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FEATURE ANALYSIS OF SELFTUNING PI LEVEL CONTROL IN A CASCADE OF TANKS

ANALÝZA VLASTNOSTÍ PI REGULÁTORŮ S AUTOTUNINGEM PŘI ŘÍZENÍ KASKÁDY NÁDRŽÍ

Abstract

When testing controller features with the use of simulation modelling, it is important to establish conditions for controller operation that are as close as possible to reality. In the paper it is presented a model of a controlled plant that takes into account not only the existing non-linear character of the physical phenomena in the steady-state relations between the inputs and the outputs but also other phenomena such as saturation, accumulation stopping, technological limits, etc. Such types of models are suitable tools in controller testing, especially when the ability to master various operating conditions is being examined. In the presented case, we wish to evaluate the selftuning capabilities of the PI controller with parameter adaptation via the continuous gradient method. Instead of the usual methodology, where some parameter changes in a transfer function representation of the controlled object are proposed by the experimenters themselves with no correspondence to real causes, in the presented non-linear model of a two-tank cascade changes can be made in the physical and operating conditions with all the real consequences.

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Abstrakt

Při testování vlastností regulátorů, které je založeno na simulaci pomocí modelu, je důležité nastavit podmínky řízení co nejvěrohodněji. V příspěvku je představen model zařízení, který respektuje jak nelineární charakter fyzikálních vztahů mezi akčními a řízenými veličinami, tak i další jevy jako saturace, technologické meze atd. Takovéto modely jsou vhodnými nástroji pro testování regulátorů. V uvedeném případě jsou ověřovány možnosti samonastavení PI regulátoru s adaptací parametrů založenou na spojité gradientové metodě. Místo obvykle užívané metody, kdy jsou měněny některé parametry přenosu reprezentujícího řízené zařízení bez jakýchkoli souvislostí s reálným provozem, v uváděném nelineárním modelu kaskády dvou nádrží je možné provádět změny parametrů, které reálně ovlivňují i další vlastnosti kaskády podle skutečných souvislostí.

Tato práce byla realizována jako součást zkoumané hybridní koncepce návrhu řídicích algoritmů, která je podporována grantem 101/04/1182 Grantové agentury České republiky.

1 INTRODUCTION

Testing the optimal parameter setting for a controller in control loops by means of simulation is a standard technique. It is not difficult to employ this technique provided that the control loop can be described by linear equations. In this linear case, there are many tools for achieving an optimal

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setting with the use of simulation and computer testing. There are programs, that support such a tuning procedure, including the well-known Matlab/Simulink program package. Some problems may occur if phenomena such as strong non-linear and time-varying properties of the controlled object, the presence of technological limits with possible wind-up effects, etc., start to play a significant role in the operation of real control loops.

These problems mostly arise from properties of the controlled plant whose simulation models must take into account all phenomena that are of principal importance in order to avoid wrong conclusions and results. The commonly used approaches include:

- linear models, that change parameters according to the operating range,
- non-linear models, that take into account the behaviour of the real controlled object as credibly as possible.

Therefore in the following text, we will concentrate on two tasks:

- choosing a model of a plant which is suitable for testing controllers with selftuning features
- verifying the selftuning properties of a PI controller whose parameter adaptation is newly based on the use of an control error reference model and the gradient search for a minimum of the quadratic criterion.

Our interest in controllers with selftuning stems from our experience gained from controlling complex devices such as power plants, and from the expected need for more sophisticated control algorithms for remote control of an airship. The choice of a two-tank cascade for the controlled plant model is explained by its relative simplicity, by the existence of a description in (Šulc, Vítečková, 2004), and finally by some experience of programming tricks from applying antiwind-up precautions and with ways of determining stationary states in the model before simulation begins.

2 CONTROLLED PLANT MODEL

For the reasons mentioned above, a model of a two-tank cascade was chosen. This model enables reliable modelling of the most important physical phenomena, with some variety in the impact on the dynamics in accordance with the known types of transfer functions.

The kinds of influence on the dynamics linked with the need for changes in the controller parameter setting are demonstrated in Fig. 1. The figure shows a high level of non-linear influence on the behaviour of the controlled plant. Both the manipulated variable u_1 and the disturbance value u_3 influence the setting of the operating point. There are areas where small changes in the setting of a valve opening have only a small impact (low sensitivity of the controlled variable to the input changes), and other areas where some small changes in the valve opening have a large impact (high sensitivity of the controlled variable to the input changes).

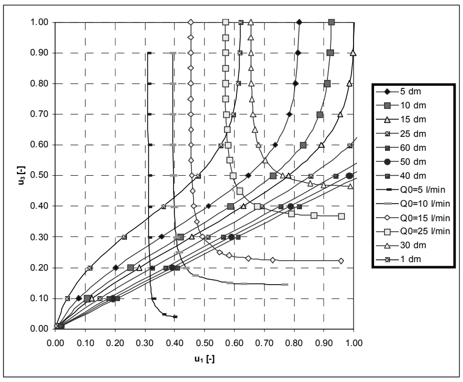

Fig. 1 Steady-state characteristics of a two-tank cascade

3 MODIFICATION OF THE CONTINUOUS GRADIENT METHOD

A PI controller is self-tuned using the continuous gradient method to evaluate the difference between the reference time course of the control error from a reference model and the values of the control error course obtained from the control circuit.

The desired time behaviour of the control error can be expressed in several ways. Two methods have been tested. In the first method, the required behaviour is defined by the transfer function of the so-called standard form. The second tested alternative is based on a transcendent transfer function representing the desired model of behaviour (Vítečková, 1998), (Šulc, Vítečková 2004).

The control loop transfer function of the standard form is characterized by a three-term polynomial in the denominator of the second degree with the damping factor $\xi = \sqrt{2}/2$. Then the time response of the control error can be described by a differential equation of the second order

$$\tau^2 \ddot{e}(t) + \tau\sqrt{2}\dot{e}(t) + e(t) = 0,\ e(t_0) = e_0,\ \dot{e}(t_0) = \dot{e}_0 \tag{1}$$

where $e_0, \dot{e}_0$ means the initial conditions at time instant t_0.

If the desired time behaviour of the control loop is defined on the basis of a transfer function, the Laplace transform of the desired time course in the method of the desired model is

$$E(s) = \frac{a}{s + ae^{-T_d s}} e^{-T_d s} \tag{2}$$

Corresponding to the transform *E(s)* is the following equation, which includes elements with the time delay,

$$\dot{e}(t) + ae(t - T_d) = 0,\ e(t_0) = e_0 \tag{3}$$

where e_0 means the initial condition at time instant t_0. Because of the presence of elements with the time delay, the time behavior of e_0 on the segment of time <0, T_d> must be defined. Two parameters in (3), the gain a and the time delay T_d, are interlinked by the relation $a = (\beta . T_d)^{-1}$, where T_d represents the time delay and β determines the behaviour of the system (Šulc, Vítečková, 2004).

Parameters of T_d and β can be found, such that the responses of the transfer function of the so-called standard form and the transcendent transfer function are identical or very similar. This situation is shown in Fig. 2, where the parameters of the transcendent transfer function are $T_d = 2$ min, and $\beta = 3$. The time constant of the transfer function is $\tau = 5$ min. There is no significant difference between the two responses.

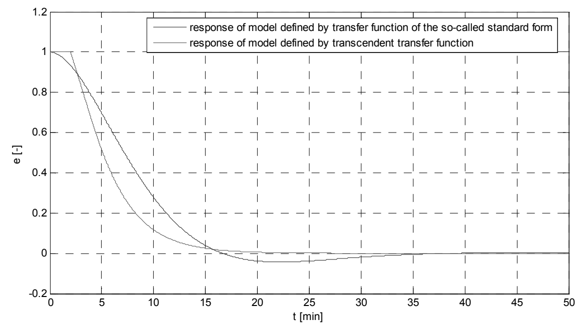

Fig. 2 Comparison of the time behaviour of chosen control error models

The criterion for the optimal controller setting is the minimum of the difference between the desired control error course and the real control error found in the control circuit. The agreement is evaluated by the integral of the squared difference between the two control errors.

$$Q(r_0, r_I, t) = \int_{t_0}^{t} \left(e_{ref}(\vartheta) - e(\vartheta)\right)^2 d\vartheta \tag{4}$$

where:

r_0, r_I - parameters of the controller,

$e(\vartheta)$ - actual value of the real control error,

$e_{ref}(\vartheta)$ - actual value of the desired time course of the control error.

In the ideal case, the value of the criterion should approach zero. In fact, the value will not directly achieve zero, but certain parameters (r_0, r_I) of the controller can be found, for which the value of the criterion will be minimal. For every operating point, i. e. for a certain desired value of the level height and the steady-state flow rate, a value of the criterion can be found for each couple of the controller parameter setting.

There are two problems with the criterion. The first is that, in some cases, a surface of the criterion values without minima can be produced. This means that the optimal values of the controller parameters may be infinite (in reality, the values will reach their maxima). The second problem is the value of the criterion. Depending on how the criterion is evaluated, the minimum of the criterion may be achieved when the integration (coefficient r_I) is very small and a permanent control error can occur.

To find the minimum values of the criterion for selftuning purposes the continuous gradient method has been applied. The controller parameter changes are defined by an equation of their motion derived from (4).

$$\frac{d}{dt}\mathbf{r}(t) = -\mathbf{K}\frac{d}{dt}\frac{d}{d\mathbf{r}}Q(\mathbf{r}(t),t) = -2\mathbf{K}\left(e_{ref}(t) - e(\mathbf{r}(t),t)\right)\frac{d}{d\mathbf{r}}e(\mathbf{r}(t),t) = -\begin{bmatrix} k_I & 0 \\ 0 & k_0 \end{bmatrix}\left(e_{ref}(t) - e(\mathbf{r}(t),t)\right)\mathbf{grad}\, Q(\mathbf{r}(t),t) \quad (5)$$

where:

$\mathbf{r}(t) = [r_I(t) \quad r_0(t)]^T$ - vector of the controller parameters,

Q - quadratic criterion evaluating the measure of accordance in the real and desired behaviour,

$\mathbf{K}$ - a diagonal matrix of elective gains of the adapting loops.

The components of the gradient vector for the quadratic criterion Q are formulated through the sensitivity functions $c_{r_I}(t)$ and $c_{r_0}(t)$. This allows us to describe the gradient method by the equations

$$\begin{aligned} \dot{r}_I(t) &= -k_I c_{r_I}(t)\left(e_{ref}(t) - e(r_0(t), r_I(t), t)\right) \\ \dot{r}_0(t) &= -k_0 c_{r_0}(t)\left(e_{ref}(t) - e(r_0(t), r_I(t), t)\right) \end{aligned} \quad (6)$$

where the sensitivity functions are defined by the relations $c_{r_I}(t) = \frac{\partial e(r_0(t), r_I(t), t)}{\partial r_I(t)}$, $c_{r_0}(t) = \frac{\partial e(r_0(t), r_I(t), t)}{\partial r_0(t)}$.

Evaluating the partial derivations $\frac{\partial e(r_0(t), r_I(t), t)}{\partial r_I(t)}$ and $\frac{\partial e(r_0(t), r_I(t), t)}{\partial r_0(t)}$, we can derive them from the relations describing a closed control loop: $e = w - y,\ y = G_S.u,\ u = G_R.e$, where the transfer function of the PI controller $G_R(s) = \frac{r_0 se + r_I e}{s}$. If the reference model for the computing sensitivity functions is of the first degree (approximate formulation only), relations can be derived for the partial derivations (sensitivity functions):

$$\begin{aligned} \frac{\partial E}{\partial r_I} &= -\frac{E}{\tau s^2 + (r_0 + 1)s + r_I} \\ \frac{\partial E}{\partial r_0} &= -\frac{sE}{\tau s^2 + (r_0 + 1)s + r_I} \end{aligned} \quad (7)$$

If the reference model is of the second degree, which better represents the cascade of two tanks, the relations for the partial derivations will be changed to:

$$\frac{\partial E}{\partial r_I} = -\frac{E}{\tau_1\tau_2 s^3 + (\tau_1+\tau_2)s^2 + (r_0+1)s + r_I}$$
$$\frac{\partial E}{\partial r_0} = -\frac{sE}{\tau_1\tau_2 s^3 + (\tau_1+\tau_2)s^2 + (r_0+1)s + r_I} \qquad (8)$$

When the method of the desired model is used as the reference model, the relations will be changed to:

$$\frac{\partial E}{\partial r_I} = -\frac{ae^{-T_d s}E}{s^2 + (r_0+1)ae^{-T_d s}s + r_I ae^{-T_d s}}$$
$$\frac{\partial E}{\partial r_0} = -\frac{sae^{-T_d s}E}{s^2 + (r_0+1)ae^{-T_d s}s + r_I ae^{-T_d s}} \qquad (9)$$

where:

E - Laplace transform of the control error,

r_0, r_I - parameters of the controller,

τ, τ_1, τ_2 - time constants of the system,

T_d - time delay.

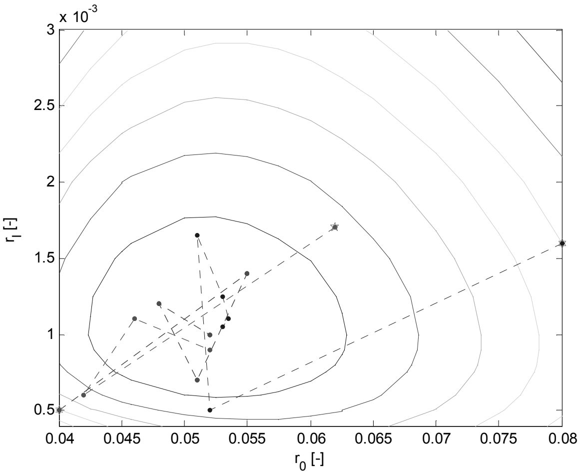

Fig. 3 Controller parameter changes (starting state values of r_0 and r_I are marked.)

When the model generating the sensitivity functions is added to the simulation model of the tank cascade control, factors c_{r_0}, c_{r_I} in the adaptation loops must be set so that the proportional gain value r_0 and the integrative gain value r_I will converge to values for which the quadratic criterion value is minimal. Because of the remaining control error that occurs with small values of the integral gain, the computed values are not the searched values, but are only in the near of the searched values. The difference between the computed and searched values depends on the frequency of computing new parameters. Rounding errors also occur.

4 IMPACT OF PARAMETER SETTING

If we can find parameters for which the value of the criterion is minimal, the time behaviour of the liquid level will be near to, but not identical the desired time behaviour. This can be seen in Fig. 4 for a different liquid level and the same disturbance.

h_{20} = 10 dm, u_3 = 0.4 $\qquad$ h_{20} = 20 dm, u_3 = 0.4

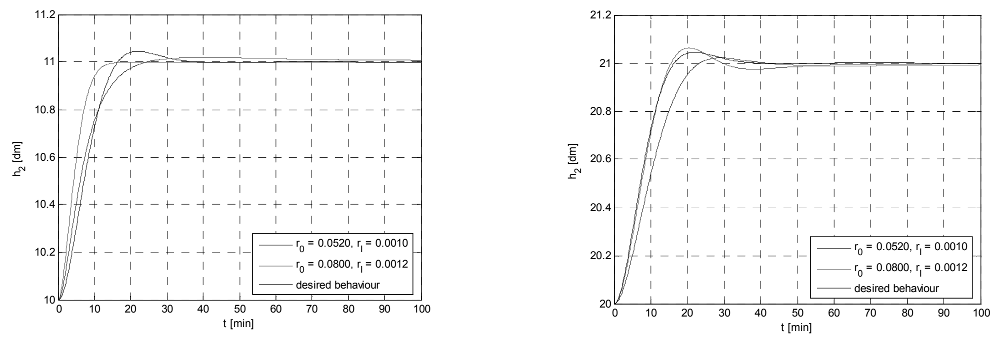

Fig. 4 Comparison of liquid level time behaviour for different steady-states

The PI level control with selftuning is held at its setting value with a smaller control error than when the liquid level is controlled using the PI controller without selftuning.

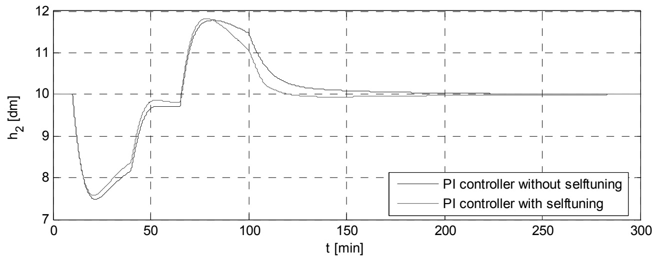

Fig. 5 Comparison of liquid level time behaviour controlled using the PI controller with selftuning, and controlled using the PI controller without selftuning during disturbances

5 CONCLUSIONS

The above described modification of the continuous gradient method has been shown to be a suitable utility for selftuning of PI controllers. This method works even when the controlled systems are non-linear. However, for successful application of the method, the existence of a minimum of the criterion is needed. In further development, modifications will be made to this method for non-linear reference models for deriving of sensitivity functions.

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