluid temperature was through the measuring constant $T = 26$ °C.

At Fig. 6 it is shown one variant of measurement. The curves on the left and right side represent the steady states and in the middle it is obvious hydraulic shock wave, where we can read the period, then compute sound velocity and observe dumping.

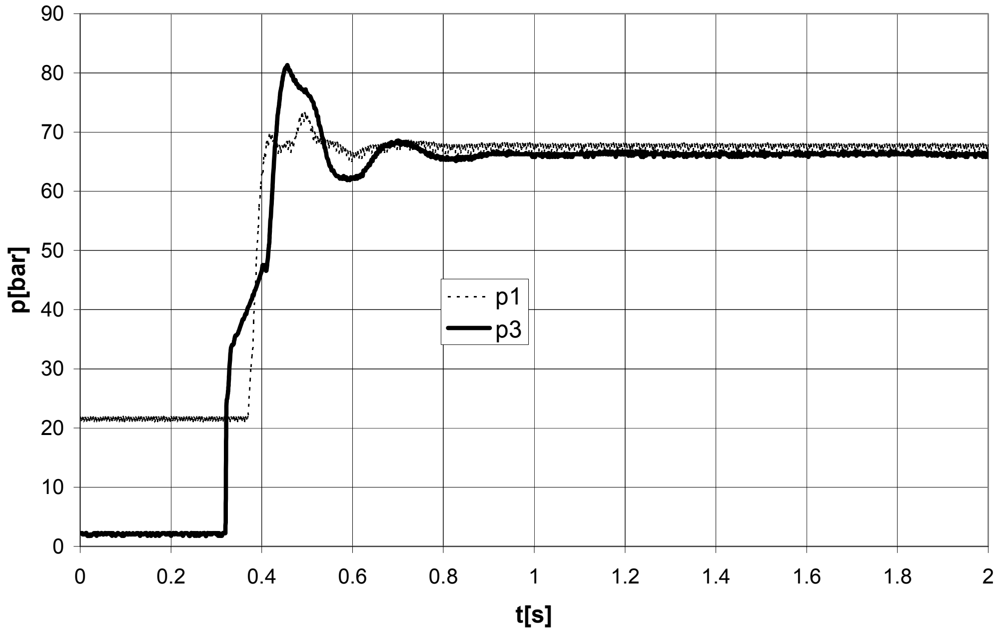


Fig. 6 Measured values of pressure on manometers P1 a P2.

- period $T_{per} = 0{,}246 \text{ s}$
- time of wave running $T_{BV} = \frac{T_{per}}{2} = 0{,}123 \text{ s}$
- sound velocity in the fluid $a = \frac{2 \cdot l}{T_{BV}} = 962 \text{ m.s}^{-1}$
- own frequency $f = \frac{1}{T_{per}} = 4{,}07 \text{ Hz}$
- elasticity modules of fluid $K = a^2 \rho = 0{,}8.10^9 \text{ Pa}$
- velocity $\upsilon = 2.21 \text{ m.s}^{-1}$
- Reynolds number $\mathrm{Re} = \frac{\upsilon d}{\nu} = 348$
- friction coefficient $\lambda = \frac{64}{\mathrm{Re}}$
- closing time $t_{uz} = 0{,}02 \text{ s}$

Because of the value of Reynolds number the **laminar flow** is supposed. It is very important information due to choice of the mathematical models of the fluid flow.

The increase of pressure using simplest Žukovskij equation is

$$\Delta p = \rho \cdot a \cdot \upsilon$$
$$\Delta p = 870 \cdot 962 \cdot 2{,}21$$
$$\Delta p = 1{,}85 \; MPa$$

4.2 Computation using method of characteristics (Flowmaster)

Program grid model (see Fig. 7) is supposed to correspond to the real circuit, where the small changes for more accurate solution are made [4], [8].

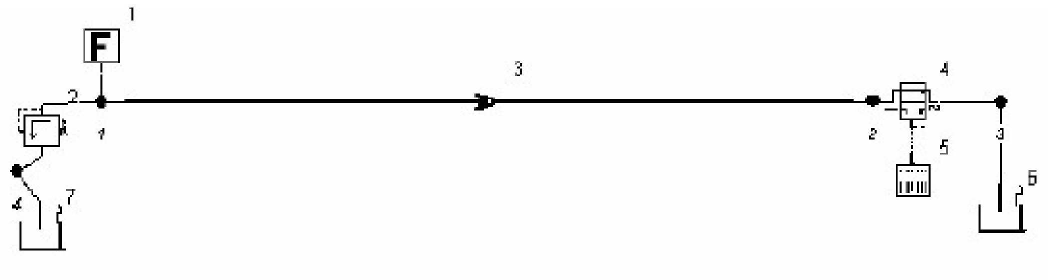


Fig. 7 Scheme of the circuit grid using program Flowmaster

- component n .1 – hydrogenerator used as flow source
- component n .2 – relief valve, after finishing of the transient state it is set to defined pressure
- component n .3 – pipe: length 59,18 m , diameter 12 mm. In the pipe the pressure response on the step input signal is measured.
- component n .4 – distributor
- component n .5 – distributor controlling
- component n .6 and 7 – tank

4.3 Computation using finite volume method (Fluent)

The fluid flow was solved in relatively simple region, i.e. in the pipe, where the axi-symetry can be supposed [9]. Therefore the case was solved as two-dimensional axi-symetric one. The grid was defined using rectangular elements. Physical properties were set by physical experiment, the dependence of the density on the pressure was tested using any variants, so that the form $d\rho = \frac{\rho}{K} dp$ was approximated:

- using UDF
- multiphase model
- approximation using equation of ideal gas

Boundary conditions were defined by step closing the valve at the pipe end, i.e. the flow rate or velocity changes from the steady value to zero and at the pipe inlet the pressure changes from the steady value to the steady value after closing the valve. The flow was time dependent.

At Fig. 8 the pressure vs. time dependence measured and calculated using method characteristics (Flowmaster) and finite volume methods (Fluent) are described. Žukovskij calculated value is overpredict (180 bar approximately).

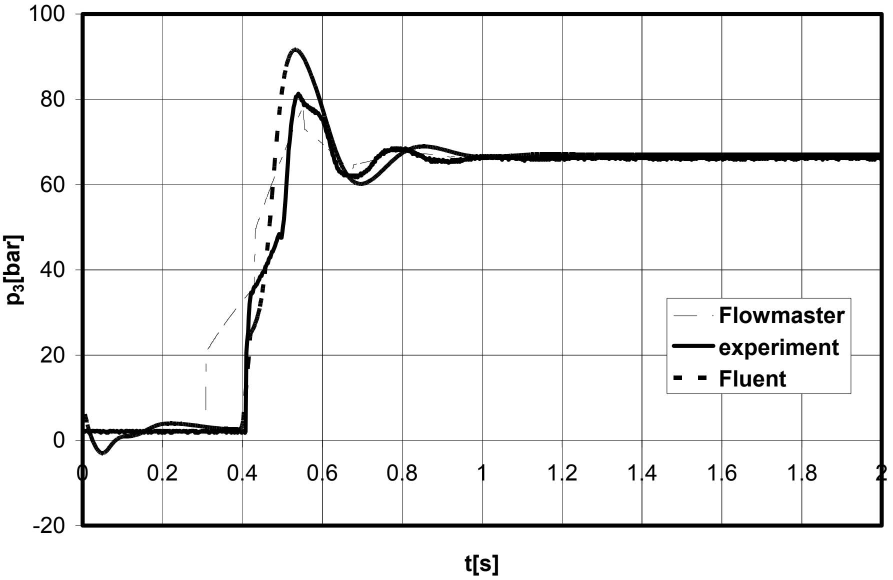


Fig. 8 Comparison of the measured and calculated pressures in front of valve

5 Conclusion

Carried out numerical simulation of the hydraulic shock using the wave equations (method of characteristics) and Navier-Stokes equations (method of finite volume) established with good accuracy the congruity of numerically and physically gained results. All approaches depend on elasticity modulus (so on air content in the oil), defined from physical experiment. Experiment was used for fluid physical properties and boundary conditions determination. Software Flowmaster is good applicable on the flow of compressible liquid. Program Fluent is more general, it is developed into the gas fluid flow including shock waves modeling, in case of liquid flow the compressibility is not supported. Therefore the approximation of density depending on pressure changes using elasticity modulus was created. After it software Fluent gave similar results. Good agreement of numerical and physical results in case of hydraulic shock is a presumption for using the same approach in the general geometry flow, when the influence of liquid flow compressibility is supposed.

Reviewer: Doc. Ing. Drábková Sylva, PhD.

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