

Petr FERFECKI^{*}, Jan ONDROUCH^{}, Tomáš LUKAS^{***}**

INVESTIGATION OF VIBRATION OF A ROTOR SYSTEM SUPPORTED BY ABSOLUTELY RIGID BEARINGS WITH A SHAFT CONTAINING A NOTCH

VYŠETŘOVÁNÍ KMITAVÉHO POHYBU ROTOROVÉ SOUSTAVY ULOŽENÉ V ABSOLUTNĚ TUHÝCH LOŽISKÁCH S HŘÍDELEM ZESLABENÝM VRUBEM

Abstrakt

Předložený příspěvek zkoumá dynamické vlastnosti rotorové soustavy s hřídelem zeslabeným vrubem, který se nachází uprostřed meziložiskové vzdálenosti. Na základě analytických vztahů odvozených pro rotorové soustavy s neokružným hřídelem je vypočítána amplitudová charakteristika, tvary orbit středu kotouče pro různé hodnoty otáček a Fourierova transformace časového průběhu posunutí středu kotouče.

Abstract

The presented work investigates dynamics of a rotor system with shaft weakened by a notch, which is situated in the midspan. On the basis of analytic relations derived for rotor systems with non-circular cross-section of the shaft the unbalance vibration response, shapes of orbits for different values of speed and the Fourier transformation of time domain response of the centre of the disc are computed.

1 Introduction

Rotor systems with the shaft weakened by a notch is possible to investigate on the basis of theory for rotors with a non-circular cross-section of the shaft [1], because the stiffness of the shaft with a notch can be formulated in two perpendicular directions. The investigated problem is very similar to rotor systems with the shaft containing a fatigue crack, see [2]. It is known, that the crack can open and close during one shaft rotation or it can be opened or closed, see [2] and [3]. The influence of an open crack and a non-circular cross-section of the shaft on dynamical behaviour of a rotor are qualitatively the same, because in both cases the stiffness of the shaft changes periodically during rotation.

2 Equations of motion of a rotor system

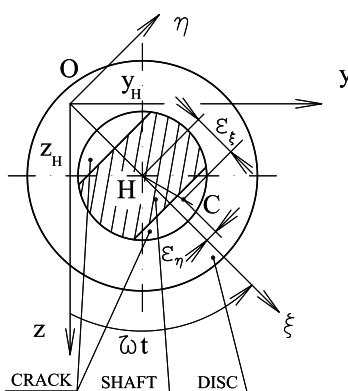


Fig. 1 Frame of reference

The model rotor system has the following properties:

- (i) the mass of disc is concentrated into its centre, (ii) the shaft is assumed to be massless and linearly flexible, (iii) gyroscopic effects of the shaft with disc are neglected, (iv) bearings are absolutely rigid, (v) a notch in the shaft is located in the midspan, (vi) rotor is loaded by a gravitational force and a centrifugal force due to disc unbalance and (vii) the rotor rotates at constant angular speed.

Equations of motion of lateral vibration of a rotor system in rotor-fixed co-ordinate system have the following form, see [1]

$$\begin{aligned} \ddot{\xi} + 2\delta(\dot{\xi} - \omega\eta) - 2\omega\dot{\eta} + (\omega_{\xi}^2 - \omega^2)\xi &= \varepsilon_{\xi}\omega^2 + g\cos(\omega t), \\ \ddot{\eta} + 2\delta(\dot{\eta} + \omega\xi) + 2\omega\dot{\xi} + (\omega_{\eta}^2 - \omega^2)\eta &= \varepsilon_{\eta}\omega^2 - g\sin(\omega t) \end{aligned} \quad (1)$$

^{*} Petr FERFECKI, CPIT, CISS – IT ASCR, branch Ostrava, VSB – TU of Ostrava

^{**} Jan ONDROUCH, CISS – IT ASCR, branch Ostrava, VSB – TU of Ostrava

^{***} Tomáš LUKAS, Department of Mechanics, VSB – TU of Ostrava

and $\omega_\zeta^2 = k_\zeta/m$, $\omega_\eta^2 = k_\eta/m$, $\delta = b/2m$, where ζ and η are displacements of the centre of disc in rotor fixed co-ordinate system (see Fig. 1), $\dot{\xi} = d\xi/dt$ is time derivative of ζ , m is the weight of disc, b is damping coefficient, k_ζ, k_η are cross stiffness coefficients of the shaft at a location of the disc in rotor fixed co-ordinate system, ω is angular speed of the shaft, $\varepsilon_\zeta, \varepsilon_\eta$ are values of unbalance of the disc in ζ, η directions and g is gravitational acceleration. The solution of equations of motion is obtained by means of a superposition principle, because system of equations of motion is a system of linear differential equations.

2.1 Natural oscillations of a rotor system

The general solution is obtained by solving a homogeneous set of equations (1), which is corresponding to natural oscillations of a rotor system. The solution has the following form, see [1]

$$\xi = A e^{\lambda t}, \quad \eta = B e^{\lambda t}, \quad (2)$$

where A, B are integration constants, λ is base of natural logarithms. An area of angular velocities, in which the motion of a rotor system is stable, is obtained by means of Hurwitz criterion.

2.2 The influence of eccentricity of the centre of gravity

Multiplying the second equation in (1) by i (imaginary unit) and adding to the first equation in (1) yields

$$\ddot{\zeta} + 2(\delta + i\omega)\dot{\zeta} + \left(\frac{\omega_\zeta^2 + \omega_\eta^2}{2} - \omega^2 + 2i\omega\delta \right) \zeta + \frac{\omega_\zeta^2 - \omega_\eta^2}{2} \bar{\zeta} = \omega^2 (\varepsilon_\zeta + i\varepsilon_\eta) + g e^{-i\omega t}, \quad (3)$$

where $\zeta = \xi + i\eta$ and $\bar{\zeta} = \xi - i\eta$. The form of particular solution for $g=0$ is assumed to be $\zeta_e = A + iB$ and $\bar{\zeta}_e = A - iB$, where constants A, B are as follows (see [1])

$$A = \omega^2 \frac{(\omega_\eta^2 - \omega^2)\varepsilon_\zeta + 2\delta\omega\varepsilon_\eta}{(\omega_\zeta^2 - \omega^2)(\omega_\eta^2 - \omega^2) + 4\delta^2\omega^2}, \quad B = \omega^2 \frac{(\omega_\zeta^2 - \omega^2)\varepsilon_\eta - 2\delta\omega\varepsilon_\zeta}{(\omega_\zeta^2 - \omega^2)(\omega_\eta^2 - \omega^2) + 4\delta^2\omega^2}. \quad (4)$$

The steady state motion of the centre of disc due to unbalance is described by the following equations

$$(y_H)_e = R_\varepsilon \sin(\omega t - \varphi), \quad (z_H)_e = \varepsilon \sin(\omega t - \varphi), \quad (5)$$

where $R_\varepsilon = \sqrt{A^2 + B^2}$, $\tan(\varphi) = \frac{B}{A} = \frac{\varepsilon_\eta (\omega_\zeta^2 - \omega^2)}{\varepsilon_\zeta (\omega_\eta^2 - \omega^2)}$ and y_H, z_H are displacements of the centre of a disc in stationary co-ordinate system (Fig. 1).

2.3 The influence of the weight of disc

The particular solution of equation (3) for $\varepsilon_\zeta = \varepsilon_\eta = 0$ is assumed to have the following form

$$\zeta_g = U e^{i\omega t} + V e^{-i\omega t} \quad \text{and} \quad \bar{\zeta}_g = \bar{U} e^{-i\omega t} + \bar{V} e^{i\omega t}, \quad (6)$$

where U, V are complex integration constants and \bar{U}, \bar{V} are their complex conjugates. U, V have the following form, see [1]

$$U = U_1 + iU_2, \quad V = V_1 + iV_2, \quad (7)$$

where

$$U_1 = -\frac{1}{2} \frac{g}{\Delta} (\omega_\xi^2 - \omega_\eta^2) [\omega_\xi^2 \omega_\eta^2 - 2\omega^2 (\omega_\xi^2 + \omega_\eta^2)], \quad U_2 = \frac{g \delta \omega}{\Delta} (\omega_\xi^2 - \omega_\eta^2) (\omega_\xi^2 + \omega_\eta^2),$$

$$V_1 = \frac{g}{\Delta} \left\{ \left[\frac{1}{2} (\omega_\xi^2 + \omega_\eta^2) - 4\omega^2 \right] [\omega_\xi^2 \omega_\eta^2 - 2\omega^2 (\omega_\xi^2 + \omega_\eta^2)] + 8\delta^2 \omega^2 (\omega_\xi^2 + \omega_\eta^2) \right\},$$

$$V_2 = \frac{2g \delta \omega}{\Delta} \left\{ \left[\frac{1}{2} (\omega_\xi^2 + \omega_\eta^2) - 4\omega^2 \right] (\omega_\xi^2 + \omega_\eta^2) - 2 [\omega_\xi^2 \omega_\eta^2 - 2\omega^2 (\omega_\xi^2 + \omega_\eta^2)] \right\},$$

$$\Delta = [\omega_\xi^2 \omega_\eta^2 - 2\omega^2 (\omega_\xi^2 + \omega_\eta^2)]^2 + 4\delta^2 \omega^2 (\omega_\xi^2 + \omega_\eta^2)^2.$$

The steady state motion of the centre of disc due to weight of disc is described by the following equations

$$(y_H)_g = U_1 \sin(2\omega t) + U_2 \cos(2\omega t) + V_2, \quad (8)$$

3 Analysis of a resulting motion of a rotor system

The centre of a disc is moving in stationary co-ordinate system on a circle with radius R_ϵ at angular velocity ω (the influence of disc unbalance) and on a circle with radius $|U|$ at angular velocity 2ω (the influence of the weight of disc)

$$y_H = R_\epsilon \sin(\omega t - \varphi) + U_1 \sin(2\omega t) + U_2 \cos(2\omega t) + V_2,$$

$$z_H = R_\epsilon \cos(\omega t - \varphi) + U_1 \cos(2\omega t) - U_2 \sin(2\omega t) + V_1. \quad (9)$$

The shaft with non-circular cross-section has an area of unstable speed in range $(\omega_\xi, \omega_\eta)$. Because of disc unbalance the values ω_ξ and ω_η are resonant frequencies of the first order. Resonant frequency of the second order ω_S exists because of the weight of a disc.

4 Numerical experiments with Rotor Kit 4 (RK 4)

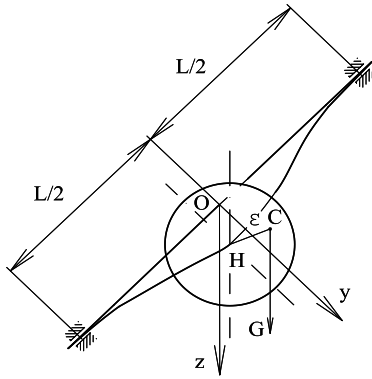


Fig. 2 Scheme of the rotor system RK 4

Introduced computational procedure was built up in computer system MATLAB and tested by numerical experiments on Rotor Kit 4 with following parameters: weight of disc $m = 0.8 \text{ kg}$, damping coefficient $b = 4 \text{ kg s}^{-1}$, cross stiffness coefficients of the shaft $k_\xi = 7.230 \cdot 10^5 \text{ N m}^{-1}$ and $k_\eta = 6.196 \cdot 10^5 \text{ N m}^{-1}$, length of the shaft $L = 0.4 \text{ m}$, notch is located in the midspan, shaft diameter $D = 0.02 \text{ m}$, disc unbalance $\epsilon_\xi = 0.1 \cdot 10^{-3} \text{ m}$, $\epsilon_\eta = 0 \text{ m}$ and acceleration of gravity $g = 9.81 \text{ m s}^{-2}$.

The unstable response of a rotor system RK 4 has been computed by means of Runge-Kutta method of the 4th order.

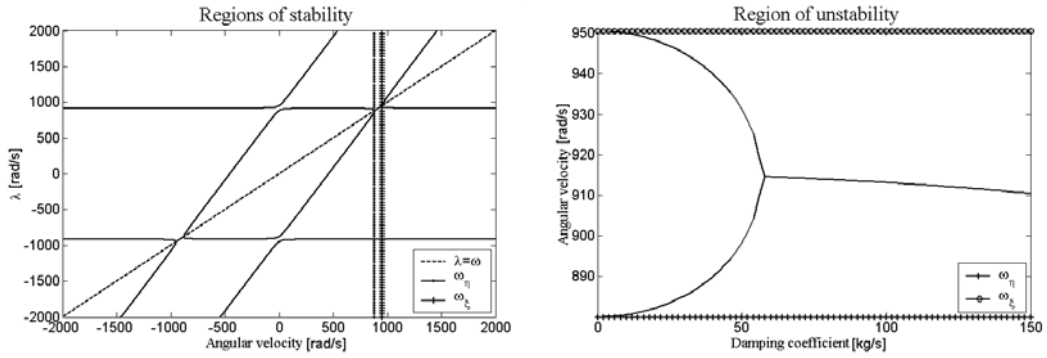


Fig. 3: Stability range of motion (left) and change of the width of an unstable speed range with damping coefficient (right)

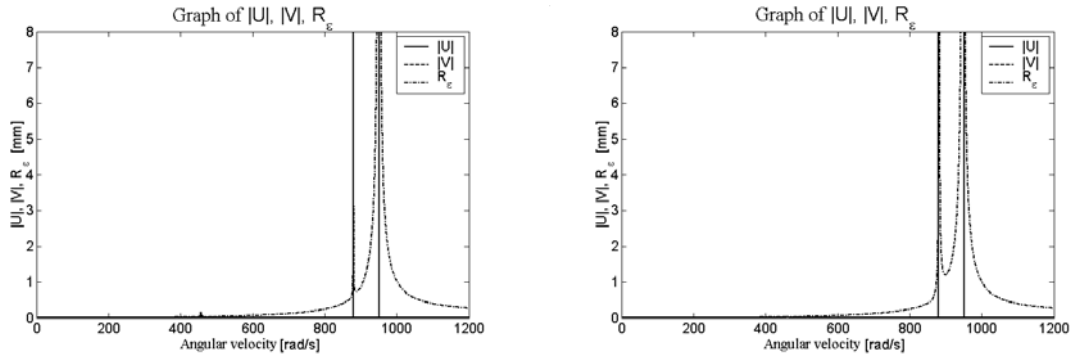


Fig. 4: Graph of $|U|, |V|, |R_c|$ for damping coefficient $b = 4 \text{ kg s}^{-1}$ (left) and $b = 20 \text{ kg s}^{-1}$ (right)

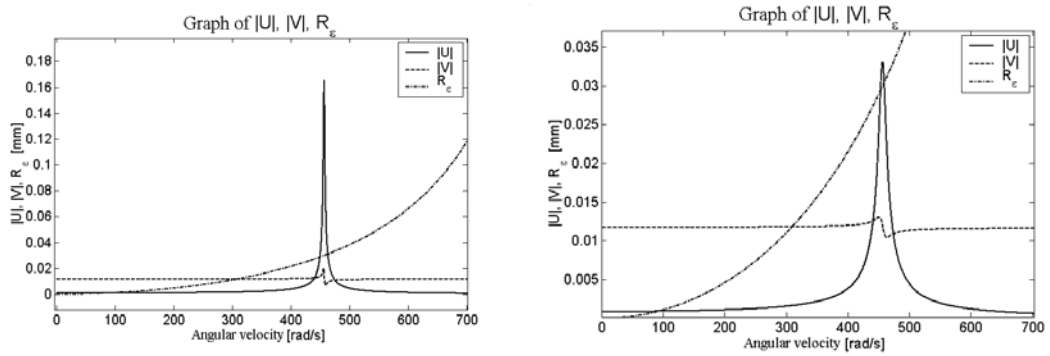


Fig. 5: Detail of lower values of angular velocities from graph of $|U|, |V|, |R_c|$ (Fig. 4) for damping coefficient $b = 4 \text{ kg s}^{-1}$ (left) and $b = 20 \text{ kg s}^{-1}$ (right)

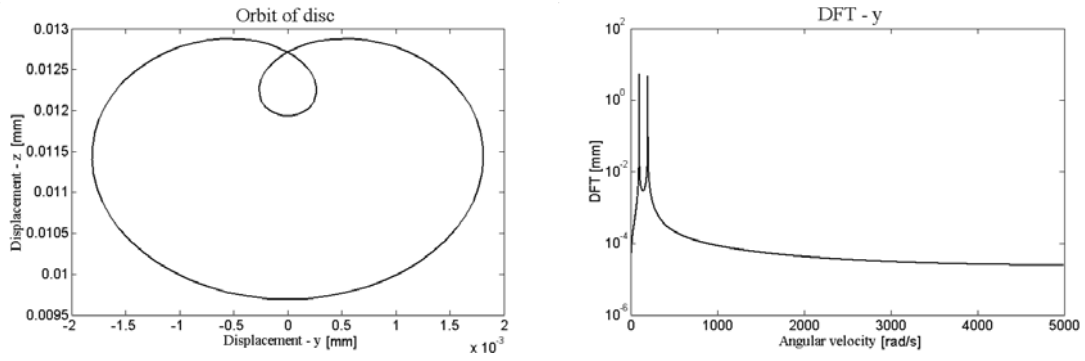


Fig. 6: Orbit plot of the centre of the disc at angular speed 100 rad s^{-1} (left) and Fourier transformation of time domain response in horizontal direction (right)

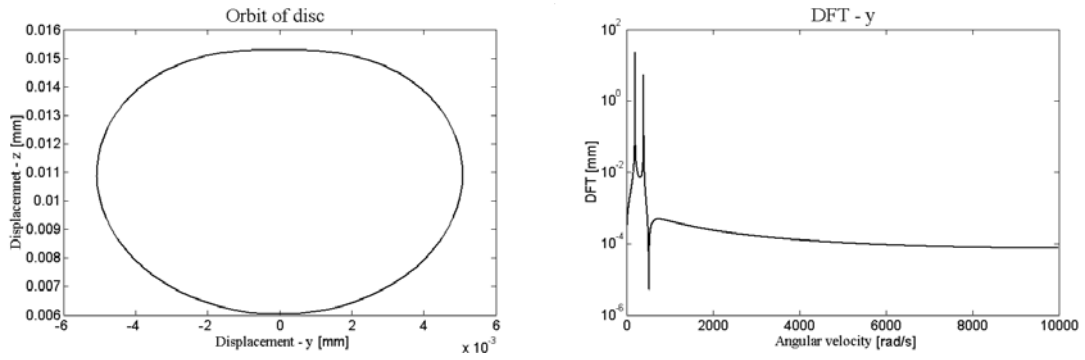


Fig. 7: Orbit plot of the centre of the disc at angular speed 200 rad s^{-1} (left) and Fourier transformation of time domain response in horizontal direction (right)

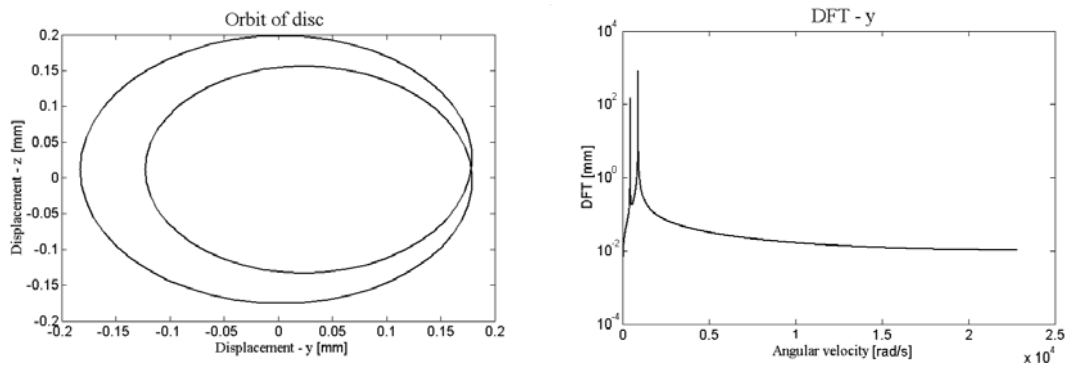


Fig. 8: Orbit plot of the centre of the disc at angular speed 456.6 rad s^{-1} in a region of the second resonance (left) and Fourier transformation of time domain response in horizontal direction (right)

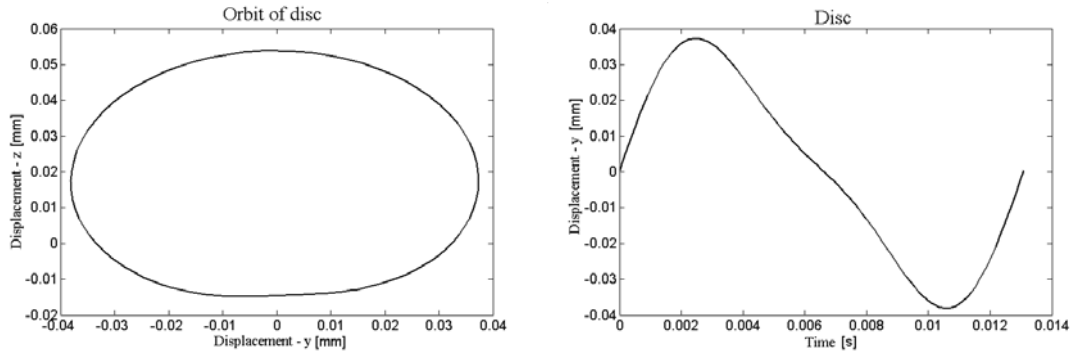


Fig. 9: Orbit plot of the centre of the disc at angular speed 480 rad s^{-1} (left) and time domain response in horizontal direction (right)

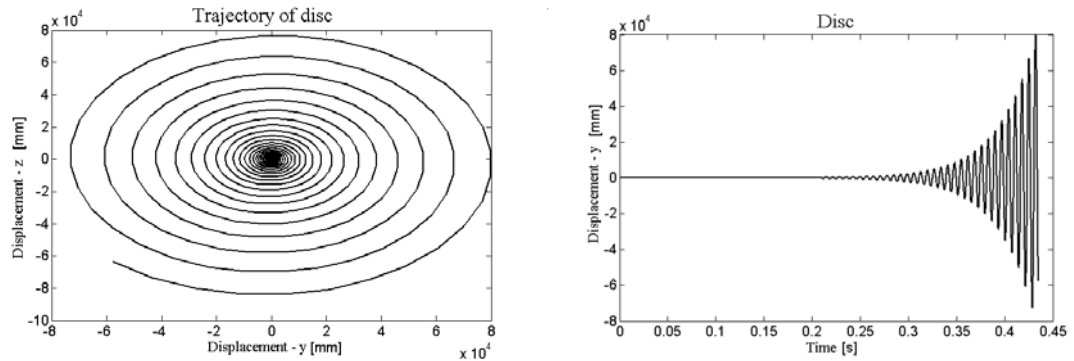


Fig. 10: Trajectory of the centre of the disc at angular speed 895 rad s^{-1} , which lies in the unstable speed range (left) and time domain response of a disc in horizontal direction (right)

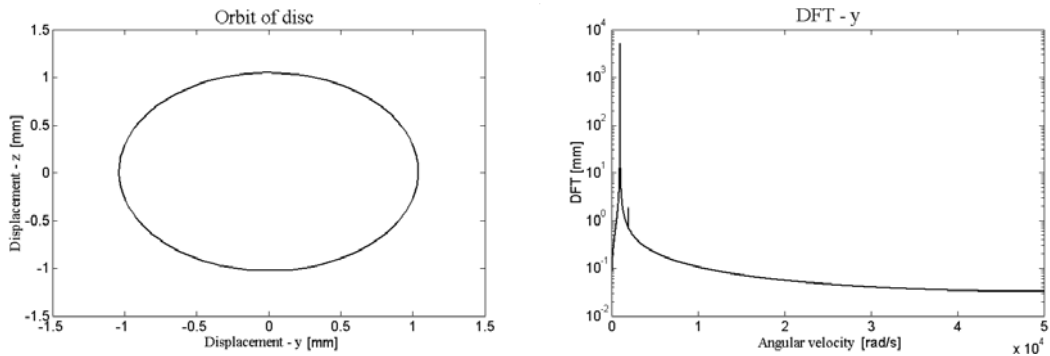


Fig. 11: Orbit plot of the centre of the disc at angular speed 1000 rad s^{-1} (left) and Fourier transformation of time domain response in horizontal direction (right)

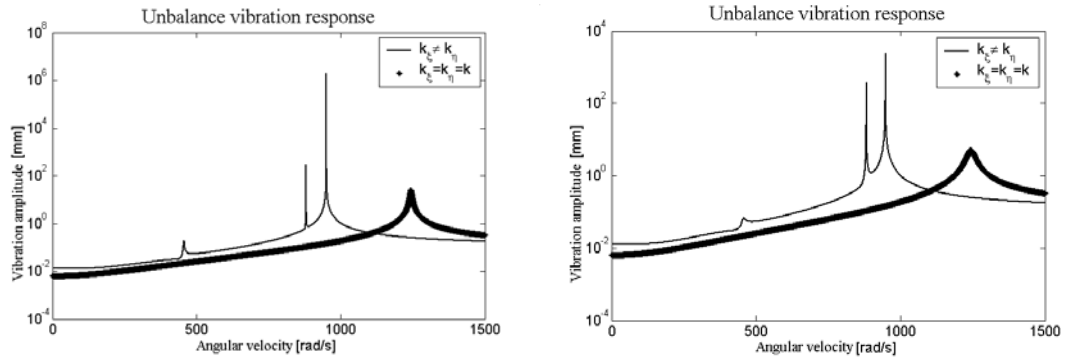


Fig. 12: Unbalance vibration response for damping coefficient $b=4 \text{ kg s}^{-1}$ (left) and $b=20 \text{ kg s}^{-1}$ (right)

The dependence of roots of characteristic equation $\lambda_{1,2,3,4}$ on angular speed ω is shown on Fig. 3 (left). The range of unstable angular velocities ($\omega_\zeta, \omega_\eta$) is located between two intersections of synchronous excitation with roots of characteristic equation. This range is decreasing with increasing value of damping coefficient b , see Fig. 3 (right). The influence of a notch on dynamical behaviour of a rotor system can be expected up to angular velocity slightly crossing the value of the second resonance, see graph of $|U|, |V|, |R_\zeta|$ (Fig. 4 and 5). The complicated shape of orbit plot (Fig. 6-9 left) clearly indicate the presence of a notch in the shaft. Another characteristic sign of the presence of a notch in the shaft is second harmonic component (Fig. 6-8 right) in the frequency spectra. Displacements of a rotor system are increasing ad infinitum (Fig. 10) for chosen unstable angular velocity 895 rad s^{-1} , which lies at the edge of the range of unstable angular velocities ($\omega_\zeta, \omega_\eta$). The range of resonant frequencies of a rotor system with a notch lies under the range of resonant frequencies of a rotor system without a notch, see unbalance vibration response (Fig. 12).

5 Conclusions

From numerical experiments it can be concluded, that the presence of a notch or an open crack in a shaft can be detected from the shape of orbit plot and the presence of second harmonic component in frequency spectra. This detection of the presence of a notch (open crack) in a shaft is applicable for angular velocities up to the value corresponding to the second resonant frequency (Fig. 4-11).

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6 References

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Reviewer: prof. Ing. Petr Horyl, CSc., Department of Mechanics, VSB – TU of Ostrava

