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MODIFICATION OF SYMMETRIC OPTIMUM METHOD

MODIFIKOVANÁ METODA SYMETRICKÉHO OPTIMA

Abstract

This contribution deals with a modification of the symmetric optimum method. This method is designated for the synthesis of linear one-dimensional control systems whose structure can be divided into a controller and a controlled system. It enables the design of continuous controllers only. New equations for specific combinations of controllers and controlled systems have been derived, which provide an approach for designing both the continuous and discrete controllers. Derivations have been carried out based on delta models principle.

Abstrakt

Příspěvek se zabývá modifikovanou metodou symetrického optima. Tato metoda je určena pro syntézu lineárních jednorozměrových regulačních obvodů jejichž strukturu můžeme rozčlenit na regulátor a regulovanou soustavu. Původní metoda umožňuje návrh pouze spojitých regulátorů. Byly odvozené nové rovnice pro určité kombinace regulátor a regulovaná soustava, které umožňují návrh, jak spojitých, tak diskrétních regulátorů. Odvození těchto rovnic bylo provedeno na základě delta modelů.

1 INTRODUCTION

First we have to specify what we understand with the terms control system and synthesis of control systems. In this case we will be dealing with the synthesis of linear one-dimensional control systems whose structure can be divided into a controller and a controlled system [Balátě, 2003]. The synthesis process deals with designing adjustable controller parameters.

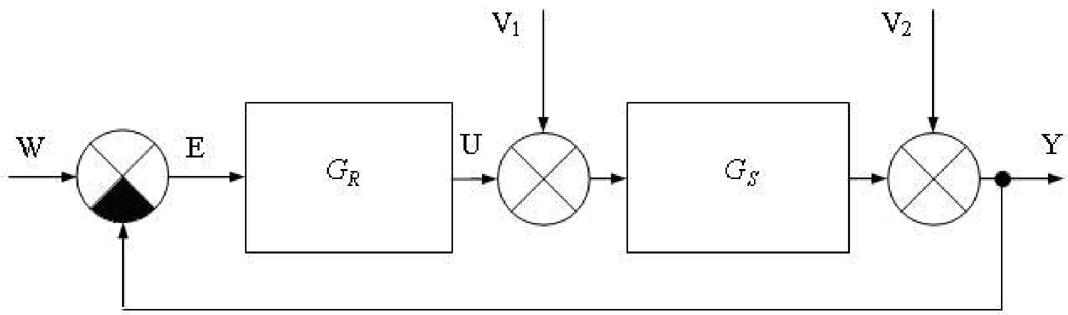

Fig. 1 Control system structure

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2 THE SYMMETRIC OPTIMUM METHOD

The symmetric optimum method is especially suitable for a case when the transfer function of an open control system has a third degree multinominal polynomial in the denominator and the number of integrators $q = 2$. First, we will describe the derivation process of an equation for the calculation of adjustable controller parameters by L-transform.

We will have a controlled system which will be expressed by the transfer function:

$$G_S(s) = \frac{k_1}{s(T_1 s + 1)} \quad , \tag{1}$$

where:

k_1 – gain of system,

T_1 – time constant of system [s],

s – complex variable in L – transform,

and we will choose a PI controller for this controlled system:

$$G_R(s) = k_P \left(1 + \frac{1}{T_I s}\right), \tag{2}$$

where:

k_P – gain of controller,

T_I – integral time constant of the controller [s].

Then the transfer function of this opened control system is

$$G_o(s) = \frac{k_1 k_P T_I s + k_1 k_P}{s^2 (T_1 T_I s + T_I)} = \frac{a_1 s + a_0}{s^2 (a_3 s + a_2)} , \tag{3}$$

and the transfer function of the closed control system is

$$G_w(s) = \frac{k_1 k_p T_I s + k_1 k_p}{T_1 T_I s^3 + T_I s^2 + k_1 k_P T_I s + k_1 k_P} = \frac{a_1 s + a_0}{a_3 s^3 + a_2 s^2 + a_1 s + a_0} . \tag{4}$$

The symmetric optimum method is based on the general equation

$$A_i = a_i^2 + 2 \sum_{j=1}^{i} (-1)^j a_{i-j} a_{i+j} . \tag{5}$$

It means, the determination of the adjustable controller parameters will be based on the system of equations

$$A_i = 0 , \qquad i = 1, 2, ..., m , \tag{6}$$

where:

m – number of adjustable parameters of chosen controller.

In our case $m = 2$ and so we will solve these equations

$$\begin{aligned} A_1 &= (k_1 k_P T_I)^2 - 2 k_1 k_P T_I = a_1^2 - 2 a_0 a_2 = 0 \\ A_2 &= T_I^2 - 2 k_1 k_P T_1 T_I^2 = a_2^2 - 2 a_1 a_3 = 0 \end{aligned} , \tag{7}$$

where:

a_0, a_1, a_2 and a_3 – coefficients of characteristic multinominal.

When we solve these equations we obtain two equations for calculating the optimal adjustable controller parameters

$$k_P^* = \frac{1}{2k_1 T1}, \tag{8}$$

$$T_I^* = 4T_1. \tag{9}$$

After substitution into (3) and (4) we obtain the transfer function of the open control system in standard form

$$G_o(s) = \frac{4T_1 s + 1}{8T_1^2 s^2 (T_1 s + 1)} \tag{10}$$

and the transfer function of the closed control system in standard form for symmetric optimum method

$$G_w(s) = \frac{4T_1 s + 1}{8T_1^3 s^3 + 8T_1^2 s^2 + 4T_1 s + 1} = \frac{4T_1 s + 1}{(2T_1 s + 1)(4T_1^2 s^2 + 2T_1 s + 1)}. \tag{11}$$

The step response of closed control system is shown in Fig. 2.

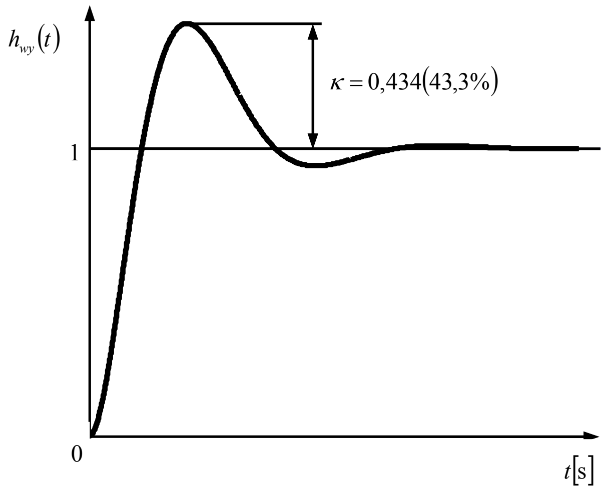

Fig. 2 Step response of closed control system designed by symmetric optimum method

3 DERIVATION OF EQUATIONS FOR CALCULATION ADJUSTABLE PARAMETERS OF CONTROLLER BASED ON δ MODELS

We will have the same transfer functions of a controlled system and controller as in previous case.

We will carry out discretization of equation (1). Discretization is based on this equation

$$G_w(\gamma) = \frac{\gamma}{1 + T\gamma} D\left\{ L^{-1}\left\{ \frac{1}{s} G_w(s) \right\} \bigg|_{t = kT} \right\}. \tag{12}$$

where:

kT – discreet time,

T – sampling period [s],

γ – complex variable in D – transform.

Then we will obtain discrete D – transfer function of the controlled system

$$G_S(\gamma) = k_1 \frac{(T - bT_1)\gamma + b}{\gamma(T\gamma + b)}, \qquad b = 1 - e^{-\frac{T}{T_1}}. \tag{13}$$

Now we need to obtain D – the transfer function of the controller. The relation between the input and output of the PS controller using backward rectangle summation is

$$u(kT) = k_P \left\{ e(kT) + \frac{T}{T_I} \sum_{i=0}^{k} e(iT) \right\}. \tag{14}$$

For transfer we will utilize this property of D – transformation

$$D\left\{ T \sum_{i=0}^{k} x(iT) \right\} = \frac{1 + T\gamma}{\gamma} X(\gamma). \tag{15}$$

After transfer we will obtain

$$U(\gamma) = k_P \left(1 + \frac{1 + \gamma T}{T_I \gamma} \right) E(\gamma). \tag{16}$$

We express D – transfer function of the PS controller

$$G_R(\gamma) = \frac{U(\gamma)}{E(\gamma)} = k_P \left(1 + \frac{1 + \gamma T}{T_I \gamma} \right). \tag{17}$$

In order to determine the characteristic multinominal and its coefficients we need to express either the transfer function of the closed control system or the transfer function of the open control system. We will use second way

$$G_o(\gamma) = G_R(\gamma) G_S(\gamma) = \frac{M_o}{N_o} = k_1 k_P \frac{[(T - bT_1)\gamma + b](T_I\gamma + T\gamma + 1)}{T_I \gamma^2 (T\gamma + b)}. \tag{18}$$

Now we can express the characteristic multinominal which is generally defined

$$N(\gamma) = M_o + N_o = a_n \gamma^n + a_{n-1}\gamma^{n-1} + \ldots + a_1 \gamma + a_0. \tag{19}$$

In our case we obtain

$$\begin{aligned} N(\gamma) = {} & TT_I\gamma^3 + \left[bT_I + k_1 k_P \left(TT_I - bT_1T_I + T^2 - bT_1T \right) \right] \gamma^2 + \\ & + k_1 k_P (T - bT_1 + bT_I + bT)\gamma + bk_1k_P \end{aligned}. \tag{20}$$

We express coefficients a_0, a_1, a_2 and a_3 from this equation

$$a_0 = bk_1k_P, \tag{21}$$

$$a_1 = k_1k_P(T - bT_1 + bT_I + bT), \tag{22}$$

$$a_2 = bT_I + k_1k_P(TT_I - bT_1T_I + T^2 - bT_1T), \tag{23}$$

$$a_3 = TT_I. \tag{24}$$

We can substitute coefficients a_0, a_1 and a_2 into the equation for calculating A_1. We will express the general equation for calculating the gain k_P^* of the controller from this equation.

$$k_P^* = \frac{2b^2 T_I^*}{k_1 \left(b^2 T_I^{*2} + 2b^2 T T_I^* + b^2 T_I^{*2} - 2bTT_1 + b^2T^2 + T^2 \right)}. \tag{25}$$

We substitute this equation into (22), (23) and (24) and coefficients a_1, a_2 and a_3 substitute into equation for calculation A_2. We will express general equation for calculation T_I^* from this equation

$$T_I^* = \frac{1}{b}\left[\frac{O}{3} - \frac{3\left(\frac{8}{9}T^2 - \frac{8}{3}bTT_1\right)}{O} + bT_1 - bT + \frac{1}{3}T\right]$$
$$O = \sqrt[3]{-72bT^2T_1 + 108b^2TT_1^2 + 28T^3 + P} \quad . \qquad (26)$$
$$P = 12\sqrt{-204b^3T^3T_1^3 + 174b^2T^4T_1^2 - 60bT^5T_1 + 9T^6 + 81b^4T^2T_1^4}$$

From equations (25) and (26) it is evident that the derivation of these equations is quite complicated and doing that manually takes too much time. The symbolic mathematic toolbox in MATLAB was used for this derivation.

In order to simplify these equations we used the approximation

$$b = 1 - e^{-\frac{T}{T_1}} \approx \frac{2T}{(2T_1 + T)} . \qquad (27)$$

We obtained the equation for calculating k_P^* in this form after approximation (27) into (25)

$$k_P^* = \frac{8T_I}{k_1\left(4T_I^2 + 8T_IT + 5T^2\right)} \qquad (28)$$

and equation for calculation T_I^* in this form after approximation (27) into (26)

$$T_I^* = \frac{1}{6\sqrt[3]{Q}}\left[\sqrt[3]{2}\sqrt[3]{Q^2}\sqrt[3]{(2T_1 + T)} + 16\sqrt[3]{2}RT_1 - 4\sqrt[3]{2}RT + 8T_1\sqrt[3]{Q} - 5\sqrt[3]{Q}T\right]$$
$$Q = 64T_1^2 - 8T_1T + 7T^2 + 3\sqrt{3}T\sqrt{64T_1^2 - 16T_1T + 3T^2} \quad . \qquad (29)$$
$$R = \sqrt[3]{(2T_1 + T)^2}$$

Now we need to simplify equations (28) and (29). We can rewrite equation (29) into this form

$$T_I^* = \frac{\sqrt[3]{8192}T_1R}{6Q} - \frac{\sqrt[3]{128}TR}{6Q} + \frac{\sqrt[3]{4}\sqrt[3]{2T_1 + T}}{6} + \frac{4}{3}T_1 - \frac{5}{6}T . \qquad (30)$$

After analysis of the first three members of equation (30) we can express this equation in following form

$$T_I^* = \frac{4}{3}T_1 + \frac{5}{24}T - \frac{2}{6}T + \frac{4}{3}T_1 + \frac{11}{24}T + \frac{4}{3}T_1 - \frac{5}{6}T . \qquad (31)$$

We will obtain the simplified equation for calculation T_I^* after last modification

$$T_I^* = 4T_1 - \frac{T}{2} . \qquad (32)$$

Now we simplify (28). First we neglect $5T^2$ and then we substitute (32) and we will obtain simplified equation for calculation k_P^*

$$k_P^* = \frac{4}{k_1(8T_1 + 3T)} . \qquad (33)$$

If we consider the limit case $T \to 0$ then we obtain system of equation as same as we obtained in derivation when we used L - transformation

$$k_P^* = \frac{1}{2k_1T_1}, \tag{34}$$

$$T_I^* = 4T_1. \tag{35}$$

4 CONCLUSIONS

In this contribution we dealt with the modification of the symmetric optimum method. We explained what we understand with the term control system and synthesis of control systems. Subsequently we explained when the symmetric optimum method is suitable. Then we showed the derivation of the equations for calculating the adjustable controller parameters using L – transformation and subsequently by the means of delta models. The next goal is to derive the equations for calculating the adjustable controller parameters for other combinations of controlled system and controller.

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